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EXISTENCE OF POSITIVE PERIODIC SOLUTIONS FOR NEUTRAL FUNCTIONAL DIFFERENTIAL EQUATIONS

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Abstract. We find sufficient conditions for the existence of positive periodic solutions of two kinds of neutral differential equations. Using Krasnoselskii's fixed-point theorem in cones, we obtain results that extend and improve previous results. These results are useful mostly when applied to neutral equations with delay in bio-mathematics.

1. Introduction

In this paper, we investigate the existence of positive periodic solutions of the following two kinds of nonlinear neutral functional differential equations

$$\frac{d}{dt}(x(t) - cx(t-\tau(t))) = -a(t)x(t) + g(t, x(t-\tau(t))), \tag{1.1}$$

and

$$\frac{d}{dt}(x(t) - c\int_{-\infty}^{0} K(r)x(t+r)dr) = -a(t)x(t) + b(t)\int_{-\infty}^{0} K(r)g(t, x(t+r))dr, \tag{1.2}$$

where $a, \tau \in C(\mathbb{R};\mathbb{R})$, $\int_0^\omega a(t)dt > 0$, $b \in C(\mathbb{R};(0,\infty))$, $g \in C(\mathbb{R}\times[0,\infty),[0,\infty))$, and $a(t)$, $b(t)$, $\tau(t)$, $g(t,x)$ are ω-periodic functions. $\omega > 0$ and $c \in [0,1)$ are two constants. Moreover, $K \in C((-\infty,0],[0,\infty))$ and $\int_{-\infty}^0 K(r)dr = 1$. The function $a(t)$ admits negative values in bad conditions, since the environment fluctuates randomly.

Our work is motivated by [8, 9, 14], where the equations

$$\frac{d}{dt}x(t) = -a(t)x(t) + g(t, x(t-\tau(t))),$$
$$\frac{d}{dt}x(t) = -a(t)x(t) + b(t)\int_{-\infty}^{0} K(s)g(t, x(t+s))ds,$$

are considered. Since these equations include many important models in mathematical biology, such as Hematopoiesis models, blood cell production and the Nicholson's blowflies models in [2, 3, 6, 8, 9, 10, 11, 12, 13, 14, 15], the sufficient conditions

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for the existence of positive periodic solutions of these equations in [8, 9, 14] are interesting.

Meanwhile, since a growing population is likely to consume more (or less) food than a matured one, depending on individual species, this leads to the neutral functional differential equations. Moreover, it is well-known that periodic solutions of differential equations describe the important modality of the systems. So it is important to study the existence of periodic solutions to (1.1) and (1.2).

Equations (1.1) and (1.2) include many mathematical ecological models and population models (directly or after some transformation). For example, there are many Hematopoiesis models, which are modifications from models in [2, 8, 9, 11, 12, 14, 15]:

$$\frac{d}{dt}(x(t)-cx(t-\tau(t))) = -a(t)x(t)+b(t)e^{-\beta(t)x(t-\tau(t))}, \tag{1.3}$$

$$\frac{d}{dt}(x(t)-c\int_{-\infty}^{0}K(r)x(t+r)dr) = -a(t)x(t)+b(t)\int_{-\infty}^{0}K(r)e^{-\beta(t)x(t+r))}dr. \tag{1.4}$$

There are more general models for blood cell production, which are variations of models in [2, 3, 8, 9, 11, 12, 14, 15]:

$$\frac{d}{dt}(x(t)-cx(t-\tau(t))) = -a(t)x(t)+b(t)\frac{1}{1+x^n(t-\tau(t))}, n>0, \tag{1.5}$$

$$\frac{d}{dt}(x(t)-cx(t-\tau(t))dr) = -a(t)x(t)+b(t)\frac{x(t-\tau(t))}{1+x^n(t-\tau(t))}, n>0, \tag{1.6}$$

$$\begin{aligned}&\frac{d}{dt}(x(t)-c\int_{-\infty}^{0}K(r)x(t+r)dr)\\&= -a(t)x(t)+b(t)\int_{-\infty}^{0}K(r)\frac{1}{1+x^n(t+r)}dr, n>0,\end{aligned} \tag{1.7}$$

$$\begin{aligned}&\frac{d}{dt}(x(t)-c\int_{-\infty}^{0}K(r)x(t+r)dr)\\&= -a(t)x(t)+b(t)\int_{-\infty}^{0}K(r)\frac{x(t+r)}{1+x^n(t+r)}dr, n>0.\end{aligned} \tag{1.8}$$

