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An existence theorem for Volterra integrodifferential equations with infinite delay *

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Abstract

Using Schauder's fixed point theorem, we prove an existence theorem for Volterra integrodifferential equations with infinite delay. As an appplication, we consider an n species Lotka-Volterra competitive system.

1 Introduction

Vrabie [10, page 265] studied the partial integrodifferential equation

$$\begin{gathered}\dot{u}(t) = -Au(t) + \int_a^t k(t-s)g(s,u(s))\mathrm{d}s \\ u(a) = u_0,\end{gathered} \tag{1.1}$$

where $u : [a,b] \to X$, X is a Banach space, $A : \mathcal{D}(A) \subset X \to X$ is an M-accretive operator; $t \in [a,b]$, $g : [a,b] \times X \to X$, $k : [0,a] \to \mathcal{L}(X)$ are continuous functions. The result, existence of solutions on some interval $[a,c)$ was obtained by using the Schauder's fixed point theorem.

Schauder's fixed point theorem is a usual tool for proving existence theorems in infinite delay case. In [8], Teng applied it to prove existence theorems for Kolmogorov systems. Another frequently used method (especially for integrodifferential equations) is the Leray-Schauder alternative, see [5] and its references.

Modifying (1.1) we investigate the case when the initial function is given on $(-\infty, 0]$, which means infinite delay, moreover in the right-hand side we take a function of the integral. This form allows us proving existence theorems for systems. In this case g, k in the right hand side have to be also modified. The spirit of the proof is similar to [10, pages 265–268] but we need some assumptions on k and g and additional spaces and operators have to be introduced to carry out the proof.

In section 3 we apply the result to a system (a competition model arising from population dynamics); existence of global solution will be proved. In the compactness arguments we need the following definition.

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Definition A family of functions $H \subset L^1([a,b];X)$ is 1-equiintegrable if the following two conditions are satisfied:

- For all $\epsilon > 0$, there exists δ such that for all $f \in H$, $\lambda(E) < \delta \to \int_E \|f(t)\|\mathrm{d}t < \epsilon)$
- For all $\epsilon > 0$, there exists $h > 0$ such that for all $f \in H$ and all $h_0 < h$,
$$\int_a^{b-h_0} \|f(t+h_0) - f(t)\|\mathrm{d}t < \epsilon.$$

In this paper, let X be a Banach space, $A : \mathcal{D}(A) \subset X \to X$ an M-accretive operator [10, page 21]. Further, the spaces equipped with the supremum-norm are denoted by denoted by $\mathcal{C}$. We study of the abstract Cauchy problem ([7, page 90], [2, pages 390–398])

$$\begin{gathered}\dot{u}^f(t) + Au^f(t) = f(t) \quad \text{if} \quad t \geq a \\ u^f(a) = u(a).\end{gathered} \tag{1.2}$$

Here u^f denotes the f dependence of the solution. We also use the following theorem [10, page 65] which is the basis of the compactness method employing in the following section.

Theorem 1.1 *Let $A : X \to X$ be an M-accretive operator, and $(I-\lambda A)^{-1}$ compact for each $\lambda > 0$. Let $u_0 \in \mathcal{D}(A)$ and $K \subset L^1([a,b];X)$ be 1-equiintegrable. Then the set $M(K) = \{u^f : u^f \text{ is the mild solution of (1.2)}, f \in K\}$ is relatively compact in $\mathcal{C}([a,b];X)$.*

2 An existence result for a class of Volterra-type integrodifferential equations

A class of Volterra-type integrodifferential equations

Let U be an open subset of X, and $U_A = U \cap \mathcal{D}(A)$, with $(I - \lambda A)^{-1}$ compact. Let $b > a$ and $g = (g_1, g_2, \dots, g_n)$ be Lipschitz-continuous functions in the second variable, where $g_i : (-\infty, b] \times U_A \to X$ are bounded and continuous. Let $k = (k_1, k_2, \dots, k_n)$ be a function such that $k_i \in L_1([0,\infty), \mathcal{L}(X))$ and

$$k(t)g(s,u(s)) = (k_1(t)g_1(s,u(s)), k_2(t)g_2(s,u(s)), \dots, k_n(t)g_n(s,u(s))). \tag{2.1}$$

