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CONVERGENCE OF COHEN-GROSSBERG NEURAL NETWORKS WITH DELAYS AND TIME-VARYING COEFFICIENTS

QIYUAN ZHOU, JIANYING SHAO

Abstract. In this paper presents sufficient conditions for all solutions of the Cohen-Grossberg neural networks with delays and time-varying coefficients to converge exponentially to zero.

1. Introduction

Consider the Cohen-Grossberg neural network (CGNN), with delay and time-varying coefficients,

$$\begin{aligned}\dot{x}_i(t) = -a_i(t, x_i(t))\Big[&b_i(t, x_i(t)) - \sum_{j=1}^{n} c_{ij}(t) f_j(x_j(t-\tau_{ij}(t))) \\ &- \sum_{j=1}^{n} d_{ij}(t) g_j\Big(\int_0^{\infty} K_{ij}(u) x_j(t-u)du\Big) + I_i(t)\Big], \quad i = 1, 2, \dots, n,\end{aligned} \tag{1.1}$$

where a_i and b_i are continuous functions on $\mathbb{R}^2$, f_j, g_j, c_{ij}, d_{ij} and I_i are continuous functions on $\mathbb{R}$; n corresponds to the number of units in a neural network; $x_i(t)$ denotes the potential (or voltage) of cell i at time t; a_i represents an amplification function; b_i is an appropriately behaved function; $c_{ij}(t)$ and $d_{ij}(t)$ denote the strengths of connectivity between cell i and j at time t respectively; the activation functions $f_i(\cdot)$ and $g_i(\cdot)$ show how the ith neuron reacts to the input, $\tau_{ij} \geq 0$ corresponds to the transmission delay of the ith unit along the axon of the jth unit at the time t, and $I_i(t)$ denotes the ith component of an external input source introduced from outside the network to cell i at time t for $i, j = 1, 2, \dots, n$.

Since the model CGNNs was introduced by Cohen and Grossberg [3], the dynamical characteristics (including stable, unstable and periodic oscillatory) of CGNNs have been widely investigated for the sake of theoretical interest as well as application considerations. Many good results on the problem of the existence and

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stability of the equilibriums for system (1.1) are given out in the literature. We refer the reader to the references in this article and the references cited therein. Suppose that the following conditions are satisfied.

(H0) $a_i(t, x_i) = a_i(0, x_i)$ and $b_i(t, x_i) = b_i(0, x_i)$ for all t, and $c_{ij}, d_{ij}, I_j, : \mathbb{R} \to \mathbb{R}$ are constants, where $i, j = 1, 2, \dots, n$.

(H0*) For each $j \in \{1, 2, \dots, n\}$, there exist nonnegative constants $\tilde{L}_j$ and L_j such that

$$|f_j(u) - f_j(v)| \le \tilde{L}_j|u - v|, \quad |g_j(u) - g_j(v)| \le L_j|u - v|, \quad \forall u, v \in \mathbb{R}.$$

Most authors of bibliographies listed above obtained that all solutions of system (1.1) converge to the equilibrium point. However, to the best of our knowledge, no author has considered the convergence of all solutions without assumptions (H0) and (H0*). Thus, it is worth to investigate the convergence for (1.1) in this case. The main purpose of this paper is to give a new criteria for the convergence for all solutions of (1.1). By applying mathematical analysis techniques, without assuming (H0) and (H0*), we derive some sufficient conditions ensuring that all solutions of (1.1) converge exponentially to zero, which are new and complement of previously known results. Moreover, we provide an example that illustrates our results.

Throughout this paper, for $i, j = 1, 2, \dots, n$, it will be assumed that $K_{ij} : [0, +\infty) \to \mathbb{R}$ are continuous functions, and there exists a constant τ such that

$$\tau = \max_{1 \le i,j \le n} \big\{ \sup_{t \in \mathbb{R}} \tau_{ij}(t) \big\}. \tag{1.2}$$

We also assume that the following conditions.