Meanwhile, there are more Nicholson's blowflies models, which are modifications from models in [2, 6, 8, 9, 12, 14]:

$$\frac{d}{dt}(x(t)-cx(t-\tau(t))) = -a(t)x(t)+b(t)x(t-\tau(t))e^{-\beta(t)x(t-\tau(t))}, \tag{1.9}$$

$$\begin{aligned}&\frac{d}{dt}(x(t)-c\int_{-\infty}^{0}K(r)x(t+r)dr)\\&= -a(t)x(t)+b(t)\int_{-\infty}^{0}K(r)x(t+r)e^{-\beta(t)x(t+r)}dr.\end{aligned} \tag{1.10}$$

In this paper, we obtain sufficient conditions for the existence of positive periodic solutions for the neutral delay differential equations (1.1) and (1.2). Our results improve and generalize the corresponding results of Jiang and Wei [8, 9] and Wan [14], when $c = 0$ in (1.1) and (1.2). In fact, Theorem 2.1 extends and improves the corresponding results in [14, Theorem 2.1] and [9, Theorem 2.1]. Meanwhile, Theorem 2.2 improves the corresponding results in [8, Theorem 2.1]. For $a(t) > 0$ in [14] and $g(t,x)$ sub-linear or super-linear in [9], the assumptions in Theorem 2.1 and Theorem 2.2 are weaker than theirs. When $c \neq 0$, our main results are new.

Due to $c \neq 0$, the methods used by the authors [8, 9, 14] can not be directly applied to (1.1) and (1.2). The proofs of the main results in our paper are based on an application of Krasnoselskii's fixed point theorem in cones (See [1, 4, 5]). To make use of fixed point theorem in a cone, firstly, we introduce the definition of a cone in a Banach space.

Definition. Let X be a Banach space. K is called a cone if K is a closed nonempty subset of X and satisfies

(i) $\alpha x + \beta y \in K$, for all $x, y \in K$ and $\alpha, \beta > 0$;
(ii) $x, -x \in K$ implies $x = 0$.

The following Lemma is due to Krasnoselskii (See [1, 4, 5]).

Lemma 1.1. *Let X be a Banach space, and let $K \subset X$ be a cone in X. Assume Ω_1, Ω_2 are open subsets of X with $0 \in \Omega_1, \bar{\Omega}_1 \subset \Omega_2$, and let*

$$\Phi : K \cap (\bar{\Omega}_2 \setminus \Omega_1) \to K$$

be a completely continuous operator that satisfies one of the following conditions:

(i) $\|\Phi x\| \geq \|x\|, \forall x \in K \cap \partial\Omega_1$ *and* $\|\Phi x\| \leq \|x\|, \forall x \in K \cap \partial\Omega_2$*;*
(ii) $\|\Phi x\| \geq \|x\|, \forall x \in K \cap \partial\Omega_2$ *and* $\|\Phi x\| \leq \|x\|, \forall x \in K \cap \partial\Omega_1$.

Then Φ has a fixed point in $K \cap (\bar{\Omega}_2 \setminus \Omega_1)$.

For convenience, we need to introduce a few notations and assumptions. Let

$$\begin{gathered} G(t,s) = \frac{\exp(\int_t^s a(r)dr)}{\exp(\int_0^\omega a(r)dr) - 1}, \\ A := \min\{G(t,s) : 0 \leq t, s \leq \omega\} = G(t,t) > 0, \\ B := \max\{G(t,s) : 0 \leq t, s \leq \omega\} = G(t, t+\omega) > 0, \\ 0 < \sigma = \frac{A}{B} < 1, \\ m(y) = \max_{(t,x) \in [0,\omega] \times [0,y]} g(t,x), y \geq 0. \end{gathered}$$

For (1.1), we assume that

(H1) $\liminf_{x \to 0} \frac{g(t,x)}{x} = \alpha(t)$ and $\limsup_{x \to \infty} \frac{g(t,x)}{x} = \beta(t)$, where $\alpha(t), \beta(t)$ are continuous ω-periodic functions on $\mathbb{R}$.
(H2) $\int_0^\omega \alpha(t)dt > c\int_0^\omega a(t)dt + \frac{1}{A\sigma}(1 - c\sigma)$ and $\int_0^\omega \beta(t)dt < c\int_0^\omega a(t)dt + \frac{1}{B}(1-c)$.
(H3) $g(t,x) \geq ca(t)x, \forall (t,x) \in \mathbb{R} \times [0, r_2]$.