Let the space X^n be equipped with the maximum norm, $\|\mathbf{x}\| = \max_{1\leq i\leq n} \|x_i\|$, where $\mathbf{x} = (x_1, x_2, \dots, x_n)$. Let $F : X^n \to X$ be a function such that for some constant $M_F \in \mathbb{R}$,

$$\|F(x)\| \leq M_F\|x\| \quad \text{and} \quad M_F \int_{-\infty}^0 \|k(-\tau)\|\mathrm{d}\tau \leq 1. \tag{2.2}$$

Consider the problem

$$\dot{u}(t) = -Au(t) + F\Big(\int_{-\infty}^{t} k(t-s)g(s,u(s))\mathrm{d}s\Big) \quad \text{for} \quad t \geq a \tag{2.3}$$

$$u(t) = u_0(t-a) \quad \text{for} \quad t \leq a, \tag{2.4}$$

where $u_0 \in \mathcal{C}((-\infty,0],X)$ is a given bounded, equiintegrable function which fulfills the matching condition

$$u_0(0) = F\Big(\int_{-\infty}^{0} k(-s)g(a+s,u_0(s))\mathrm{d}s\Big). \tag{2.5}$$

Theorem 2.1 *Under assumptions* (2.1) *and* (2.2)*, there is a value* c *in* (a,b) *such that* (2.3)-(2.4) *has a weak solution on* $(-\infty,c]$.

Proof: Note that $k_i \in L_1([0,\infty),\mathcal{L}(X))$ implies $k \in L_1([0,\infty),\mathcal{L}(X^n,\mathbb{R}^n))$ and (2.2) makes sense. This is only a technical supposition because (2.3) could be rewrite with k/M and Mg (instead of k, g, resp.; $M \in \mathbb{R}$ is sufficiently big) fulfilled (2.3). Let

$$P : \mathcal{C}((-\infty,b],U) \mapsto \mathcal{C}((-\infty,b],U)$$

defined by

$$Pf(t) = \begin{cases} F\Big(\int_{-\infty}^{t} k(t-s)g(s,u^f(s))\mathrm{d}s\Big) & \text{if } t \geq a \\ f(t) & \text{if } t \leq a, \end{cases} \tag{2.6}$$

where u^f is the weak solution of (1.2).

Observe that $Pf = f$ holds if and only if u^f is the weak solution of the equation (2.3)-(2.4). Let us choose $\rho > 0$ such that

$$B(u(a),\rho) := \{v \in X : \|v - u(a)\| \leq \rho\} \subset U. \tag{2.7}$$

Since g is bounded there is $M \in \mathbb{R}$ such that

$$\|g(s,v)\| \leq M \quad \text{for} \quad (s,v) \in ([-\infty,b] \times [U_A \cap B(u_0,\rho)]). \tag{2.8}$$

Denote by $S(t)$ the semigroup generated by $-A$ on $\mathcal{D}(A)$. Let us choose further $b \geq c_0 \geq a$ such that for all $t \in [a,c_0]$

$$\|S(t-a)u_0 - u_0\| + (c_0 - a)M \leq \rho, \tag{2.9}$$

and $c \in [a,c_0]$ such that

$$(c-a)M_F\|k\|_{L_1} \leq 1. \tag{2.10}$$

Let us define

$$\mathcal{C}_{u_0}((-\infty,b],U) = \{u \in \mathcal{C}((-\infty,b],U) : u(t) = u_0(t-a) \quad \text{for} \quad t \leq a\}. \tag{2.11}$$

Let

$$H : \mathcal{C}_{u_0}((-\infty, b], U) \mapsto \mathcal{C}([a, b], U)$$

be a natural homeomorphism with $(Hf)(t) = f(t)$ for $t \in [a, b]$ and let

$$K^r_{u_0} := \{f \in \mathcal{C}([-\infty, c], X) : \|Hf(t)\|_\infty \le r \,\&\, f(b) = u_0(d - a) \quad \text{for} \quad d \le a\}. \tag{2.12}$$

Obviously $K^r_{u_0}$ is nonempty, bounded, closed and convex subset of the space $\mathcal{C}_{u_0}([-\infty, c], X)$.

