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# SOME PROPERTIES OF SHEAF-SOLUTIONS OF SHEAF FUZZY CONTROL PROBLEMS

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Abstract. In this paper we present some results on the dependence of sheaf-solutions of sheaf fuzzy control problems on variation of fuzzy controls.

## 1. Introduction

The problems of differential equations, differential inclusions and control differential equations have been studying in both theory and application. See [1, 11, 12, 13, 18]. The concept sheaf solution of classical control problems was introduced in [10]. Instead of studying each solution, one studies sheaf-solution, that means, a set of solutions. In [14, 15, 16], we presented some results of sheaf-solutions in fuzzy control problems whose variables are crisp and controls are fuzzy. In this paper, we study the fuzzy control problems whose variables and controls are fuzzy sets and the dependence of sheaf-solutions of sheaf fuzzy control problems on variation of fuzzy controls. The paper is organized as follows: Section 2 reviews some concepts of fuzzy sets, Hausdorff distance and fuzzy diferential equations. The sheaf fuzzy control problem and some new results of the dependence of sheaf-solutions on variation of fuzzy controls are presented in Section 3.

## 2. Preliminaries and notation

We recall some notation and concepts which were presented in detail in [4]-[7], [9]. Let $K_c(\mathbb{R}^n)$ denote the collection of all nonempty, compact, convex subsets of $\mathbb{R}^n$. Let $A, B$ be two nonempty bounded subsets of $\mathbb{R}^n$. The Hausdorff distance between $A$ and $B$ is defined as

$$D[A,B] = \max\{\sup_{a\in A}\inf_{b\in B}\|a-b\|,\ \sup_{b\in B}\inf_{a\in A}\|a-b\|\}. \tag{2.1}$$

Note that $K_c(\mathbb{R}^n)$ with the metric $D$ is a complete metric space. See [17]. It is known that if the space $K_c(\mathbb{R}^n)$ is equipped with the natural algebraic operations of addition and nonnegative scalar multiplication, then $K_c(\mathbb{R}^n)$ becomes a semilinear metric space which can be embedded as a complete cone into a corresponding Banach space.

Set $\mathbb{E}^n = \{u : \mathbb{R}^n \to [0,1] : u$ satisfies (i)–(iv) stated below $\}$

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(i) $u$ is normal, that is, there exists an $x_0 \in \mathbb{R}^n$ such that $u(x_0) = 1$;
(ii) $u$ is fuzzy convex, that is, for $0 \le \lambda \le 1$

$$u(\lambda x_1 + (1-\lambda)x_2) \ge \min\{u(x_1), u(x_2)\};$$

(iii) $u$ is upper semicontinuous;
(iv) $[u]^0 = \mathrm{cl}\{x \in \mathbb{R}^n : u(x) > 0\}$ is compact. The element $u \in \mathbb{E}^n$ is called a fuzzy number or fuzzy set.

For $0 < \alpha \le 1$, the set $[u]^\alpha = \{x \in \mathbb{R}^n : u(x) \ge \alpha\}$ is called the $\alpha$-level set. From (i)-(iv), it follows that the $\alpha$-level sets are in $K_c(\mathbb{R}^n)$, for $0 \le \alpha \le 1$.
Let us denote

$$D_0[u,v] = \sup\{D\big[[u]^\alpha, [v]^\alpha\big] : 0 \le \alpha \le 1\}$$

the distance between $u$ and $v$ in $\mathbb{E}^n$, where $D\big[[u]^\alpha, [v]^\alpha\big]$ is Hausdorff distance between two sets $[u]^\alpha, [v]^\alpha$ of $K_c(\mathbb{R}^n)$. Then $(\mathbb{E}^n, D_0)$ is a complete space. Some properties of metric $D_0$ are as follows.

$$\begin{gathered} D_0[u+w, v+w] = D_0[u,v], \\ D_0[\lambda u, \lambda v] = \mid \lambda \mid D_0[u,v], \\ D_0[u,v] \le D_0[u,w] + D_0[w,v], \end{gathered} \tag{2.2}$$

for all $u, v, w \in \mathbb{E}^n$ and $\lambda \in \mathbb{R}$. Let us denote $\theta^n \in \mathbb{E}^n$ the zero element of $\mathbb{E}^n$ as follows

$$\theta^n(z) = \begin{cases} 1 & \text{if } z = \hat{0}, \\ 0 & \text{if } z \ne \hat{0}, \end{cases}$$

where $\hat{0}$ is the zero element of $\mathbb{R}^n$.