(H1) For each $j \in \{1, 2, \dots, n\}$, there exist nonnegative constants $\tilde{L}_j$ and L_j such that

$$|f_j(u)| \le \tilde{L}_j|u|, \quad |g_j(u)| \le L_j|u|, \quad \forall u \in \mathbb{R}. \tag{1.3}$$

(H2) For $i = 1, 2, \dots, n$, there exist positive constants such that $\underline{a_i}$, $\overline{a_i}$ and T_1 such that

$$\underline{a_i} \le a_i(t, u) \le \overline{a_i}, \quad \text{for all } t > T_1, \ u \in \mathbb{R}.$$

(H3) For $i = 1, 2, \dots, n$, there exist positive constants $\underline{b_i}$ and T_2 such that

$$\underline{b}_i|u| \le \operatorname{sign}(u) b_i(t, u), \quad \text{for all } t > T_2, \ u \in \mathbb{R}.$$

(H4) There exist constants $T_3 > 0, \eta > 0, \lambda > 0$ and $\xi_i > 0$, $i = 1, 2, \dots, n$, such that for all $t > T_3$,

$$-[\underline{a}_i\underline{b}_i - \lambda]\xi_i + \sum_{j=1}^n |c_{ij}(t)|\overline{a_i} e^{\lambda\tau}\tilde{L}_j\xi_j + \sum_{j=1}^n |d_{ij}(t)|\overline{a_i} \int_0^\infty |K_{ij}(u)|e^{\lambda u} du L_j \xi_j < -\eta < 0,$$

where $i = 1, 2, \dots, n$.

(H5) $I_i(t) = O(e^{-\lambda t})$, $i = 1, 2, \dots, n$.

The initial conditions associated with (1.1) are

$$x_i(s) = \varphi_i(s), s \in (-\infty, 0], \quad i = 1, 2, \dots, n, \tag{1.4}$$

where $\varphi_i(\cdot)$ denotes a real-valued bounded continuous function defined on $(-\infty, 0]$. For $Z(t) = (x_1(t), x_2(t), \dots, x_n(t))^T$, we define the norm

$$\|Z(t)\|_\xi = \max_{i=1,2,\dots,n} |\xi_i^{-1} x_i(t)|.$$

The remaining part of this paper is organized as follows. In Section 2, we present sufficient conditions to ensure that all solutions of (1.1) converge exponentially to the zero. In Section 3, we shall give some examples and remarks to illustrate the results obtained in the previous sections.

2. Main Results

Theorem 2.1. *Assume that* (H1)–(H5) *hold. Then every solution*
$$Z(t) = (x_1(t), x_2(t), \dots, x_n(t))^T$$
of (1.1)*, corresponding to any initial value* $\varphi = (\varphi_1(t), \varphi_2(t), \dots, \varphi_n(t))^T$*, satisfies*
$$x_i(t) = O(e^{-\lambda t}), \quad i = 1, 2, \dots, n.$$

Proof. From (H5), we can choose constants $F > 0$ and $T > \max\{T_1, T_2, T_3\}$ such that
$$\overline{a_i}|I_i(t)| < \frac{1}{2}Fe^{-\lambda t}, \quad \text{for all } t \geq T,\ i = 1, 2, \dots, n. \tag{2.1}$$
Set $Z(t) = (x_1(t), x_2(t), \dots, x_n(t))^T$ be a solution of (1.1) with any initial value $\varphi = (\varphi_1(t), \varphi_2(t), \dots, \varphi_n(t))^T$, and let i_t be an index such that
$$\xi_{i_t}^{-1}|x_{i_t}(t)| = \|Z(t)\|_\xi. \tag{2.2}$$
Calculating the upper right derivative of $e^{\lambda s}|x_{i_s}(s)|$ along (1.1), in view of (2.1), (H1), (H2) and (H3), we have
$$\begin{aligned}
&D^+(e^{\lambda s}|x_{i_s}(s)|)|_{s=t}\\
&= \lambda e^{\lambda t}|x_{i_t}(t)| + e^{\lambda t}\operatorname{sign}(x_{i_t}(t))\{-a_{i_t}(t, x_{i_t}(t))[b_{i_t}(t, x_{i_t}(t))\\
&\quad - \sum_{j=1}^n c_{i_t j}(t) f_j(x_j(t - \tau_{i_t j}(t)))\\
&\quad - \sum_{j=1}^n d_{i_t j}(t) g_j(\int_0^\infty K_{i_t j}(u) x_j(t-u) du) + I_{i_t}(t)]\}\\
&\leq e^{\lambda t}\{-(\underline{a}_{i_t}\underline{b}_{i_t} - \lambda)|x_{i_t}(t)|\xi_{i_t}^{-1}\xi_{i_t} + \sum_{j=1}^n |c_{i_t j}(t)|\overline{a_i}\tilde{L}_j|x_j(t - \tau_{i_t j}(t))|\xi_j^{-1}\xi_j\\
&\quad + \sum_{j=1}^n |d_{i_t j}(t)|\overline{a_i}L_j\int_0^\infty |K_{i_t j}(u)||x_j(t-u)|\xi_j^{-1} du\xi_j\} + \frac{1}{2}Fe^{-\lambda t}e^{\lambda t},
\end{aligned} \tag{2.3}$$
where $t > T$. Let
$$M(t) = \max_{s \leq t}\{e^{\lambda s}\|Z(s)\|_\xi\}. \tag{2.4}$$
It is obvious that $e^{\lambda t}\|Z(t)\|_\xi \leq M(t)$, and $M(t)$ is non-decreasing. Now, we consider two cases.