Observe that $P = P_1 \circ P_2$, where (using the matching condition (2.5)) we define $P_1 : \mathcal{C}_{u_0}((-\infty, b], U) \to \mathcal{C}_{u_0}((-\infty, b], U)$ as

$$P_1 v(t) = \begin{cases} F\Big(\int_{-\infty}^t k(t-s) g(s, v(s)) \mathrm{d}s \Big) & \text{if } t \ge a \\ v(t) & \text{if } t \le a \end{cases} \tag{2.13}$$

and $P_2 : \mathcal{C}_{u_0}((-\infty, b], U) \to \mathcal{C}_{u_0}((-\infty, b], U)$ is defined as $P_2 = H^{-1} P_2^* H$, where

$$P_2^* : \mathcal{C}([a, b], U) \to \mathcal{C}([a, b], U)$$

and $P_2^* g(t)$ is the weak solution of the abstract Cauchy problem

$$\begin{gathered} \dot{u}(t) + Au(t) = g(t) \quad \text{for} \quad t \ge a \\ u(a) = g(a) = u_0(0). \end{gathered} \tag{2.14}$$

For details on this problem, we refer the reader to Barbu [1, page 124] and for some applications of this result to [10, page 35].

Let $f, h \in L_1([a, b], X)$ and let u, v be solutions, in the weak sense, of

$$\begin{aligned} \dot{u}(t) + Au(t) &= f(t) \\ \dot{v}(t) + Av(t) &= h(t) \end{aligned} \tag{2.15}$$

with some initial conditions $u(a), v(a)$. Then for $s, t \in [a, b]$ we have

$$\|u(t) - v(t)\| \le \|u(s) - v(s)\| + \int_s^t \|f(\tau) - h(\tau)\| \mathrm{d}\tau. \tag{2.16}$$

From this inequality, it follows that

$$\begin{aligned} \|P_2^* h_1(t) - P_2^* h_2(t)\| &\le \|h_1(a) - h_2(a)\| + \int_a^t \|h_1(\tau) - h_2(\tau)\| \mathrm{d}\tau \\ &\le \|h_1 - h_2\|_\infty (t - a + 1), \end{aligned}$$

which implies the continuity of P_2^* on $\mathcal{C}([a, b], U)$ and so P_2 on $K^r_{u_0}$. Using (2.8), (2.9) and (2.16) for $u \in K^r_{u_0}$, $t \in [a, c_0]$ we get

$$\begin{aligned} \|P_2^* u(t) - u(a)\| &\le \|P_2 u(t) - S(t-a)u(a)\| + \|S(t-a)u(a) - u(a)\| \\ &\le \|S(t-a)u(a) - u(a)\| + \int_a^{c_0} \|g(t)\| \mathrm{d}t \\ &\le \|S(t-a)u(a) - u(a)\| + (c_0 - a)M \le \rho. \end{aligned} \tag{2.17}$$

Then we conclude that $P_2^* u(t) \in B(u(a),\rho) \cap \mathcal{D}(A)$. Consequently, $P_2 u(t) \in \mathcal{D}(g)$ for $t \geq a$. By (2.2), (2.8), (2.10) and (2.13), for $t \geq a$ we have

$$\begin{aligned}
\|Pu(t)\| &= \|P_1 P_2 u(t)\| = \|F\big(\int_{-\infty}^{t} k(t-s) g(s, P_2 u(s)) \mathrm{d}s\big)\| \\
&\leq M_F \sup_{s\in(-\infty,t]} \|g(s,P_2u(s))\| \int_{-\infty}^{0} \|k(-\tau)\| \mathrm{d}\tau \\
&\leq M_F M \int_{-\infty}^{0} \|k(-\tau)\| \mathrm{d}\tau \leq M\,.
\end{aligned}$$