Suppose that $Q, G \subset \mathbb{E}^l$ for some positive integer $l$. Here is some useful notation:

$$\begin{gathered} d^*[Q,G] = \sup\{D_0[q,g] : q \in Q, g \in G\}, \\ \mathrm{diam}[Q] = \sup\{D_0[q,q'] : q, q' \in Q\}, \\ d[Q] = d^*[Q, \theta^l], \end{gathered} \tag{2.3}$$

where $\theta^l$ is zero element of $\mathbb{E}^l$ which is regarded as a one point set. Let $u, v \in \mathbb{E}^n$. The set $z \in \mathbb{E}^n$ satisfying $u = v + z$ is known as the geometric difference of the sets $u$ and $v$ and is denoted by the symbol $u - v$. The mapping $F : \mathbb{R}_+ \supset I = [t_0, T] \to \mathbb{E}^n$ is said to have a Hukuhara derivative $D_H F(\tau)$ at a point $\tau \in I$, if

$$\lim_{h \to 0^+} \frac{F(\tau + h) - F(\tau)}{h} \quad \text{and} \quad \lim_{h \to 0^+} \frac{F(\tau) - F(\tau - h)}{h}$$

exist and are equal to $D_H F(\tau)$. Here limits are taken in the metric space $(E^n, D_0)$. If $F : I \to \mathbb{E}^n$ is continuous, then it is integrable and

$$\int_{t_0}^{t_2} F(s)ds = \int_{t_0}^{t_1} F(s)ds + \int_{t_1}^{t_2} F(s)ds. \tag{2.4}$$

If $F, G : I \to \mathbb{E}^n$ are integrable, $\lambda \in \mathbb{R}$, then some properties below hold

$$\int_{t_0}^{t} (F(s)+G(s))ds = \int_{t_0}^{t} F(s)ds + \int_{t_0}^{t} G(s)ds; \tag{2.5}$$

$$\int_{t_0}^{t} \lambda F(s)ds = \lambda \int_{t_0}^{t} F(s)ds, \quad \lambda \in \mathbb{R}, t_0 \le t \le T; \tag{2.6}$$

$$D_0\Big[\int_{t_0}^{t} F(s)ds, \int_{t_0}^{t} G(s)ds\Big] \le \int_{t_0}^{t} D_0\big[F(s), G(s)\big]ds. \tag{2.7}$$

Let $F : I \to \mathbb{E}^n$ be continuous. Then integral $\int_{t_0}^{t} F(s)ds$ is differentiable and $D_H G(t) = F(t)$.
Recently, the study of fuzzy differential equation (FDE)

$$D_H x = f(t, x(t)), \quad x(t_0) = x_0 \in \mathbb{E}^n,$$

where $f \in C[\mathbb{R}_+ \times \mathbb{E}^n, \mathbb{E}^n]$, and $t \in \mathbb{R}_+, x(t) \in \mathbb{E}^n$ has gained much attention. Some important results on the existence and comparison of solutions of FDE have been obtained by Prof. V. Lakshmikantham and other authors in [2, 3], [6]- [8] and [9].

## 3. Main results

We consider the initial valued problem (IVP) for a fuzzy control differential equation (FCDE) as follows

$$D_H x = f(t, x(t), u(t)), \quad x(t_0) = x_0 \in \mathbb{E}^n, \tag{3.1}$$

where $f \in C[\mathbb{R}_+ \times \mathbb{E}^n \times \mathbb{E}^p, \mathbb{E}^n]$, $t \in \mathbb{R}_+$, state $x(t) \in \mathbb{E}^n$ and fuzzy control $u(t) \in \mathbb{E}^p$.

