

STABILITY OF THE INTERFACE BETWEEN TWO IMMISCIBLE FLUIDS IN POROUS MEDIA

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ABSTRACT. A generalized model has been recently proposed in [3] to describe deformations of the mobile interface separating two immiscible and compressible fluids in a deformable porous medium. This paper deals with a few applications of this model in realistic situations where it can be supposed that gravity perturbations are propagating much slower than elastic perturbations. Among these applications, one can include the classical well-known case of groundwater flow with free surface, but also more complex phenomena, as gravitational instability with finger growth.

1. INTRODUCTION

Evolution of the interface between two immiscible fluids is one of the more complicated problem in mathematical analysis. It is usually described by a system of partial differential equations which are true on either side of the interface. The closure of the system is assured by some dynamic and kinetic conditions on the interface. The principal difficulty of this kind of problem is to transform such a system into an explicit closed differential equation which governs the interface movement.

For instance, if we deal with two superposed fluids and if $h(x_1, x_2; t)$ denotes the thickness of the lower fluid, then the problem is to deduce the closed differential equation for the function $h(x_1, x_2; t)$ (here x_1, x_2 are the coordinates in horizontal plane, t is the time variable and $z = h(x_1, x_2; t)$ is the equation of the interface considered as a mobile surface).

In natural gas-oil reservoirs, such a model is basic to describe behavior of the stratified pair "gas-oil" or "oil-water". Indeed, before any recovery, the natural non perturbed system is generally a porous medium composed of three superposed layers (gas, oil and water, in downward direction). The pumping of oil leads to an ascent of water from below or/and to an overlapping of the oil layer by broken gas which is coming from above. Then, for an optimized recovery, it is absolutely necessary to have a good modelling of the evolution of both interfaces (gas-oil and oil-water).

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Explicit differential equations describing the evolution of such an interface have been already obtained for the case when the viscosity of one of the two fluids can be neglected, as for instance in the case of water-air flow [10] in the framework of shallow water theory, or in the case of the groundwater flow in unconfined aquifers [1], [2], [4], [5], [7]. Whitham [10] has considered the case where the viscous forces can be neglected for both fluids, but this situation is rarely encountered in porous media. When both fluids are viscous, some explicit equations were deduced in [9].

In all previous studies, some hypotheses (as vanishing of vertical flow velocity (gravity equilibrium condition), constant horizontal flow velocity along z , or steady-state flow) are imposed. Such assumptions seem to be reasonable if the interface deformation is rather small, if the boundary is free and if the transition phenomena are not taken into account, but become excessively strong in the case of two-liquids flow.

A new model has been recently proposed in [3] for two viscous fluids in porous media. It generalizes previous models by removing the classical condition of gravity equilibrium and by considering a non-stationary flow (the compressibility of the fluids and of the porous medium is not neglected). Instead of previous hypotheses, a more general assumption about linear behavior of the vertical velocity along vertical coordinate is used. This generalized model has been already applied to describe the flow of rather compressible fluids, when an elastic perturbation is propagating in the domain much slower than a gravity perturbation.

However, the previous model is able to describe many other situations, corresponding, for instance, to the case when an elastic perturbation is propagating much faster than a gravity perturbation. This paper deals with a few applications of the model in this last case. The outline is as follows: in Section 1, we give a brief description of the physical problem and recall the general model deduced in [3]. For the particular case investigated here, i.e. fast elastic perturbations and slow gravity perturbations, a reduced model is obtained in Section 2. It can be converted into one nonlinear equation of third order. In particular, a generalization of Boussinesq equation is obtained for groundwater flow with free surface. Stable flow is taken into account when the upper fluid is more light. If this condition is not true, the evolution of instability is described.

2. GENERAL MODEL

We consider an orthogonal coordinate system (x_1, x_2, z) , where z is the vertical coordinate and (x_1, x_2) are coordinates of the horizontal plane. In this system, a horizontal porous medium with constant height H is considered. Its inferior and superior limits are some impermeable media. The voids of the porous medium are filled with two immiscible fluids which are separated one from other by an interface. Let the value $h(x_1, x_2; t)$ denotes the height of the interface with respect to the bottom of the porous stratum (Figure 1).

