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ON SOLUTIONS OF A VOLTERRA INTEGRAL EQUATION WITH DEVIATING ARGUMENTS

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ABSTRACT. In this article, we establish the existence and asymptotic characterization of solutions to a nonlinear Volterra integral equation with deviating arguments. Our proof is based on measure of noncompactness and the Schauder fixed point theorem.

1. INTRODUCTION

Several authors have studied nonlinear Volterra integral equations with deviating arguments and functional integral equations; see for example [1, 4, 5, 11, 13, 14]. Banaś [3] proved an existence theorem for functional integral equation while Balachandran and Illamaran [1] proved an existence theorem for Volterra integral equation with a deviating argument. Existence of solutions to nonlinear integral equations, which contain particular cases of important integral and functional equations such as nonlinear Volterra integral equation, Urysohn integral equation and integral equations of Chandrasekhar type, have been considered in many papers and monographs [8, 12, 15].

In this paper we study the nonlinear Volterrra integral equation

$$x(t) = g\big(t, x(h_1(t)), \dots, x(h_n(t))\big) + \int_0^t k\big(t, s, x(H_1(s)), \dots, x(H_m(s))\big)ds, \quad (1.1)$$

where $t \geq 0$. In particular, we prove the existence and asymptotic stability of solutions for this equation. The investigation is done on the space of continuous and tempered functions on $\mathbb{R}^n$. The main tool used in our considerations is the measure of noncompactness and the Schauder fixed point theorem. The results obtained in this paper generalize several results presented in [6, 9, 10, 16].

2. NOTATION AND AUXILIARY RESULTS

Suppose E is a real Banach space with the norm $\|\cdot\|$ and zero element θ. Denote by $B(x, r)$ the closed ball centered at x and with radius r. We write B_r for the ball $B(\theta, r)$. If X is a subset of E then the symbols $\overline{X}$ and $\operatorname{Conv} X$ stand for the closure and convex closure of X, respectively. Further, let $\mathcal{M}_E$ denote the family

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of all nonempty and bounded subsets of E and $\mathcal{N}_E$ its subfamily consisting of all relatively compact sets.

We define the the following notion of measure of noncompactness.

Definition 2.1. A mapping $\mu : \mathcal{M}_E \to \mathbb{R}_+$ is said to be a measure of noncompactness in the space E if it satisfies the following conditions:

(i) The family $\ker\mu = \{X \in \mathcal{M}_E : \mu(X) = 0\}$ is nonempty and $\ker\mu \subset \mathcal{N}_E$;
(ii) $X \subset Y \Rightarrow \mu(X) \le \mu(Y)$;
(iii) $\mu(\overline{X}) = \mu(\operatorname{Conv} X) = \mu(X)$;
(iv) $\mu(\lambda X + (1-\lambda)Y) \le \lambda\mu(X) + (1-\lambda)\mu(Y)$ for $\lambda \in [0,1]$;
(v) If (X_n) is a sequence of closed sets from $\mathcal{M}_E$ such that $X_{n+1} \subset X_n$ for $n = 1,2,3\dots$ and if $\lim_{n\to\infty}\mu(X_n) = 0$, then the set $X_\infty = \cap_{n=1}^{\infty} X_n$ is nonempty.

The family $\ker\mu$ defined in axiom (i) is called the kernel of the measure of noncompactness μ.

Let us remark that the intersection set X_∞ from (v) is a member of the kernel of the measure of noncompactness μ. Indeed, from the inequality $\mu(X_\infty) \le \mu(X_n)$ for $n = 1,2,\dots$, we infer that $\mu(X_\infty) = 0$, so $X_\infty \in \ker\mu$. This property of the intersection set X_∞ will be crucial in our study.

Further facts concerning measures of noncompactness and their properties may be found in [2, 7].

Now, let us assume that $p = p(t)$ is a given function defined and continuous on the interval $\mathbb{R}_+$ with real positive values. We will denote by $C\big(\mathbb{R}_+, p(t); \mathbb{R}^n\big) = C_p$, the Banach space consisting of all continuous functions from $\mathbb{R}_+$ into $\mathbb{R}^n$ such that

$$\|x\| = \sup\{|x(t)|p(t) : t \ge 0\} < \infty.$$

Now we recall the definition of the measure of noncompactness in the space C_p which will be used in the sequel [2, 7]. Let X be a nonempty and bounded subset of the space C_p. Fix a positive number T. For $x \in X$ and $\epsilon > 0$ denote by $\omega^T(x,\epsilon)$ the modulus of continuity of the function x(tempered by the function p) on the interval $[0,T]$, i.e.