The function $u : I \to \mathbb{E}^p$ is integrable, is called an admissible control. Let $U \subset \mathbb{E}^p$ be the set of all admissible fuzzy controls. The mapping $x \in C^1[I, \mathbb{E}^n]$ is said to be a solution of (3.1) on $I$ if it satisfies (3.1) on $I$. Since $x(t)$ is continuously differentiable, we have

$$x(t) = x_0 + \int_{t_0}^{t} D_H x(s)ds, \quad t \in I.$$

Thus, we associate with the initial value problem (IVP) (3.1) the function

$$x(t) = x_0 + \int_{t_0}^{t} f(s, x(s), u(s))ds, \quad t \in I \tag{3.2}$$

where the integral is the Hukuhara integral. Observe that $x(t)$ is a solution of (3.1) if and only if it satisfies (3.2) on $I$.

In [16], the authors proved the following theorem (which is an adaptation of Theorems 4.1-4.2 of the set differential equations in [6] to FCDE) on the existence of solutions of FCDE.

**Theorem 3.1** ([16]). *Assume that $f \in C[\mathbb{R}_+ \times \mathbb{E}^n \times \mathbb{E}^p, \mathbb{E}^n]$ and*

(i) $D_0\big[f(t, x(t), u(t)), \theta^n\big] \le g(t, D_0[x(t), \theta^n])$ *for* $(t, x, u) \in I \times \mathbb{E}^n \times U$, *where* $g \in C[I \times \mathbb{R}_+, \mathbb{R}_+]$, $g(t, w)$ *is nondecreasing in* $(t, w)$.

(ii) *The maximal solution $r(t, w_0)$ of the scalar differential equation*

$$w' = g(t, w), \quad w(0) = w_0 \ge 0 \quad \textit{exists on } I.$$

*Then there exists a solution* $x(t) = x(t, t_0, x_0, u(t))$ *to* (3.1) *which satisfies*

$$D_0[x(t), x_0] \leq r(t, w_0) - w_0, \quad t \in I,$$

*where* $w_0 = D_0[x_0, \theta^n], u(t) \in U$.

In [10], the author presented the concept of sheaf-solution of a classical control differential equation. Instead of studying each solution, one studies sheaf-solution, that means, a set of solutions. In this paper, we use this concept for FCDE.

**Definition 3.1.** The sheaf-solution (or sheaf-trajectory) of (3.1) which gives at the time $t$ a set

$$H_{t,u} = \{x(t) = x(t, x_0, u(t)) - \text{solution of (3.1)} : x_0 \in H_0\},$$

where $u(t) \in U \subset \mathbb{E}^p$, $t \in I$, $H_0 \subset \mathbb{E}^n$.

$H_{t,u}$ is called a $cross - area$ at $(t, u)$ (or $(t, u)$-cut) of the above sheaf-solution. System (3.1) with its sheaf-solutions are called a *sheaf fuzzy control problem.* For two admissible controls $u(t)$ and $\bar{u}(t)$, we have two sheaf-solutions whose cross-areas are

$$H_{t,u} = \big\{x(t) = x(t, x_0, u(t)) - \text{solution of (3.1)} : x_0 \in H_0\big\}, \tag{3.3}$$

$$\bar{H}_{t,\bar{u}} = \big\{\bar{x}(t) = x(t, \bar{x}_0, \bar{u}(t)) - \text{solution of (3.1)} : \bar{x}_0 \in \bar{H}_0\big\} \tag{3.4}$$

where $t \in I$; $u(t), \bar{u}(t) \in U$; $H_0, \bar{H}_0 \subset \mathbb{E}^n$. In this paper, instead of comparison two sheaf-solutions, we compare their cross-areas.

An immediate consequence of Theorem 3.1 for estimate of one sheaf-solution is the following result.

**Corollary 3.1.** *Under assumptions of Theorem 3.1, one has*

$$d^*[H_{t,u}, H_0] \leq r(t, w_0) - \alpha_0$$

*where* $\alpha_0 = \inf\{D_0[x_0, \theta^n] : x_0 \in H_0\}$ *for all* $u(t) \in U$.