The initial position of the interface is supposed to be known (as for instance, a horizontal surface) and denoted by h_0 . At this time, we suppose there is no flow in the domain. The initial state is perturbed by an external factor (as pumping by a well or seismic waves). The objective is to find the evolution of the interface, i.e. of the function $h(x_1, x_2; t)$, generated by such a perturbation. It is supposed that the interface does not cross the top and bottom boundaries of the domain and it remains always non perturbed at the lateral extremities.

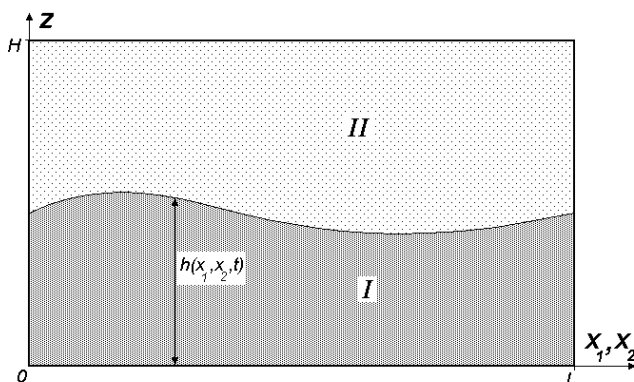


FIGURE 1. Schematization of the physical situation

2.1. Physical parameters. The porous medium is assumed to be homogeneous (its constant porosity is denoted by m), but anisotropic, the permeability tensor $K \equiv \{K_{ij}\}_{i,j=1}^3$ being diagonal with $K_x \equiv K_{11} = K_{22}$, $K_z \equiv K_{33}$.

Let the indexes $i = I, II$ correspond to the lower and the upper fluid. For each fluid, ρ^i denotes the density, μ^i denotes the viscosity and β_*^i is a measure of fluid/medium compressibility.

We need also to introduce the following parameters: L is the horizontal scale of the domain, \mathcal{P}_0 and $\mathcal{P}(x_1, x_2; t)$ are respectively the common pressure of the two fluids on the interface at initial state and for an arbitrary instant time, g is the gravity acceleration.

2.2. Closed system in dimensionless formulation. The time variable and the space variables are replaced by some dimensionless variables:

$$\tau \equiv t/t_*, \quad y_1 \equiv x_1/L, \quad y_2 \equiv x_2/L$$

and the unknowns are the thickness of the two fluids and the common pressure on the interface, all of them in dimensionless form:

$$\varphi \equiv h/h_0, \quad \psi \equiv (H-h)/(H-h_0), \quad \xi \equiv \mathcal{P}/\mathcal{P}_0$$

Then the dimensionless system deduced in [3] is:

$$\left(\tau_* \frac{\partial}{\partial \tau} - \Delta_{yy} \right) \left(\varphi^2 + \frac{\lambda_1^2 \omega \tau_*}{3 \varepsilon_z} \varphi^2 \frac{\partial \varphi}{\partial \tau} + \lambda_2 \xi \right) = -\omega \tau_* \frac{\partial \varphi}{\partial \tau}, \quad (2.1a)$$

$$\left(\frac{\tau_* \bar{\beta}}{\bar{\mu}} \frac{\partial}{\partial \tau} - \Delta_{yy} \right) \left(-\psi^2 + \frac{\lambda_1^2 \bar{\rho} \omega \tau_*}{3 \varepsilon_z \bar{\mu} \lambda_0} \psi^2 \frac{\partial \psi}{\partial \tau} + \lambda_2 \lambda_0 \bar{\rho} \xi \right) = -\frac{\bar{\rho} \lambda_0 \omega \tau_*}{\bar{\mu}} \frac{\partial \psi}{\partial \tau}, \quad (2.1b)$$