$$\omega^T(x,\epsilon) = \sup\{|x(t)p(t) - x(s)p(s)| : t,s \in [0,T],\ |t-s| \le \epsilon\}.$$

Further, let us put

$$\omega^T(X,\epsilon) = \sup\{\omega^T(x,\epsilon) : x \in X\}, \quad \omega_0^T(X) = \lim_{\epsilon\to 0}\omega^T(X,\epsilon),$$
$$\omega_0(X) = \lim_{T\to\infty}\omega_0^T(X).$$

Moreover, we put

$$\alpha(X) = \lim_{T\to\infty}\{\sup_{x\in X}\{\sup\{|x(t)|p(t) : t \ge T\}\}\}.$$

Finally, let us the define the function μ on the family $\mathcal{M}_{C_p}$ by formula $\mu(X) = \omega_0(X) + \alpha(X)$.

It may be shown that the function μ is the measure of noncompactness in the space C_p [2, 7]. The kernel $\ker\mu$ is the family of all nonempty and bounded sets X such that functions belonging to X are locally equicontinuous on $\mathbb{R}_+$ and such that $\lim_{t\to\infty} x(t)p(t) = 0$ uniformly with respect to the set X, i.e. for each $\epsilon > 0$ there exists $T > 0$ with the property that $|x(t)|p(t) \le \epsilon$ for $t \ge T$ and for $x \in X$.

Finally, let us assume that $x \in C_p$. For $T > 0$ and denote by $\nu^T(x, \epsilon)$ the usual modulus of continuity of the function x on the interval $[0, T]$:

$$\nu^T(x, \epsilon) = \sup\{|x(t) - x(s)| : t, s \in [0, T],\ |t - s| \le \epsilon\}.$$

3. Main Result

We assume the following conditions:

(H1) The function $g : \mathbb{R}_+ \times R^{n^2} \to \mathbb{R}^n$ is continuous and there exists a constant $K \ge 0$ such that

$$|g(t, x_1, \dots, x_n) - g(t, y_1, \dots, y_n)| \le K \sum_{i=1}^{n} |x_i - y_i|$$

for all $t \in \mathbb{R}_+$ and $(x_1, x_2, \dots, x_n)$, $(y_1, y_2, \dots, y_n) \in R^{n^2}$;

(H2) There exists a continuous function $a : \mathbb{R}_+ \to \mathbb{R}_+$ such that $|g(t, 0, \dots, 0)| \le a(t)$ for all $t \in \mathbb{R}_+$;

(H3) Let $\Delta = \{(t, s) : 0 \le s \le t < \infty\}$. The kernel $k : \Delta \times R^{nm} \to \mathbb{R}^n$ is continuous and there exists a continuous function $L_0 : \mathbb{R}_+ \to \mathbb{R}_+$, a continuous nonincreasing function $L_1 : \mathbb{R}_+ \to R$ and a continuous nondecreasing function $q : \mathbb{R}_+ \to \mathbb{R}_+$ such that

$$|k(t, s, x_1, \dots, x_m)| \le L_0(s) + \exp(L_1(t)) q(s) \sum_{i=1}^{m} |x_i|$$

for all $t, s \in \mathbb{R}_+$ and $(x_1, x_2, \dots, x_m) \in R^{nm}$;

(H4) For $i = 1, 2, \dots n$, $j = 1, 2, \dots m$ the functions $h_i, H_j : \mathbb{R}_+ \to \mathbb{R}_+$ are continuous and $h_i(t) \le t$, $H_j(t) \le t$;

(H5) $\lim_{t\to\infty} t q(t) \exp L_1(t) = 0$ and $\lim_{t\to\infty} [a(t) + b(t)] \exp(-M b(t)) = 0$ where $b(t) = \int_0^t L_0(s) ds$;

Let us define the function $L(t)$ as

$$L(t) = \int_0^t [L_0(s) + q(s) \exp L_1(s)] ds.$$

Fix a constant $M > 1$ and denote by $C_L = C\big(\mathbb{R}_+, \exp(-ML(t)); \mathbb{R}^n\big)$. It is obvious that the function $L(t)$ is nondecreasing and continuous on $\mathbb{R}_+$.

Theorem 3.1. *Under assumptions* (H1)-(H5), *Equation* (1.1) *has at least one solution $x \in C_L$ such that $x(t) = o(\exp(ML(t))$ as $t \to \infty$, provided* $(Kn + m/M) < 1$.