and (2.5) implies that

$$Pu(t) = u(t) \quad \text{for } t \leq a\,;$$

i.e., P maps $K_{u_0}^M$ into itself. Since

$$\begin{aligned}
&\|(P_1 v - P_1 w)(t)\| \\
&= \|F\big(\int_{-\infty}^{t} k(t-s)[g(s,v(s)) - g(s,w(s))] \mathrm{d}s\big)\| \\
&= M_F \| \int_{-\infty}^{a} k(t-s)[g(s,v(s)) - g(s,w(s))] \mathrm{d}s \\
&\quad + \int_{a}^{t} k(t-s)[g(s,v(s)) - g(s,w(s))] \mathrm{d}s\| \\
&\leq M_F [v(a) - w(a) \\
&\quad + \max_{s\in[a,t]} [g(s,v(s)) - g(s,w(s))](t-a)\|k(t-s)\|_{\mathcal{L}_1}],
\end{aligned} \tag{2.18}$$

the function P_1 is continuous from $\mathcal{C}_{u_0}((-\infty,b];U)$ into itself. Using the continuity of P_2 we have that $P : K_{u_0}^M \to K_{u_0}^M$ is continuous. Since

$$\int_E Pf(t)\mathrm{d}t \leq \lambda(E) \max_t Pf(t) \leq \lambda(E)\|k\|_{L^1} M$$

and

$$\begin{aligned}
&\int_a^{b-h_0} \|Pf(t+h_0) - Pf(t)\| \mathrm{d}t \\
&\leq \|F\|\, \|a-b\| \big(\int_{-\infty}^{t+h_0} k(t-s) g(s,u^f(s)) \mathrm{d}s - \int_{-\infty}^{t} k(t-s) g(s,u^f(s)) \mathrm{d}s \big) \\
&\leq h_0 \|F\|\, \|a-b\|\, \|k\|_{L_1} M
\end{aligned}$$

we get that $HP(K_{u_0}^M)$ is 1-equiintegrable. Let us define

$$K_{u_0} := \mathrm{cl}(\mathrm{conv}\, P(K_{u_0}^M)).$$

Easy calculations shows that $H(K_{u_0}) = \mathrm{cl}(\mathrm{conv}\, HP(K_{u_0}^r))$ is equiintegrable and Theorem 1.1 implies the relative compactness of $P_2^* H(K_{u_0}) = HP_2(K_{u_0})$.

Since H is homeomorphism, $P_2(K_{u_0})$ and $P(K_{u_0}) = P_1P_2(K_{u_0})$ are relative compact. Since $P(K_{u_0})$ is a subset of the closed, bounded and convex set K_{u_0}, the Schauder fixed point theorem ensures the existence of a fixed point of P.

3 Application to an n species Lotka-Volterra competitive system

We prove local existence of solutions for a system, which is a model of an n species competition arising in the population dynamics. Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with smooth boundary. Feng [3] studied the system ($i = 1, \dots, N$)

$$\begin{gathered}
(u_i)_t = D_i\big[\Delta u_i + u_i(a_i - u_i - \sum_{j\neq i}^{N} \kappa_{ij} u_j^{\tau_{ij}})\big] \quad \text{on } (0,\infty)\times\Omega \\
u_i = 0 \quad \text{in } (0,\infty)\times\partial\Omega \\
u_i(s,x) = \eta_i(s,x) \quad \text{on } [-\tau,0]\times\Omega,
\end{gathered} \tag{3.1}$$

where $u_i(t,x)$ denotes the density of the i-th species at time t and position x (inside a bounded domain Ω of $\mathbb{R}^3$), $u_j^{\tau_{ij}}(t,x) = u_j(t-\tau_{ij},x)$, $\tau_{ij} > 0$, $\tau = \max\{\tau_{ij}\}$, D_i, a_i are positive, and κ_{ij} are nonnegative real numbers. Supposing the existence of a solution (a sufficient condition for this - using upper and lower semisolutions - is formulated in [6]) the authors describe the attractors of (3.1).