$$\psi = -\lambda_0 \varphi + \lambda_0 + 1 \quad (2.1c)$$

with the following notation

$$\begin{aligned} \lambda_0 &\equiv \frac{h_0}{H - h_0}, \quad \lambda_1 \equiv \frac{h_0}{L}, \quad \lambda_2 \equiv \frac{2\mathcal{P}_0}{\rho^I g h_0} \\ \bar{\beta} &\equiv \frac{\beta_*^I}{\beta_*^{II}}, \quad \bar{\mu} \equiv \frac{\mu^I}{\mu^{II}}, \quad \bar{\rho} \equiv \frac{\rho^I}{\rho^{II}}, \quad \varepsilon_z \equiv \frac{K_z}{K_x} \\ t_{el} &= \frac{\mu^I L^2}{K_x \beta_*^I}, \quad t_{gr} = \frac{2\mu^I m L^2}{K_x \rho^I g h_0}, \quad \omega \equiv \frac{2m\beta_*^I}{\rho^I g h_0} = \frac{t_{gr}}{t_{el}}, \quad \tau_* \equiv \frac{t_{el}}{t_*}, \end{aligned}$$

where Δ_{yy} denotes the Laplace’s operator with respect to the variable $y = (y_1, y_2)$.

Two characteristic times have been introduced: t_{el} signifies the time of propagation of an elastic perturbation within the scale L and t_{gr} is the time required for the complete emptying of the medium occupied by a fluid (say the fluid I) if the gravity drop is the only phenomenon taken into account.

In the general case, the scale of the time (t_*) used to get the dimensionless time variable (τ) can be arbitrary chosen. However, for two particular but important cases, it can be defined according to the relationship between previous characteristic times, as follows:

$$t_* = \begin{cases} t_{el}, & \text{when } t_{gr} \ll t_{el}, \text{ or } \omega \ll 1 \\ t_{gr}, & \text{when } t_{el} \ll t_{gr}, \text{ or } \omega \gg 1 \end{cases} \quad (2.2)$$

The first case, i.e. $t_{gr} \ll t_{el}$ has been already investigated in [3].

3. GRAVITY DOMINATED FLOW. FAST ELASTIC PERTURBATIONS

In this section, we deal with the case where the time of elastic wave propagation is very small with respect to the gravity time, i.e. $t_{el} \ll t_{gr}$. Therefore, we have: $\omega \gg 1$ In this case, compressibility of fluid/medium does not play any role and then, the non-stationarity is involved only by gravity phenomena.

According to (2.2), the scale of the time t_* is chosen as equal to t_{gr} . Then:

$$\omega \tau_* = 1, \quad \tau_* = \omega^{-1} \ll 1$$

3.1. General nonlinear equation for gravitational waves. Taking into account previous relations, the general model (2.1a)-(2.1c) becomes:

$$\Delta_{yy} \left(\varphi^2 + \frac{\lambda_1^2}{3\varepsilon_z} \varphi^2 \frac{\partial \varphi}{\partial \tau} + \lambda_2 \xi \right) = \frac{\partial \varphi}{\partial \tau}, \quad (3.1a)$$

$$\Delta_{yy} \left(-\psi^2 + \frac{\lambda_1^2 \bar{\rho}}{3\varepsilon_z \bar{\mu} \lambda_0} \psi^2 \frac{\partial \psi}{\partial \tau} + \lambda_2 \lambda_0 \bar{\rho} \xi \right) = \frac{\bar{\rho} \lambda_0}{\bar{\mu}} \frac{\partial \psi}{\partial \tau}, \quad (3.1b)$$

$$\psi = -\lambda_0 \varphi + \lambda_0 + 1 \quad (3.1c)$$

This system can be converted into one equation with respect to the function φ , after excluding the functions ξ and ψ :

$$\Delta_{yy} \left(\varphi^2 - 2\alpha\varphi + \gamma_1 \left[\varphi^2 + \gamma_2 (1 + \lambda_0 - \lambda_0 \varphi)^2 \right] \frac{\partial \varphi}{\partial \tau} \right) = \beta \frac{\partial \varphi}{\partial \tau} \quad (3.2)$$

where

$$\alpha \equiv \frac{(1 + \lambda_0)}{\lambda_0 + \bar{\rho}}, \quad \beta \equiv \frac{\bar{\rho}(\lambda_0 + \bar{\mu})}{\bar{\mu}(\bar{\rho} + \lambda_0)}, \quad \gamma_1 \equiv \frac{\lambda_1^2 \bar{\rho}}{3\varepsilon_z (\lambda_0 + \bar{\rho})}, \quad \gamma_2 \equiv \frac{1}{\lambda_0 \bar{\mu}} \quad (3.3)$$