From (H1), there exist two constants r_1 and n with $0 < r_1 < n$ such that

$$\begin{gathered} g(t,x) \geq \alpha(t)x, \quad 0 \leq x \leq r_1, \\ g(t,x) \leq \beta(t)x, \quad x > n. \end{gathered}$$

Let $r_2 > \max\{\frac{Bm}{1-c-B\int_0^\omega [\beta(t) - ca(t)]dt}, n\} > r_1$, where $m = \omega(m(n) + cn\|a(t)\|)$. For (1.2), we suppose that (H1) holds and

(P2) $\int_0^\omega b(t)\alpha(t)dt > c\int_0^\omega a(t)dt + \frac{1}{A\sigma}(1 - c\sigma)$ and $\int_0^\omega b(t)\beta(t)dt < c\int_0^\omega a(t)dt + \frac{1}{B}(1-c)$.
(P3) $g(t,x) \geq \frac{ca(t)}{b(t)}x$, for all $(t,x) \in \mathbb{R} \times [0, R_2]$.

From (H1) there exist two constants R_1 and N with $0 < R_1 < N$ such that

$$g(t,x) \geq \alpha(t)x, \quad 0 \leq x \leq R_1,$$
$$g(t,x) \leq \beta(t)x, \quad x > N.$$

Let

$$R_2 > \max\{\frac{BM}{1 - c - B\int_0^\omega [\beta(t) - ca(t)]dt}, N\} > R_1,$$

where $M = \omega(m(N) + cN\|a(t)\|)$.

The rest of this paper is organized as follows. In the second section, we give and prove our main results. As applications, in the final section, we apply our main results to some population models and several new results are obtained.

2. Existence of Positive Periodic Solutions

Now we state our main results.

Theorem 2.1. *Assume that (H1)-(H3) hold, then* (1.1) *has at least one positive ω-periodic solution.*

Theorem 2.2. *Assume that (H1),(P2) and (P3) hold, then* (1.2) *has at least one positive ω-periodic solution.*

Remark 2.3. *When $c = 0$, (H3) and (P3) hold obviously. In this case, Theorem 2.1 extends and improves the corresponding results in* [14, Theorem 2.1] *and* [9, Theorem 2.1]*, Meanwhile, Theorem 2.2 improves the corresponding results in* [8, Theorem 2.1]. *If assumes $a(t) > 0$ in* [14] *and $g(t,x)$ is sub-linear or super-linear in* [9]*, clearly, then the assumptions in Theorem 2.1 and Theorem 2.2 are weaker than theirs.*

We remark that when $c \neq 0$, our main results are new.

Now, we should construct a Banach space X and a cone K. Let $X = \{x(t) : x(t) \in C(\mathbb{R},\mathbb{R}), x(t) = x(t+\omega), \text{for all } t \in \mathbb{R}\}$ and defining $\|x(t)\| = \sup_{t\in[0,\omega]} |x(t)|$, for all $x \in X$. Then X is a Banach space with the norm $\|\cdot\|$. Let $K = \{x \in X : x(t) \geq 0, x(t) \geq \sigma\|x(t)\|\}$, it is not difficult to verify that K is a cone in X.

First, we consider the integral equation

$$x(t) = \int_t^{t+\omega} G(t,s)[g(s,x(s-\tau(s))) - ca(s)x(s-\tau(s))]ds + cx(t-\tau(t)). \quad (2.1)$$

It is easy to see that $\varphi(t)$ is an ω-periodic solution of (1.1) if and only if $\varphi(t)$ is an ω-periodic solution of (2.1).

Define an operator on X, $x = \Phi x$, for $x \in X$, where Φ is given by

$$(\Phi x)(t) = \int_t^{t+\omega} G(t,s)[g(s,x(s-\tau(s))) - ca(s)x(s-\tau(s))]ds + cx(t-\tau(t)). \quad (2.2)$$

Clearly, Φ is not a completely continuous operator on X, since cx is not a completely continuous operator on X. Since Ω_1 and Ω_2 defined in [8, 9, 14] are not suitable to here, we should construct two different sets Ω_1 and Ω_2.