Case (i). If
$$M(t) > e^{\lambda t}\|Z(t)\|_\xi \quad \text{for all } t \geq T. \tag{2.5}$$
Then, we claim that
$$M(t) \equiv M(T) \quad \text{is constant for all } t \geq T. \tag{2.6}$$

Assume, by way of contradiction, that (2.6) does not hold. Then, there exists $t_1 > T$ such that $M(t_1) > M(T)$. Since

$$e^{\lambda t}\|Z(t)\|_\xi \le M(T) \quad \text{for all } t \le T,$$

there must exist $\beta \in (T, t_1)$ such that

$$e^{\lambda\beta}\|Z(\beta)\|_\xi = M(t_1) \ge M(\beta),$$

which contradicts to (2.5). This contradiction implies that (2.6) holds. It follows that

$$e^{\lambda t}\|Z(t)\|_\xi < M(t) = M(T) \text{ for all } t \ge T. \tag{2.7}$$

Case (ii). If there is a point $t_0 \ge T$ such that $M(t_0) = e^{\lambda t_0}\|Z(t_0)\|_\xi$. Then, in view of (2.3) and (H4), we obtain

$$\begin{aligned}
& D^+(e^{\lambda s}|x_{i_s}(s)|)|_{s=t_0} \\
& \le e^{\lambda t_0}\{-(\underline{a}_{i_{t_0}}\underline{b}_{i_{t_0}} - \lambda)|x_{i_{t_0}}(t_0)|\xi_{i_{t_0}}^{-1}\xi_{i_{t_0}} \\
& \quad + \sum_{j=1}^n |c_{i_{t_0}j}(t_0)|\overline{a_{i_{t_0}}}\tilde{L}_j|x_j(t_0 - \tau_{i_{t_0}j}(t_0))|\xi_j^{-1}\xi_j \\
& \quad + \sum_{j=1}^n |d_{i_{t_0}j}(t_0)|\overline{a_{i_{t_0}}}L_j\int_0^\infty |K_{i_{t_0}j}(u)||x_j(t_0-u)|\xi_j^{-1}du\xi_j\} + \frac{1}{2}F \\
& = -(\underline{a}_{i_{t_0}}\underline{b}_{i_{t_0}} - \lambda)|x_{i_{t_0}}(t_0)|e^{\lambda t_0}\xi_{i_{t_0}}^{-1}\xi_{i_{t_0}} \\
& \quad + \sum_{j=1}^n |c_{i_{t_0}j}(t_0)|\overline{a_{i_{t_0}}}\tilde{L}_j|x_j(t_0 - \tau_{i_{t_0}j}(t_0))|e^{\lambda(t_0-\tau_{i_{t_0}j}(t_0))}\xi_j^{-1}e^{\lambda\tau_{i_{t_0}j}(t_0)}\xi_j \\
& \quad + \sum_{j=1}^n |d_{i_{t_0}j}(t_0)|\overline{a_{i_{t_0}}}L_j\int_0^\infty |K_{i_{t_0}j}(u)|e^{\lambda u}|x_j(t_0-u)|e^{\lambda(t_0-u)}\xi_j^{-1}du\xi_j + \frac{1}{2}F \\
& \le \{-(\underline{a}_{i_{t_0}}\underline{b}_{i_{t_0}} - \lambda)\xi_{i_{t_0}} + \sum_{j=1}^n |c_{i_{t_0}j}(t_0)|\overline{a_{i_{t_0}}}\tilde{L}_j e^{\lambda\tau}\xi_j \\
& \quad + \sum_{j=1}^n |d_{i_{t_0}j}(t_0)|\overline{a_{i_{t_0}}}L_j\int_0^\infty |K_{i_{t_0}j}(u)|e^{\lambda u}du\xi_j\}M(t_0) + \frac{1}{2}F \\
& < -\frac{1}{2}\eta M(t_0) + F.
\end{aligned} \tag{2.8}$$