Due to the presence of a mixed derivative of third order, it is difficult to determine the type of this equation, but in some particular cases it can be analyzed.

3.2. One-fluid flow with free boundary. The following generalization of the classical nonlinear Boussinesq equation can be deduced from equation (3.2) for single-phase flow of the lower liquid with free boundary. In fact, assuming that the upper fluid has no density and viscosity, i.e. $\bar{\rho} \rightarrow \infty$ and $\bar{\mu} \rightarrow \infty$, we get: $\alpha = 0$, $\beta = 1$, $\gamma_1 = \lambda_1^2 / (3\varepsilon_z)$, $\gamma_2 = 0$, and then

$$\Delta_{yy} \left(\varphi^2 + \gamma_1 \varphi^2 \frac{\partial \varphi}{\partial \tau} \right) = \frac{\partial \varphi}{\partial \tau} \tag{3.4}$$

When the porous layer is very thin, i.e. its horizontal dimension is much larger than the vertical dimension, we have

$$\lambda_1 \ll 1 \tag{3.5}$$

and then, the Boussinesq equation is obtained:

$$\Delta_{yy} \varphi^2 = \frac{\partial \varphi}{\partial \tau} \tag{3.6}$$

which is the classical object of the theory of shallow water flow in porous layers [1, 2, 5, 9].

3.3. Two fluids, thin layer. Gravitational instability. Let us examine two fluids in a thin porous layer, such as the assumption (3.5) holds. Then (3.2) becomes

$$\Delta_{yy} (\varphi^2 - 2\alpha\varphi) = \beta \frac{\partial \varphi}{\partial \tau}, \quad \text{or} \quad \frac{\partial}{\partial y_i} \left((\varphi - \alpha) \frac{\partial \varphi}{\partial y_i} \right) = \frac{\beta}{2} \frac{\partial \varphi}{\partial \tau} \tag{3.7}$$

where parameters α and β are defined according to (3.3), and the summation is made on the index i .

This equation is parabolic when $\varphi > \alpha$. Otherwise, when $\varphi < \alpha$, this is a nonlinear anti-diffusion equation, which can describe the evolution of instability.

To check the nature of these phenomena, let us examine stability of the unperturbed state of the interface, basing on the 1D self-similar solutions of (3.7). In fact, in 1D case, (3.7) has solutions in the form of $\varphi = \varphi(y/\sqrt{\tau})$, which correspond to a boundary-value problem of the interface evolution in an infinite horizontal domain. It is easy to show that self-similar solutions satisfy the following ordinary differential equation

$$\frac{d^2}{d\xi^2} (\varphi^2 - 2\alpha\varphi) = -\frac{1}{2} \beta \xi \frac{d\varphi}{d\xi}, \quad \xi \equiv \frac{y}{\sqrt{\tau}} \tag{3.8}$$

Let us examine small perturbation of the unperturbed state, i.e., $\varphi = 1 + \varepsilon\zeta$, where ε is assumed to be small. Then, using the perturbation method, we get:

$$\zeta'' = -\frac{\beta\xi}{4 - 4\alpha} \zeta', \quad \text{or} \quad \zeta(\xi) = C_1 + C_2 \int_{\infty}^{\xi} e^{\frac{\beta u^2}{8(\alpha-1)}} du$$

where the prime denotes derivation with respect to ξ , and C_1, C_2 are some integration constants.