Proof of Theorem 2.1. We define

$$\Omega_1 := \{x \in X : \|x\| < r_1, \|x'\| < \bar{r}_1\},$$
$$\Omega_2 := \{x \in X : \|x\| < r_2, \|x'\| < \bar{r}_2\},$$

where $\bar{r}_1 = \frac{\|a(t)\|r_1+m(r_1)}{1-c}$ and $\bar{r}_2 = \frac{\|a(t)\|r_2+m(r_2)}{1-c}$, where r_1 and r_2 are given in above. Obviously, $0 \in \Omega_1, \bar{\Omega}_1 \subset \Omega_2$.

We will show that Φ is a completely continuous operator on Ω_1 and Ω_2, respectively. It is not difficult to see $\Phi(\Omega_1)$ is a uniformly bounded set and Φ is continuous on Ω_1, so it suffices to show $\Phi(\Omega_1)$ is equi-continuous by Ascoli-Arzela theorem. For any $x \in \Omega_1$, by (2.2), we have

$$\|(\Phi x)'(t)\| \le \|a(t)\|r_1 + \|g(t,x)\| + c\|x'\| \le \|a(t)\|r_1 + m(r_1) + c\bar{r}_1 \le \bar{r}_1.$$

This implies $\Phi(\Omega_1)$ is equi-continuous. So Φ is a completely continuous operator on Ω_1.

Thus, if $x \in K \cap \partial\Omega_1$, then $x(t) \ge \sigma r_1$ and $\|x\| = r_1, \|x'\| \le \bar{r}_1$ or $\|x\| \le r_1, \|x'\| = \bar{r}_1$. It follows from (2.2) and $(H_1), (H_2)$, either $\|x\| = r_1, \|x'\| \le \bar{r}_1$ or $\|x\| \le r_1, \|x'\| = \bar{r}_1$, we all have

$$\begin{aligned}
(\Phi x)(t) &\ge A\int_t^{t+\omega} (g(s, x(s-\tau(s))) - ca(s)x(s-\tau(s))]ds + cx(t-\tau(t)) \\
&\ge A\int_0^{\omega} [\alpha(s) - ca(s)]x(s-\tau(s))ds + cx(t-\tau(t)) \\
&\ge A\sigma r_1 \int_0^{\omega} [\alpha(s) - ca(s)]ds + c\sigma r_1 > r_1,
\end{aligned}$$

which implies that $\|\Phi x\| > \|x\|$ for $x \in K \cap \partial\Omega_1$.

On the other hand, by using the same type of argument as in above, we will obtain that Φ is a completely continuous operator on Ω_2.

Thus, if $x \in K \cap \partial\Omega_2$, then $\|x\| = r_2, \|x'\| \le \bar{r}_2$ or $\|x\| \le r_2, \|x'\| = \bar{r}_2$. It follows from (2.2) and $(H3)$, either $\|x\| = r_2$, $\|x'\| \le \bar{r}_2$ or $\|x\| \le r_2$, $\|x'\| = \bar{r}_2$. We have

$$\begin{aligned}
(\Phi x)(t) &\le B\int_t^{t+\omega} (g(s, x(s-\tau(s))) - ca(s)x(s-\tau(s))]ds + cx(t-\tau(t)) \\
&\le B\int_{x(t-\tau(t))\le n} [g(t, x(s-\tau(s))) - ca(s)x(s-\tau(s))]ds \\
&\quad + B\int_{x(t-\tau(t))>n} [g(t, x(s-\tau(s))) - ca(s)x(s-\tau(s))]ds + cx(t-\tau(t)) \\
&\le Bm + Br_2\int_0^{\omega} [\beta(t) - ca(t)]dt + cr_2 < r_2.
\end{aligned}$$

This implies $\|\Phi x\| < \|x\|$ for $x \in K \cap \partial\Omega_2$ and $\Phi(\Omega_2) \subseteq \bar{\Omega}_2$. Next, we prove that

$$\Phi : K \cap (\bar{\Omega}_2 \setminus \Omega_1) \to K.$$

For any $x \in K \cap (\bar{\Omega}_2 \setminus \Omega_1)$, we have

$$\|\Phi x\| \le B\int_t^{t+\omega} [g(s, x(s-\tau(s))) - ca(s)x(s-\tau(s))]ds + cx(t-\tau(t))$$

and

$$(\Phi x)(t) \ge A\int_t^{t+\omega} [g(s, x(s-\tau(s))) - ca(s)x(s-\tau(s)))]ds + cx(t-\tau(t)).$$