Now, we consider the following assumption on $f$: The vector function $f : \mathbb{R}^+ \times \mathbb{E}^n \times \mathbb{E}^p \to \mathbb{E}^n$ satisfies the condition

$$D_0\big[f(t, \bar{x}(t), \bar{u}(t)), f(t, x(t), u(t))\big] \leq c(t)\big[D_0\big[\bar{x}(t), x(t)\big] + D_0[\bar{u}(t), u(t)]\big] \tag{3.5}$$

for all $t \in I; u(t), \bar{u}(t) \in U; x(t), \bar{x}(t) \in \mathbb{E}^n$, where $c(t)$ is a positive and integrable on $I$.

Let $C = \int_0^T c(t)dt$. Because $c(t)$ is integrable on $I$, it is bounded almost everywhere (a.e) by some positive constant $K$, that is, $c(t) \leq K$ a.e. $t \in I$. If we consider the RHS of assumption (3.5) as a scalar function $g$, then $g$ depends on states and controls and the structure of $g$ is simple. The scalar function $g$ is used to estimate the variation of function $f$ whose value is in $\mathbb{E}^n$. In the following theorem, we prove that the solutions of (3.1) depend continuously on variation of controls and initials.

**Theorem 3.2.** *Suppose that* $f$ *is continuous and satisfies* (3.5) *and* $\bar{x}(t), x(t)$ *are two solutions of (3.1) originating at different initials* $\bar{x}_0, x_0$ *with controls* $\bar{u}(t), u(t)$, *respectively. Then for every* $\epsilon > 0$ *there exists a* $\delta(\epsilon) > 0$ *such that*

$$D_0\big[\bar{x}(t), x(t)\big] \leq \epsilon$$

*if*

$$D_0\big[\bar{x}_0, x_0\big] \leq \delta(\epsilon) \quad \textit{and} \quad D_0[\bar{u}(t), u(t)] \leq \delta(\epsilon) \tag{3.6}$$

*where* $t \in I$; $\bar{u}(t), u(t) \in U$.

*Proof.* The solutions of (3.1) for controls $u(t)$ and $\bar{u}(t)$ originating at the points $x_0$ and $\bar{x}_0$, are equivalent to the following integral forms

$$\begin{aligned} x(t) &= x_0 + \int_{t_0}^{t} f(s, x(s), u(s))ds, \\ \bar{x}(t) &= \bar{x}_0 + \int_{t_0}^{t} f(s, \bar{x}(s), \bar{u}(s))ds. \end{aligned}$$

Estimating $D_0[\bar{x}(t), x(t)]$, using (2.2) and (2.7), we get

$$D_0[\bar{x}(t), x(t)] \le D_0[\bar{x}_0, x_0] + \int_{t_0}^{t} D_0\big[f(s, \bar{x}(s), \bar{u}(s)), f(s, x(s), u(s))\big]ds.$$

By assumption (3.5), we have

$$\begin{aligned} D_0[\bar{x}(t), x(t)] &\le D_0[\bar{x}_0, x_0] + \int_{t_0}^{t} c(s)\big[D_0\big[\bar{x}(s), x(s)\big] + D_0\big[\bar{u}(s), u(s)\big]\big]ds \\ &\le D_0[\bar{x}_0, x_0] + \int_{t_0}^{t} c(s)D_0\big[\bar{x}(s), x(s)\big]ds + K\int_{t_0}^{t} D_0\big[\bar{u}(s), u(s)\big]ds, \end{aligned} \tag{3.7}$$

then using $D_0\big[\bar{u}(t), u(t)\big] \le \delta(\epsilon)$ and $D_0[\bar{x}_0, x_0] \le \delta(\epsilon)$, we obtain

$$D_0[\bar{x}(t), x(t)] \le [1 + K(T - t_0)]\delta(\epsilon) + \int_{t_0}^{t} c(s)D_0[\bar{x}(t), x(t)]ds.$$

By Gronwall's inequality, we have the estimate

$$D_0[\bar{x}(t), x(t)] \le [1 + K(T - t_0)]\delta(\epsilon)\exp(C).$$

For a given $\epsilon > 0$, if we choose

$$0 < \delta(\epsilon) \le \frac{\epsilon}{[1 + K(T - t_0)]\exp(C)}$$

then $D_0[\bar{x}(t), x(t)] \le \epsilon$. The proof is complete. $\square$

An immediate consequence of Theorem 3.2 is the following result.