In [8], Teng studies

$$\begin{aligned}
\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} =& x_i(t)[a_i(t) - g_i(t,x_i(t)) - \sum_{j=1}^{m} c_{ij} P_j(x(t-\tau_{i,j}(t))) \\
&- \sum_{j=1}^{m} \int_{-\sigma_{ij}}^{0} \kappa_{ij}(t,s) Q_j(x_j(t+s))\mathrm{d}s], \quad (i = 1,\dots,n)
\end{aligned} \tag{3.2}$$

an n-species Lotka-Volterra competitive system with delays as an application of existence result for periodic Kolmogorov systems with delay. Detailed study of the non-autonomous Lotka-Volterra models with delay (focused on existence of positive periodic solutions) can be found in [9].

We rewrite (3.1) taking into account that a bounded attractor A has a bounded neighborhood U and $B \in \mathbb{R}$ such that $u(t,x) \in U$ for $t \le t_0$ implies $\|u(t,x)\| < B$ for all $t > t_0$. B can be considered as a bound determined by the carrying capacity of the territory. Let $b : \mathbb{R} \to \mathbb{R}$ be a bounded, continuous such that $b(x) = x$ for $|x| < B$. The new form of (3.1) is

$$\begin{gathered}
(u_i)_t = D_i\big[\Delta u_i + b(u_i)(a_i - b(u_i) - \sum_{j=1}^{N} \kappa_{ij} b(u_j^{\tau_{ij}}))\big] \quad \text{on } (0,\infty)\times\Omega \\
u_i = 0 \quad \text{on } (0,\infty)\times\partial\Omega \\
u_i(s,x) = \eta_i(s,x) \quad \text{on } [-\tau,0]\times\Omega.
\end{gathered} \tag{3.3}$$

We reformulate (3.3) again in according to the notations and assumptions of Theorem 2.1. Let $\Omega \subset \mathbb{R}^3$ be a bounded open subset, $X = [L^2(\Omega)]^n$, $\mathbf{u} = (u_1, \dots, u_n) : \mathbb{R} \to X$ $\mathbf{u}(s)(x) = (u_1(s,x), \dots, u_n(s,x))$ and

$$\mathcal{D}(A) = [\mathcal{C}^2(\Omega)]^n, \quad A(u_1, u_2, \dots, u_n) = (D_1 \Delta u_1, \dots, D_n \Delta u_n).$$

Let $g = (g_1, g_2, \dots, g_{n+1})$ be such that $g_i : (-\infty, \infty] \times X \to X$ are bounded and continuous, Lipschitz-continuous in the second variable and $g_i(s, \mathbf{u}(s))\big|_{\mathbb{R}\times B} = \mathbf{u}(s)$, where B is an a priori bound of the solutions of (3.3), $k = (k_1, k_2, \dots, k_{n+1})$, where $k_i \in L_1([0,\infty), \mathcal{L}(X))$.

We rewrite (2.3)-(2.4) in the form

$$\begin{gathered}(u_i(t,x))_t = D_i \Delta u_i(t,x) + F_i\big(\int_{-\infty}^t k(t-s) g(s, \mathbf{u}(s)) \mathrm{d}s\big) \text{ on } (0,\infty)\times\Omega \\ u_i(s,x) = \eta_i(s,x) \text{ on } [-\tau, 0]\times\Omega,\end{gathered} \tag{3.4}$$

where we take A as defined above and $n+1$ instead of n. In a special case we get a perturbed version of (3.3), supposed that the right-hand side of (3.4) is approximated such that

$$\big[\int_{-\infty}^t k_i(t-s) g_i(s, \mathbf{u}(s)) \mathrm{d}s\big]_j \approx \kappa_{ij} b(u_j^{\tau_{ij}}(t)) \quad (i,j = 1, \dots, n) \tag{3.5}$$

and

$$\big[\int_{-\infty}^t k_{n+1}(t-s) g_{n+1}(s, \mathbf{u}(s)) \mathrm{d}s\big]_j \approx b(u_j(t)) \quad (j = 1, \dots, n). \tag{3.6}$$