So, we have

$$\begin{aligned}(\Phi x)(t) &\geq \frac{A}{B}[B\int_t^{t+\omega}(g(s,x(s-\tau(s)))-ca(s)x(s-\tau(s)))ds+cx(t-\tau(t))]\\ &\quad + c(1-\frac{A}{B})x(t-\tau(t))\\ &\geq \sigma\|\Phi x\|+c(1-\sigma)x(t-\tau(t))\geq \sigma\|\Phi x\|.\end{aligned}$$

Hence $(\Phi x)(t)\geq 0$ and $(\Phi x)(t)\in K$ for all $x(t)\in K\cap(\bar{\Omega}_2\setminus\Omega_1)$, i.e., $\Phi(K\cap(\bar{\Omega}_2\setminus\Omega_1))\subset K$.

From the above arguments, we know $\Phi: K\cap(\bar{\Omega}_2\setminus\Omega_1)\to K$ is a completely continuous operator. Therefore, Φ has a fixed point $x\in K\cap(\bar{\Omega}_2\setminus\Omega_1)$ by Lemma 1.1. Furthermore, $r_1\leq\|x\|\leq r_2$ and $x(t)\geq\sigma r_1>0$, which means $x(t)$ is a positive ω-periodic solution of (1.1). □

Next, we consider the integral equation

$$\begin{aligned}x(t)=\int_t^{t+\omega}&G(t,s)[b(s)\int_{-\infty}^0 K(r)g(s,x(s+r))dr\\ &-ca(s)\int_{-\infty}^0 K(r)x(s+r)dr]ds+c\int_{-\infty}^0 K(r)x(t+r)dr.\end{aligned}\tag{2.3}$$

Similarly, we see that $\varphi(t)$ is an ω-periodic solution of (1.2) if and only if $\varphi(t)$ is an ω-periodic solution of above equation.

Define an operator on X $x=\Psi x$, for $x\in X$, where Ψ is given by

$$\begin{aligned}(\Psi x)(t)=\int_t^{t+\omega}&G(t,s)[b(s)\int_{-\infty}^0 K(r)g(s,x(s+r))dr\\ &-ca(s)\int_{-\infty}^0 K(r)x(s+r)dr]ds+c\int_{-\infty}^0 K(r)x(t+r)dr.\end{aligned}\tag{2.4}$$

Proof of Theorem 2.2. We define

$$\Omega_1:=\{x\in X:\|x\|<R_1,\|x'\|<\bar{R}_1\},$$
$$\Omega_2:=\{x\in X:\|x\|<R_2,\|x'\|<\bar{R}_2\},$$

where $\bar{R}_1=\frac{\|a(t)\|R_1+m(R_1)}{1-c}$ and $\bar{R}_2=\frac{\|a(t)\|R_2+m(R_2)}{1-c}$, where R_1 and R_2 are given in above. Obviously, $0\in\Omega_1,\bar{\Omega}_1\subset\Omega_2$.

Next, by using the same arguments in the proof of Theorem 2.1, one can obtain that the operator Ψ satisfies all the conditions in Lemma 1.1. Therefore, Ψ has a fixed point $x\in K\cap(\bar{\Omega}_2\setminus\Omega_1)$. Furthermore, $R_1\leq\|x\|\leq R_2$ and $x(t)\geq\sigma R_1>0$, which means $x(t)$ is a positive ω-periodic solution of (1.2). □

3. Some Applications

In this section, we apply the results obtained in previous section to the study equations (1.3)-(1.10). In view of Theorem 2.1 and Theorem 2.2, we obtain the following results.