**Corollary 3.2.** *Suppose that $f$ is continuous and satisfies* (3.5) *and $\bar{H}_{t,\bar{u}}, H_{t,u}$ are two cross-areas of sheaf-solutions of (3.1) corresponding to $\bar{H}_0, H_0$ and controls $\bar{u}(t), u(t) \in U$. Then for every $\epsilon > 0$ there exists a $\delta(\epsilon) > 0$ such that*

$$d^*\big[\bar{H}_{t,\bar{u}}, H_{t,u}\big] \le \epsilon$$

*if*

$$d^*\big[\bar{H}_0, H_0\big] \le \delta(\epsilon) \quad \textit{and} \quad D_0[\bar{u}(t), u(t)] \le \delta(\epsilon)$$

*where $t \in I$; $\bar{u}(t), u(t) \in U$.*

In Corollary 3.2, we remark that the sheaf-solutions of sheaf fuzzy control problem concerning fuzzy control differential equation (3.1) depend continuously on variation of fuzzy controls and initials.

We have some theorems below on comparison two sheaf-solutions in case of $\bar{H}_0, H_0, U$ are bounded subsets.

**Theorem 3.3.** *Suppose that $f$ is continuous and satisfies* (3.5) *and $H_0, U$ are bounded subsets. Then*

$$d^*\big[\bar{H}_{t,\bar{u}}, H_{t,u}\big] \le \big[\operatorname{diam}[H_0] + K.\operatorname{diam}[U](T-t_0)\big]\exp(C)$$

*where $\bar{H}_{t,\bar{u}}, H_{t,u}$ are any cross-areas of sheaf-solutions of* (3.1) *with $\bar{H}_0 = H_0$, for all $t \in I$; $\bar{u}(t), u(t) \in U$.*

*Proof.* Starting as in the proof of Theorem 3.2, we arrive at (3.7) and use (2.3), then

$$\begin{aligned} D_0[\bar{x}(t), x(t)] &\le D_0[\bar{x}_0, x_0] + \int_{t_0}^{t} c(s)\big[D_0\big[\bar{x}(s), x(s)\big] + D_0\big[\bar{u}(s), u(s)\big]\big]ds \\ &\le D_0[\bar{x}_0, x_0] + \int_{t_0}^{t} c(s)D_0\big[\bar{x}(s), x(s)\big]ds + K\int_{t_0}^{t} D_0\big[\bar{u}(s), u(s)\big]ds \\ &\le \operatorname{diam}[H_0] + \int_{t_0}^{t} c(s)D_0\big[\bar{x}(s), x(s)\big]ds + K.\operatorname{diam}[U](T-t_0). \end{aligned}$$

Using Gronwall's inequality, then the proof is complete. □

The following is result more general than Theorem 3.3.

**Theorem 3.4.** *Suppose that $f$ is continuous and satisfies* (3.5) *and $\bar{H}_0, H_0, U$ are bounded subsets. Then*

$$d^*\big[\bar{H}_{t,\bar{u}}, H_{t,u}\big] \le \big[d^*[\bar{H}_0, H_0] + K.\operatorname{diam}[U](T-t_0)\big]\exp(C)$$

*where $\bar{H}_{t,\bar{u}}, H_{t,u}$ are any cross-areas of sheaf-solutions of* (3.1), *for all $t \in I; \bar{u}(t), u(t) \in U$.*

The proof is similar the proof of Theorem 3.3.
The next comparison result provides an estimate under assumption that the structure of the scalar function $g$ is more general than the one in (3.5), but $g$ does not depend on fuzzy controls.