In addition, if $M(t_0) \ge 2\frac{F}{\eta}$, then $M(t)$ is strictly decreasing in a small neighborhood $(t_0, t_0+\delta_0)$. This contradicts that $M(t)$ is non-decreasing. Hence,

$$e^{\lambda t_0}\|Z(t_0)\|_\xi = M(t_0) < 2\frac{F}{\eta}. \tag{2.9}$$

For $t > t_0$, by the same approach as the one used in the proof of (2.9), we have

$$e^{\lambda t}\|Z(t)\|_\xi < 2\frac{F}{\eta}, \quad \text{if } M(t) = e^{\lambda t}\|Z(t)\|_\xi. \tag{2.10}$$

On the other hand, if $M(t) > e^{\lambda t}\|Z(t)\|_\xi, t > t_0$, we can choose $t_0 \le t_2 < t$ such that

$$M(t_2) = e^{\lambda t_2}\|Z(t_2)\|_\xi < 2\frac{F}{\eta}, \quad M(s) > e^{\lambda s}\|Z(s)\|_\xi \quad \text{for all } s \in (t_2, t].$$

Using a similar argument as in the proof of **Case (i)**, we can show that

$$M(s) \equiv M(t_2) \text{ is constant for all } s \in (t_2, t], \tag{2.11}$$

which implies

$$e^{\lambda t}\|Z(t)\|_\xi < M(t) = M(t_2) < 2\frac{F}{\eta}.$$

In summary, there must exist $N > 0$ such that $e^{\lambda t}\|Z(t)\|_\xi < \max\{M(T), 2\frac{F}{\eta}\}$ holds for all $t > N$. The proof is complete. □

3. An Example

In this section, we give an example to demonstrate the results obtained in previous sections. Consider the CGNN with delays and time-varying coefficients

$$\begin{aligned}
x_1'(t) &= -(2 + e^{\cos t}\frac{1}{10\pi}\arctan x_1(t))[(2 - \frac{(100+|t|)\sin t}{1+2|t|})(x_1(t) + 5x_1^3(t)) \\
&\quad + \frac{1}{8}\frac{(101+|t|)\sin t}{1+4|t|}f_1(x_1(t - 2\sin^2 t)) + \frac{1}{8}\frac{(102+|t|)\sin t}{1+36|t|} \\
&\quad \times f_2(x_2(t - 3\sin^2 t)) + \frac{1}{8}\frac{(103+|t|)\sin t}{1+4|t|}\int_0^\infty e^{-u}g_1(x_j(t-u))du \\
&\quad + \frac{1}{8}\frac{(100+|t|^2)\sin t}{1+36|t|^2}\int_0^\infty e^{-u}g_2(x_j(t-u))du + e^{-3t}\sin t], \\
x_2'(t) &= -(2 + e^{\sin t}\frac{1}{10\pi}\arctan x_2(t))[(4 - \frac{(200+|t|)\cos t}{1+2|t|})(x_2(t) + 15x_2^3(t)) \\
&\quad + \frac{1}{8}\frac{(200+|t|)\cos t}{1+8|t|}f_1(x_1(t - 2\sin^2 t)) \\
&\quad + \frac{1}{8}\frac{(206+|t|)\cos t}{1+5|t|}f(x_2(t - 5\sin^2 t)) \\
&\quad + \frac{1}{8}\frac{(205+|t|)\cos t}{1+6|t|}\int_0^\infty e^{-u}g_1(x_j(t-u))du \\
&\quad + \frac{1}{8}\frac{(204+|t|)\cos t}{1+7|t|}\int_0^\infty e^{-u}g_2(x_j(t-u))du + e^{-t}\sin t],
\end{aligned} \tag{3.1}$$

where $f_1(x) = f_2(x) = g_1(x) = g_2(x) = x\sin x$. Noting that

$$a_1(t,x) = (2 + e^{\cos t}\frac{1}{10\pi}\arctan x_1), \quad a_2(t,x) = (2 + e^{\sin t}\frac{1}{10\pi}\arctan x_2),$$