The evolution of perturbations is unlimited in time, when

$$\alpha \equiv \frac{(1 + \lambda_0)}{\lambda_0 + \bar{\rho}} > 1 \tag{3.9}$$

or $\bar{\rho} < 1$. Then, when the upper liquid is heavier than the lower liquid, (3.7) can describe the classical phenomenon of gravitational instability.

3.4. **Stable flow.** In the case when the condition (3.9) is not satisfied, i.e. the upper fluid is more light, the flow is stable. In fact, since $\alpha < 1$, then the "diffusion coefficient" $\varphi - \alpha$ is positive in the vicinity of the initial state ($\varphi \equiv 1$), and then (3.7) is parabolic.

Figure 2 illustrates the evolution of an initial perturbation defined as a single loop of sinus. The initial wave tends to be dissipated, with finite rate of the wave propagation, as usual for nonlinear parabolic equations.

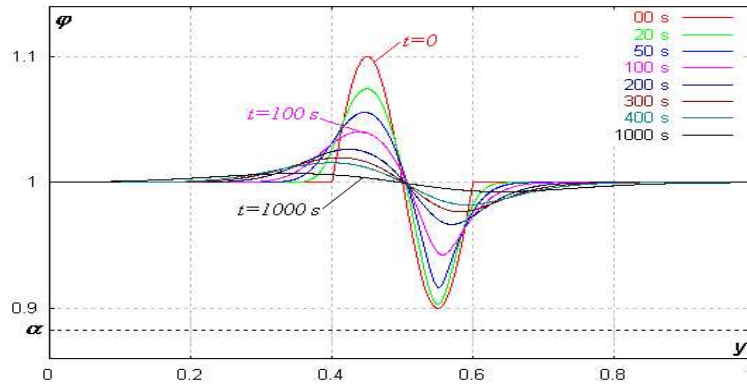


FIGURE 2. Evolution of the interface perturbation in stable case; $\alpha = 0.888888\dots$

More precisely speaking, the following initial problem was considered:

$$\beta \frac{\partial \varphi}{\partial \tau} = \frac{\partial}{\partial y} \left(2(\varphi - \alpha) \frac{\partial \varphi}{\partial y} \right), \quad y \in (0, 1), \tau > 0$$

$$\varphi|_{\tau=0} = \begin{cases} 1, & y \notin J \\ 1 + a \sin(by + c), & y \in J \end{cases}$$

$$\varphi|_{y=0;1} = 1$$

where $J \subset (0, 1)$ is a small support located far from the boundaries; a, b, c are some constant parameters.

The time evolution of φ has been obtained by numerical simulation. For the time discretization, a semi-implicit scheme has been employed. The discrete time instants are denoted by $\tau_n = n\Delta\tau$, where $\tau_0 = 0$ and $\Delta\tau$ is the time step. If the function $\varphi^n(y) \equiv \varphi(y; \tau_n)$ is already computed at time step τ_n , then one computes the corresponding values at the time instant τ_{n+1} by solving the following semi-discretized problem

$$\beta \frac{\varphi^{n+1} - \varphi^n}{\Delta\tau} = \frac{\partial}{\partial y} \left(2(\varphi^n - \alpha) \frac{\partial \varphi^{n+1}}{\partial y} \right), \quad y \in (0, 1), \forall n \geq 0 \tag{3.10}$$

For the space discretization, a conventional scheme of order 2, obtained by a finite difference method, has been used.

The numerical values used to obtain the evolution shown in Figure 2 are

$$\alpha = \frac{8}{9}, \quad \beta = \frac{55}{9}, \quad a = \frac{1}{10}, \quad b = 10\pi, \quad c = -4\pi$$

These values of α and β are obtained for the case of the pair water-oil, i.e. for $\bar{\rho} = 5/4$ ($\rho^I = 1000 \text{ kg/m}^3$, $\rho^{II} = 800 \text{ kg/m}^3$), $\bar{\mu} = 1/10$ ($\mu^I = 0.001 \text{ Pa} \cdot \text{s}$, $\mu^{II} = 0.01 \text{ Pa} \cdot \text{s}$) and by considering that the initial thickness is the same for the two fluids ($\lambda_0 = 1$). The consecutive time instants described in Figure 2 correspond to the value of the characteristic time $t_* = t_{gr} = 1$ hour.