According to the choice of g requirements (3.5) and (3.6) can be rewritten as

$$\begin{gathered}\int_{-\infty}^t k_i(t-s)(u_1(s), u_2(s), \dots, u_n(s)) \mathrm{d}s \\ \approx (\kappa_{i1} b(u_1^{\tau_{i1}}(t)), \kappa_{i2} b(u_2^{\tau_{i2}}(t)), \dots, \kappa_{in} b(u_n^{\tau_{in}}(t))) \quad (i = 1, \dots, n)\end{gathered} \tag{3.7}$$

and

$$\int_{-\infty}^t k_{n+1}(t-s)(u_1(s), \dots, u_n(s)) \mathrm{d}s \approx (b(u_1(t)), \dots, b(u_n(t))). \tag{3.8}$$

Obviously $k_1, k_2, \dots, k_{n+1}$ can be chosen such that $k_i \in L_1([0,\infty), \mathcal{L}(X))$ and approximations (3.7) and (3.8) are sharp; namely, for all $\epsilon_1, \epsilon_2, \dots, \epsilon_{n+1} > 0$ there are $k_i \in L_1([0,\infty), \mathcal{L}(X))$ such that for any bounded $(u_1, u_2, \dots, u_n)$ and for all $t > t_0$,

$$\begin{aligned}&\int_{-\infty}^t k_i(t-s)(u_1(s), u_2(s), \dots, u_n(s)) \mathrm{d}s \\ &- (\kappa_{i1} b(u_1^{\tau_{i1}}(t)), \kappa_{i2} b(u_2^{\tau_{i2}}(t)), \dots, \kappa_{in} b(u_n^{\tau_{in}}(t))) < \epsilon_i \quad (i = 1, \dots, n)\end{aligned}$$

and

$$\int_{-\infty}^{t} k_{n+1}(t-s)(u_1(s),\dots,u_n(s))\mathrm{d}s - (b(u_1(t)),\dots,b(u_n(t))) < \epsilon_{n+1}.$$

Moreover, the terms on the left-hand side of (3.7) and (3.8) lead to a more precise model than the original equation did (3.1) or (3.3) since the new terms keep track the past of the population. Finally let $F = (F_1, \dots, F_n)$ where

$$\int_{\infty}^{t} k(t-s)g(s,\mathbf{u}(s))\mathrm{d}s \in [L^2(\Omega)]^{n\times(n+1)}$$

and

$$\begin{gathered} F_i : [L^2(\Omega)]^{n\times(n+1)} \to L^2(\Omega), \\ F_i(\mathbf{x}_1,\mathbf{x}_2,\dots,\mathbf{x}_n,\mathbf{x}_{n+1}) = a_i(\mathbf{x}_{n+1})_i - (\mathbf{x}_{n+1})_i^2 - \sum_{j=1}^{n}(\mathbf{x}_{n+1})_i(\mathbf{x}_i)_j. \end{gathered} \tag{3.9}$$

Since $k = (k_1, k_2, \dots, k_{n+1})$ and $g = (g_1, g_2, \dots g_{n+1})$ fulfill every requirements listed in Theorem 2.1 we get the following

Theorem 3.1 *Let $u_i(s,x) = \eta_i(s,x)$ on $[-\tau, 0] \times \Omega$ be an initial condition with a priori bound B of the possible solutions of (3.4). Let further k, g and F be as defined by (3.5), (3.6) and (3.9) satisfying the conditions of Theorem 2.1. Then (3.4) - a modified version of (3.1) - has a global solution.*

We have to prove only the existence of a global solution. Observe that the condition $b > c_0$ (required in (2.9) and in (2.17)) plays no role here because we have not restricted the domain of g. By repeating the method for seeking local solution one can choose a constant $c - a$ in each steps, i.e. we have a local solution on $[a, c]$ and then $[a, 2c - a]$, $[a, 3c - 2a]$ and so on, where every local solution fulfills the conditions of Theorem 2.1 which ensures the existence of a global solution.

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