Proof. Consider the operator F defined on the space C_L by the formula

$$(Fx)(t) = g\big(t, x(h_1(t)), \dots, x(h_n(t))\big) + \int_0^t k\big(t, s, x(H_1(s)), \dots, x(H_m(s))\big) ds,$$

where $t \ge 0$. Obviously the function Fx is continuous on the interval $\mathbb{R}_+$. Moreover, in view of our assumptions, for arbitrarily fixed $x \in C_L$ and $t \in \mathbb{R}_+$, we obtain

$$\begin{aligned}
&|(Fx)(t)| \exp(-ML(t)) \\
&\le \Big[\big|g\big(t, x(h_1(t)), \dots, x(h_n(t))\big)\big| \\
&\quad + \int_0^t \big|k\big(t, s, x(H_1(s)), \dots, x(H_m(s))\big)\big| ds\Big] \exp(-ML(t))
\end{aligned}$$

$$
\begin{aligned}
&\le \Big[\Big|g\big(t, x(h_1(t)), \dots, x(h_n(t))\big) - g(t, 0\dots, 0)\Big| + |g(t, 0\dots, 0)|\Big] \exp(-ML(t))\\
&\quad + \Big[\int_0^t L_0(s)ds + \exp(L_1(t)) \int_0^t q(s) \sum_{i=1}^m |x(H_i(s))| ds\Big] \exp(-ML(t))\\
&\le K \sum_{i=1}^n |x(h_i(t))| \exp(-ML(t)) + [a(t) + b(t)] \exp(-ML(t))\\
&\quad + \exp(L_1(t)) \int_0^t q(s) \|x\| \sum_{i=1}^m \exp(ML(H_i(s))) ds \cdot \exp(-ML(t))\\
&\le K \sum_{i=1}^n |x(h_i(t))| \exp(-ML(h_i(t))) + [a(t) + b(t)] \exp(-ML(t))\\
&\quad + m\|x\| \int_0^t [L_0(s) + q(s) \exp(L_1(s))] \exp(ML(s)) ds \cdot \exp(-ML(t))\\
&\le Kn\|x\| + [a(t) + b(t)] \exp(-Mb(t)) + \big(\frac{m}{M}\big)\|x\|\\
&\le \big[Kn + \frac{m}{M}\big]\|x\| + D,
\end{aligned}
$$

where $D = \sup\{[a(t) + b(t)] \exp(-Mb(t)) : t \in \mathbb{R}_+\}$. Obviously $D < \infty$ in view of the hypothesis (H5). The obtained estimate shows that Fx is bounded on $\mathbb{R}_+$. This fact with the continuity of Fx on $\mathbb{R}_+$ yields that F transforms the space C_L into itself. Moreover for $r = D/(1 - Kn - m/M)$, we have that F maps the ball B_r into itself.

Let us take an arbitrary nonempty subset A of the ball B_r. Fix $T > 0$ and $\epsilon > 0$. Next, take arbitrary $t, s \in [0, T]$ with $|t - s| \le \epsilon$. Then, for arbitrarily fixed $x \in A$, we get

$$
\begin{aligned}
&|(Fx)(t) - (Fx)(s)|\\
&\le \big|g\big(t, x(h_1(t)), \dots, x(h_n(t))\big) - g\big(s, x(h_1(s)), \dots, x(h_n(s))\big)\big|\\
&\quad + \Big|\int_0^t k\big(t, \tau, x(H_1(\tau)), \dots, x(H_m(\tau))\big) d\tau\\
&\quad - \int_0^s k\big(s, \tau, x(H_1(\tau)), \dots, x(H_m(\tau))\big) d\tau\Big|\\
&\le \big|g\big(t, x(h_1(t)), \dots, x(h_n(t))\big) - g\big(t, x(h_1(s)), \dots, x(h_n(s))\big)\big|\\
&\quad + \Big|g\big(t, x(h_1(s)), \dots, x(h_n(s))\big) - g\big(s, x(h_1(s)), \dots, x(h_n(s))\big)\Big|\\
&\quad + \int_0^t \Big|k\big(t, \tau, x(H_1(\tau)), \dots, x(H_m(\tau))\big) - k\big(s, \tau, x(H_1(\tau)), \dots, x(H_m(\tau))\big)\Big| d\tau\\
&\quad + \int_s^t \Big|k\big(s, \tau, x(H_1(\tau)), \dots, x(H_m(\tau))\big)\Big| d\tau \qquad (3.1)\\
&\le K \sum_{i=1}^n |x(h_i(t)) - x(h_i(s))| + \nu_1^T(g, \epsilon) + T\nu_1^T(k, \epsilon)\\
&\quad + \epsilon \sup\Big\{\big|k\big(s, \tau, x(H_1(\tau)), \dots, x(H_m(\tau))\big)\big| : s, \tau \in [0, T],\\
&\quad |x(H_i(\tau))| \le r \exp(ML(T)),\ i = 1, 2, \dots, m\Big\},
\end{aligned}
$$