3.5. Evolution of the instability. If condition (3.9) is true, then the function $2(\varphi - \alpha)$ is negative in the vicinity of the initial state $\varphi = 1$, and Eq. (3.7) is anti-parabolic. However, if the function φ becomes larger than α , then this equation becomes parabolic, as shown in Fig. 3.

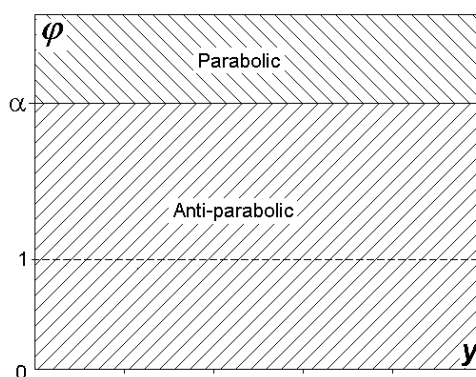


FIGURE 3. Variation of the parabolicity direction for (3.7) when $\alpha > 1$

Thus, the evolution of a perturbation follows three basic stages:

- i) the linear unstable stage, when φ is in the vicinity of the initial state ($\varphi \sim 1$); anti-diffusion equation governs the evolution;
- ii) the nonlinear unstable stage, when $\varphi \rightarrow \alpha$; the spatial derivative $\partial\varphi/\partial y$ becomes unlimited;
- iii) the final stable stage, when φ becomes greater than α ; diffusion equation describes this stage.

Equations with variation of the parabolicity direction were considered in [8], as the model of hysteresis phenomena in theory of phase transitions. However, this theory cannot be applied to (3.7), since the structure of nonlinearity is very different. On the other part, some regularization methods for equations similar to (3.7) were examined there. According to [8], the best regularization is obtained by adding the term $\Delta_{yy}\partial_\tau\varphi$. Then, the full equation (3.2) is expected to regularize unstable solutions.

The linear unstable stage. Let us examine, in 1D framework, a small perturbation of the initial plate interface, $\varphi = 1 + \varepsilon f(t, y)$, where ε is the small amplitude of the perturbation, $f(y, t)$ is a function equal to zero everywhere, excluding a small support $J = \{y : y \in [y_0, y_1]\}$. Let the function f behaves as $f \sim \varepsilon(y - y_0)^2$, when

$y \rightarrow y_0$, in such a way that the perturbation is smooth in the vicinity of the point y_0 . Since the equation (3.7) can be expressed as:

$$2(\varphi - \alpha) \frac{\partial^2 \varphi}{\partial y^2} + \left(\frac{\partial \varphi}{\partial y} \right)^2 = \beta \frac{\partial \varphi}{\partial \tau}$$

then, the evolution of such a perturbation may be easily analyzed. Consequent time phases are shown in Fig. 4.

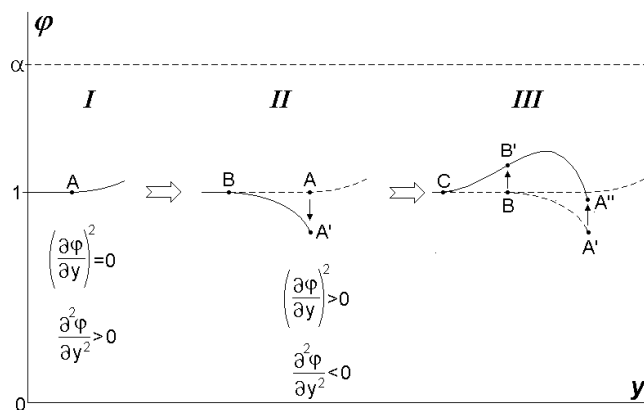


FIGURE 4. Evolution of a smooth monotonous perturbation of the interface in the unstable case

At initial time, for the point $A = (\varphi, y) = (1, y_0)$ it is true:

$$\partial \varphi / \partial y = 0, \quad \partial^2 \varphi / \partial y^2 > 0, \quad \text{then } (\varphi - \alpha) \partial^2 \varphi / \partial y^2 < 0, \quad \rightarrow \quad \partial_\tau \varphi < 0$$

Then, in Figure 4, the plot I becomes the plot II. The point A is lowered downwards and then, the extreme point with zeroth spatial derivative is displacing to the left (to the point B). The next step of the evolution leads to the plot III, etc.