Theorem 3.1. *Assume that*

(1) $a,\tau\in C(\mathbb{R};\mathbb{R})$, $\beta,b\in C(\mathbb{R};(0,\infty))$, $\int_0^\omega a(t)dt>0$, *and* $a(t),\beta(t),\tau(t)$ *are* ω*-periodic functions,* $\omega>0$ *and* $c\in[0,1)$ *are two constants.*

(2) *$b(t)e^{-\beta(t)x} \geq ca(t)x$ for all $(t,x) \in \mathbb{R} \times [0, r_2]$, where the definition of r_2 is similar to (H3) in section 1.*

Then (1.3) *has at least one positive ω-periodic solution.*

Theorem 3.2. *Assume that*

(1) *$a, \tau \in C(\mathbb{R};\mathbb{R})$, $b \in C(\mathbb{R};(0,\infty))$, $\int_0^\omega a(t)dt > 0$, and $a(t), \tau(t)$ are all ω-periodic functions, $\omega > 0$ and $c \in [0,1)$ are two constants.*

(2) *$b(t)\frac{1}{1+x^n} \geq ca(t)x$ for all $(t,x) \in \mathbb{R} \times [0, r_2]$, where the definition of r_2 is similar to (H3) in section 1.*

Then (1.5) *has at least one positive ω-periodic solution.*

Theorem 3.3. *Assume (1) in Theorem 3.2 holds and*

(2) *$b(t)\frac{1}{1+x^n} \geq ca(t)$ for all $(t,x) \in \mathbb{R} \times [0, r_2]$, where the definition of r_2 is similar to (H3) in section 1.*

Then (1.6) *has at least one positive ω-periodic solution.*

Theorem 3.4. *Assume (1) in Theorem 3.1 holds and*

(2) *$b(t)e^{-\beta(t)x} \geq ca(t)$ for all $(t,x) \in \mathbb{R} \times [0, r_2]$, where the definition of r_2 is similar to (H3) in section 1.*

Then (1.9) *has at least one positive ω-periodic solution.*

Theorem 3.5. *Assume that*

(1) *$a \in C(\mathbb{R};\mathbb{R})$, $\int_0^\omega a(t)dt > 0$, $b, \beta \in C(\mathbb{R};(0,\infty))$, and $a(t), b(t), \beta(t)$ are all ω-periodic functions, $\omega > 0$, $0 \leq c < 1$ are constants. Moreover, $K(r) \in C((-\infty,0],[0,\infty))$ and $\int_{-\infty}^0 K(r)dr = 1$.*

(2) *$e^{-\beta(t)x} \geq \frac{ca(t)}{b(t)}x$ for all $(t,x) \in \mathbb{R} \times [0, R_2]$, where the definition of R_2 is similar to (P3) in section 1.*

Then (1.4) *has at least one positive ω-periodic solution.*

Theorem 3.6. *Assume that*

(1) *$a \in C(\mathbb{R};\mathbb{R})$, $\int_0^\omega a(t)dt > 0, b \in C(\mathbb{R};(0,\infty))$ and $a(t), b(t)$ are all ω-periodic functions, $\omega > 0$ and $c[0,1)$ are two constants. Moreover, $K(r) \in C((-\infty,0],[0,\infty))$ and $\int_{-\infty}^0 K(r)dr = 1$.*

(2) *$\frac{1}{1+x^n} \geq \frac{ca(t)}{b(t)}x$ for all $(t,x) \in \mathbb{R} \times [0, R_2]$, where the definition of R_2 is similar to (P3) in section 1.*

Then (1.7) *has at least one positive ω-periodic solution.*

Theorem 3.7. *Assume (1) in Theorem 3.6 holds and*

(2) *$\frac{1}{1+x^n} \geq \frac{ca(t)}{b(t)}$ for all $(t,x) \in \mathbb{R} \times [0, R_2]$, where the definition of R_2 is similar to (P3) in section 1.*

Then (1.8) *has at least one positive ω-periodic solution.*

Theorem 3.8. *Assume (1) in Theorem 3.5 holds and*

(2) *$e^{-\beta(t)x} \geq \frac{ca(t)}{b(t)}$ for all $(t,x) \in \mathbb{R} \times [0, R_2]$, where the definition of R_2 is similar to (P3) in section 1.*

Then (1.10) *has at least one positive ω-periodic solution.*

We remark that when $c = 0$, Theorems 3.1–3.8 improve the results in [8, 9, 14].

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Addendum: Posted April 17, 2007

Professor Youssef N. Raffoul pointed out that the proof of the main result in this article is incorrect: Because the sets ω_1 and ω_2 are not open in the Banach space X, Krasnoselskii's fixed-point theorem in cones can not be applied.

We encourage the readers to find (and publish) a proof for the existence of periodic solutions to neutral functional differential equations.

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