where

$$\begin{aligned}\nu_1^T(g,\epsilon) = \sup\Big\{&|g(t,x_1,\dots,x_n) - g(s,x_1,\dots,x_n)| : t,s\in[0,T],\\ &|t-s|\le\epsilon,\ |x_i|\le r\exp(ML(T)),\ i=1,2,\dots,n\Big\},\\ \nu_1^T(k,\epsilon) = \sup\Big\{&|k(t,\tau,x_1,\dots,x_m) - k(s,\tau,x_1,\dots,x_m)| : t,s,\tau\in[0,T],\\ &|t-s|\le\epsilon,\ |x_i|\le r\exp(ML(T)),\ i=1,2,\dots,m\Big\}.\end{aligned}$$

Now let us denote

$$\begin{aligned}\beta(\epsilon) = \nu_1^T(g,\epsilon) + T\nu_1^T(k,\epsilon) + \epsilon\sup\Big\{&\big|k\big(s,\tau,x(H_1(\tau)),\dots,x(H_m(\tau))\big)\big| :\\ &s,\tau\in[0,T],\ |x(H_i(\tau))|\le r\exp(ML(T)),\ i=1,2,\dots,m\Big\}.\end{aligned}$$

Keeping in mind, the uniform continuity of $g(t,x_1,\dots,x_n)$ and $k(t,s,x_1,\dots,x_m)$ on compact subsets of $\mathbb{R}_+\times R^{n^2}$ and $\Delta\times R^{nm}$ respectively, we deduce that $\beta(\epsilon)\to 0$ as $\epsilon\to 0$. Furthermore, from (3.1), we have

$$\begin{aligned}&|(Fx)(t)\exp(-ML(t)) - (Fx)(s)\exp(-ML(s))|\\ &\le |(Fx)(t)\exp(-ML(t)) - (Fx)(s)\exp(-ML(t))|\\ &\quad + |(Fx)(s)||\exp(-ML(t)) - \exp(-ML(s))|\\ &\le K\sum_{i=1}^n |x(h_i(t)) - x(h_i(s))|\exp(-ML(t)) + \beta(\epsilon)\exp(-ML(t))\\ &\quad + |(Fx)(s)||\exp(-ML(t)) - \exp(-ML(s))|\\ &\le K\sum_{i=1}^n |x(h_i(t))\exp(-ML(h_i(t)) - x(h_i(s))\exp(-ML(h_i(s))|\\ &\quad + K\sum_{i=1}^n |x(h_i(s))\exp(-ML(h_i(s)) - x(h_i(s))\exp(-ML(h_i(t))|\\ &\quad + \beta(\epsilon)\exp(-ML(t)) + |(Fx)(s)||\exp(-ML(t)) - \exp(-ML(s))|\\ &\le K\sum_{i=1}^n \omega^T(x,\nu^T(h_i,\epsilon)) + Kr\sum_{i=1}^n \exp(ML(h_i(s)))\nu^T\big(\exp(-ML(h_i(t))),\epsilon\big)\\ &\quad + \beta(\epsilon)\exp(-ML(t)) + r\exp(ML(T))\nu^T\big(\exp(-ML(t)),\epsilon\big).\end{aligned}$$

The above estimate and the fact that $\exp(-ML(t))$ and $\exp(-ML(h_i(t)))$ are uniformly continuous on $[0,T]$, yields the inequality

$$\omega_0^T(FA)\le Kn\omega_0^T(A).$$

Hence,

$$\omega_0(FA)\le Kn\omega_0(A). \tag{3.2}$$

Next, let us assume that $t\ge T$. Then, by virtue of our assumptions we obtain

$$\begin{aligned}&|(Fx)(t)|\exp(-ML(t))\\ &\le \Big[\big|g\big(t,x(h_1(t)),\dots,x(h_n(t))\big) - g(t,0\dots,0)\big| + \big|g(t,0\dots,0)\big|\Big]\exp(-ML(t))\\ &\quad + \Big[\int_0^t L_0(s)ds + \exp(L_1(t))\int_0^t q(s)\sum_{i=1}^m |x(H_i(s))|ds\Big]\exp(-ML(t))\end{aligned}$$

$$
\begin{aligned}
&\le K\sum_{i=1}^{n}|x(h_i(t))|\exp(-ML(t))+[a(t)+b(t)]\exp(-ML(t))\\
&\quad+\exp(L_1(t))\int_0^t q(s)\|x\|\sum_{i=1}^{m}\exp(ML(H_i(s)))ds\cdot\exp(-ML(t))\\
&\le K\sum_{i=1}^{n}|x(h_i(t))|\exp(-ML(h_i(t)))+[a(t)+b(t)]\exp(-ML(t))\\
&\quad+m\|x\|\exp(L_1(t))\int_0^t q(s)\exp(ML(s))ds\cdot\exp(-ML(t))\\
&\le K\sum_{i=1}^{n}|x(h_i(t))|\exp(-ML(h_i(t)))+[a(t)+b(t)]\exp(-Mb(t))\\
&\quad+mrtq(t)\exp(L_1(t))
\end{aligned}
$$