Thus, collecting all the results obtained about the unstable evolution, it may be concluded that a small localized smooth and monotonous perturbation of the interface leads to arising of three types of phenomena:

- a) lateral propagation of the perturbation along the interface;
- b) fast development of the spatial and time oscillations of the interface;
- c) fast growth of the amplitude of the oscillations.

Figure 5 illustrates the evolution of an initial perturbation defined as a single loop of sinus.

Growth of the oscillations is qualitatively equivalent to the finger growth observed in the experimental research, as for instance in [6]. We have used the same numerical scheme (3.10) as for the stable flow, but with other numerical values, corresponding to the case when the oil is the lower fluid and the water is the upper fluid. In particular, the value of α is now $1.1111... > 1$. The support of the initial perturbation (the loop of the sinus) has been enlarged to get a better representation of the finger growth. On the other part, it must be mentioned that, using the same

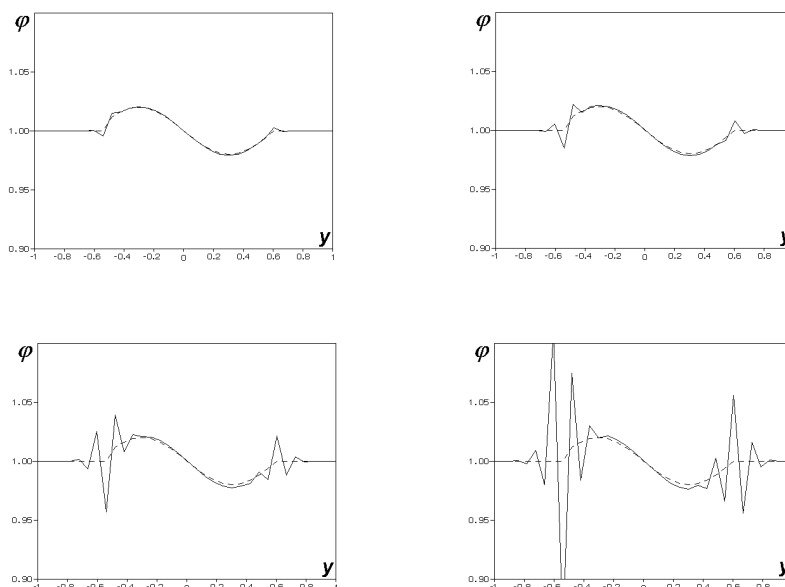


FIGURE 5. Unstable evolution of the interface perturbation

value for the gravitational time t_{gr} , the evolution of the interface is much faster. Indeed, the consequent instant times shown in Figure 5 are respectively 0.2 s, 0.4 s, 0.6 s, 0.8 s.

The nonlinear unstable stage. Growth of the fingers leads to the instant when the finger approaches to the critical value $\varphi = \alpha$. This is possible only if $|\frac{\partial \varphi}{\partial y}| \rightarrow \infty$ in this point, that follows from (3.7). Hence, each finger crossing the critical line $\varphi = \alpha$ has there an infinite spatial derivative. From this property, it follows that a finger can cross the critical line, only if it becomes a delta-function.

These results correspond well to the experimental results obtained by Maxworthy [6], where a similar behavior of fingers growth has been observed.

Conclusion. Applications of the general model developed in [3] in the case where gravity perturbations are propagating much slower than elastic perturbations have been investigated. In this case, the model can be reduced to a nonlinear evolution equation with varying sense of parabolicity. This equation becomes the well-known Boussinesq equation if the upper fluid has not viscosity and density and can describe the gravity unstable system if the lower fluid is more light.

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