$$b_1(t,x) = (2 - \frac{(100+|t|)\sin t}{1+2|t|})(x_1 + 5x_1^3),$$

$$b_2(t,x) = (4 - \frac{(200+|t|)\cos t}{1+2|t|})(x_2(t) + 15x_2^3),$$

$$L_1 = L_2 = \tilde{L}_1 = \tilde{L}_2 = 1, \quad \tau = 5, \quad K_{ij}(u) = e^{-u}, \quad i,j = 1,2,$$

$$c_{11}(t) = \frac{1}{8}\frac{(101+|t|)\sin t}{1+4|t|}, \quad d_{11}(t) = \frac{1}{8}\frac{(103+|t|)\sin t}{1+4|t|},$$

$$c_{12}(t) = \frac{1}{8}\frac{(102+|t|)\sin t}{1+36|t|}, \quad d_{12}(t) = \frac{1}{8}\frac{(100+|t|^2)\sin t}{1+36|t|^2},$$

$$
\begin{aligned}
c_{21}(t) &= \frac{1}{8}\frac{(200+|t|)\cos t}{1+8|t|}, \quad d_{21}(t) = \frac{1}{8}\frac{(205+|t|)\cos t}{1+6|t|},\\
c_{22}(t) &= \frac{1}{8}\frac{(206+|t|)\cos t}{1+5|t|}, \quad d_{22}(t) = \frac{1}{8}\frac{(204+|t|)\cos t}{1+7|t|}\,.
\end{aligned}
$$

It follows that

$$
\begin{gathered}
1 = \underline{a_i} \le a_i(t,u) \le \overline{a_i} = 4, \quad \text{for all } t,u \in \mathbb{R}, i = 1,2;\\
|u| = \underline{b}_i|u| \le \operatorname{sign}(u) b_i(t,u), \quad \text{for all } t,u \in \mathbb{R},\ i = 1,2.
\end{gathered}
$$

Then, we can choose a sufficient large constant $T_0 > 0$ and a positive constant $\bar{\eta} = \frac{1}{2}$ and $\xi_i = 1$, $i = 1,2$, such that for all $t > T_0$, there holds

$$
\begin{aligned}
&- \underline{a}_i\underline{b}_i\xi_i + \sum_{j=1}^{2}|c_{ij}(t)|\overline{a_i}\tilde{L}_j\xi_j + \sum_{j=1}^{2}|d_{ij}(t)|\overline{a_i}\int_0^\infty |K_{ij}(u)|du L_j\xi_j\\
&= -\underline{a}_i\underline{b}_i + \sum_{j=1}^{2}|c_{ij}(t)|\overline{a_i}\tilde{L}_j + \sum_{j=1}^{2}|d_{ij}(t)|\overline{a_i}L_j\\
&= -1 + 4\sum_{j=1}^{2}|c_{ij}(t)| + 4\sum_{j=1}^{2}|d_{ij}(t)| < -\frac{1}{2} = -\bar{\eta} < 0, i = 1,2.
\end{aligned}
$$

Then, we can choose constants $\eta > 0$ and $\lambda > 0$ such that

$$
\begin{aligned}
&- [\underline{a}_i\underline{b}_i - \lambda]\xi_i + \sum_{j=1}^{2}|c_{ij}(t)|\overline{a_i}e^{\lambda\tau}\tilde{L}_j\xi_j + \sum_{j=1}^{2}|d_{ij}(t)|\overline{a_i}\int_0^\infty |K_{ij}(u)|e^{\lambda u}du L_j\xi_j\\
&< -\eta < 0, \quad i = 1,2, t > T_0,
\end{aligned}
$$

which implies that (3.1) satisfies (H1)–(H5). Hence, from Theorem 2.1, all solutions of system (3.1) converge exponentially to the zero point $(0,0,\dots,0)^T$.

Remark 3.1. Since $f_1(x) = f_2(x) = g_1(x) = g_2(x) = x\sin x$ and (3.1) is a very simple form of delayed Cohen-Grossberg neural network with time-varying coefficients. It is clear that the conditions (H0) and (H0*) are not satisfied. Therefore, the results in the references of this article are not applicable for proving that the solutions to (3.1) converge exponentially to the zero. This implies that the results of this paper are essentially new.

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Qiyuan Zhou
Department of Mathematics, Hunan University of Arts and Science, Changde, Hunan 415000, China
E-mail address: zhouqiyuan65@yahoo.com.cn

Jianying Shao
College of Mathematics and Information Engineering, Jiaxing University, Jiaxing, Zhejiang 314001, China
E-mail address: shaojianying2008@yahoo.cn