**Theorem 3.5.** *Assume that $f \in C\big[I \times \mathbb{E}^n \times \mathbb{E}^p, \mathbb{E}^n\big]$ and*

$$D_0\big[f(t, \bar{x}(t), \bar{u}(t)), f(t, x(t), u(t))\big] \le g(t, D_0[\bar{x}(t), x(t)]), \tag{3.8}$$

*for $(t, \bar{x}(t), \bar{u}(t))$, $(t, x(t), u(t)) \in I \times \mathbb{E}^n \times U$, where $g \in C\big[\mathbb{R}_+ \times \mathbb{R}_+, \mathbb{R}_+\big]$ and $g(t, w)$ is nondecreasing in $w$ for each $t \in I$. Suppose further that the maximal solution $r(t) = r(t, t_0, w_0)$ of scalar differential equation*

$$w' = g(t, w), \quad w(t_0) = w_0 \ge 0$$

*exists for $t \in I$. Then, if $\bar{x}(t) = \bar{x}(t, t_0, \bar{x}_0, \bar{u}(t))$ and $x(t) = x(t, t_0, x_0, u(t))$ are any solutions of (3.1) such that $\bar{x}(t_0) = \bar{x}_0, x(t_0) = x_0$; $\bar{x}_0, x_0 \in \mathbb{E}^n$ existing for $t \in I$, one has*

$$D_0[\bar{x}(t), x(t)] \le r(t, t_0, w_0), t \in I, \tag{3.9}$$

*provided $D[\bar{x}_0, x_0] \le w_0$, for all $t \in I$; $\bar{u}(t), u(t) \in U$.*

*Proof.* The solutions of (3.1) for controls $u(t)$ and $\bar{u}(t)$ originating at $x_0, \bar{x}_0$ are equivalent to the integral forms

$$\begin{aligned} x(t) &= x_0 + \int_{t_0}^{t} f(s, x(s), u(s))ds, \\ \bar{x}(t) &= \bar{x}_0 + \int_{t_0}^{t} f(s, \bar{x}(s), \bar{u}(s))ds. \end{aligned}$$

Set $m(t) = D_0\big[\bar{x}(t), x(t)\big]$, so that $m(t_0) = D_0\big[\bar{x}_0, x_0\big] \le w_0$. Using the properties (2.2), (2.7) of the metric $D_0$, one has

$$\begin{aligned} m(t) &= D_0\big[\bar{x}_0 + \int_{t_0}^{t} f\big(s, \bar{x}(s), \bar{u}(s)\big)ds, x_0 + \int_{t_0}^{t} f\big(s, x(s), u(s)\big)ds\big] \\ &\le D_0\big[\bar{x}_0 + \int_{t_0}^{t} f\big(s, \bar{x}(s), \bar{u}(s)\big)ds, \bar{x}_0 + \int_{t_0}^{t} fb\big(s, x(s), u(s)\big)ds\big] \\ &\quad + D_0\big[\bar{x}_0 + \int_{t_0}^{t} f\big(s, x(s), u(s)\big)ds, x_0 + \int_{t_0}^{t} f\big(s, x(s), u(s)\big)ds\big] \\ &= D_0\big[\int_{t_0}^{t} f\big(s, \bar{x}(s), \bar{u}(s)\big)ds, \int_{t_0}^{t} f\big(s, x(s), u(s)\big)ds\big] + D_0[\bar{x}_0, x_0] \\ &\le m(t_0) + \int_{t_0}^{t} D_0\big[f\big(s, \bar{x}(s), \bar{u}(s)\big), f\big(s, x(s), u(s)\big)\big]ds. \end{aligned}$$

Then, using (3.8), we obtain

$$\begin{aligned} m(t) &\le m(t_0) + \int_{t_0}^{t} g\big(s, D_0[\bar{x}(s), x(s)]\big)ds \\ &= m(t_0) + \int_{t_0}^{t} g\big(s, m(s)\big)ds, \quad t \in I. \end{aligned} \tag{3.10}$$

Applying [8, Theorem 1.9.2], we conclude that $m(t) \le r(t, t_0, w_0), t \in I$, which completes the proof. ☐

In the above theorem we can dispense with the monotone character of $g(t, w)$ if we employ the theory of differential inequality instead of integral inequalities. This result and some more results on this direction will be presented in next works.

**Corollary 3.3.** *Under the assumptions of Theorem 3.5, one has*

$$d^*\big[\bar{H}_{t,\bar{u}}, H_{t,u}\big] \le r(t, t_0, w_0), \quad t \in I,$$

*provided* $d^*[\bar{H}_0, H_0] \le w_0$, *for all* $\bar{u}(t), u(t) \in U$.

It is easy to prove this corollary.

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