Now, taking into account (H5), from the above estimate we obtain

$$\alpha(FA)\le Kn\alpha(A). \tag{3.3}$$

where $\alpha(A)$ was defined previously. Next, from (3.2) and (3.3) we have

$$\mu(FA)\le C\mu(A) \tag{3.4}$$

where $C=Kn>0$ is a constant and μ denotes the measure of noncompactness defined earlier.

Furthermore, let us consider the sequence of sets (B_r^n), where $B_r^1=\operatorname{Conv}F(B_r)$, $B_r^2=\operatorname{Conv}F(B_r^1)$ and so on. Observe that all sets of this sequence are nonempty, bounded, closed and convex. Moreover, $B_r^{n+1}\subset B_r^n$ for $n=1,2,3,\dots$. Thus in virtue of (3.4) it is easily seen that

$$\mu(B_r^n)\le C^n\mu(B_r)$$

This inequality and the fact that $C<1$ yields that $\lim_{n\to\infty}\mu(B_r^n)=0$. Thus from (v) in Definition 2.1, we deduce that the set $B=\cap_{n=1}^{\infty}B_r^n$ is nonempty, bounded, closed and convex. Moreover, in view of the remark we have that $B\in\ker\mu$. It should be also noted that the operator F maps the set B into itself.

Next we show that F is continuous on the set B. To do this fix $\epsilon>0$ and take arbitrary functions $x,y\in B$ such that $\|x-y\|\le\epsilon$. Taking into account the fact that $B\in\ker\mu$ and the description of sets belonging to $\ker\mu$ we can find $T>0$ such that for each $z\in B$ and $t\ge T$ the inequality $|z(t)|\exp(-ML(t))\le\epsilon/2$ is satisfied.

Let us observe that in view of our assumptions, for an arbitrarily fixed $t\in\mathbb{R}_+$, we obtain

$$
\begin{aligned}
&|(Fx)(t)-(Fy)(t)|\exp(-ML(t))\\
&\le\Big|g\big(t,x(h_1(t)),\dots,x(h_n(t))\big)-g\big(t,y(h_1(t)),\dots,y(h_n(t))\big)\Big|\exp(-ML(t))\\
&\quad+\int_0^t\Big|k\big(t,\tau,x(H_1(\tau)),\dots,x(H_m(\tau))\big)\\
&\quad-k\big(t,\tau,y(H_1(\tau)),\dots,y(H_m(\tau))\big)\Big|d\tau\cdot\exp(-ML(t))\\
&\le K\sum_{i=1}^{n}|x(h_i(t))-y(h_i(t))|\exp(-ML(t))+T\nu^T(k,\epsilon)\exp(-ML(t)).
\end{aligned}
\tag{3.5}
$$

Now let $t \in [0, T]$, then we have

$$|(Fx)(t) - (Fy)(t)| \exp(-ML(t)) \le C\epsilon + T\nu^T(k,\epsilon)\exp(-ML(t)) \tag{3.6}$$

where the quantity $\nu^T(k,\epsilon)$ is defined as follows

$$\begin{aligned}\nu^T(k,\epsilon) = \sup\Big\{&|k(t,\tau,x_1,\dots,x_m) - k(t,\tau,y_1,\dots,y_m)| : t,\tau \in [0,T],\\ &|x_i|,\ |y_i| \le r\exp(ML(T)),\ |x_i - y_i| \le \epsilon\exp(ML(T)),\ i = 1,2,\dots,m\Big\}.\end{aligned}$$

From the definition of the function $k(t,s,x_1,\dots,x_m)$, we conclude that $\nu^T(k,\epsilon) \to 0$ as $\epsilon \to 0$.

Further, let us assume that $t > T$. Then, taking into account that $x, y \in B$ and that F transforms the set B into itself we have that Fx, $Fy \in B$. Hence by the characterization of the set B given above, we get

$$\begin{aligned}&|(Fx)(t) - (Fy)(t)| \exp(-ML(t))\\ &\le |(Fx)(t)|\exp(-ML(t)) + |(Fy)(t)|\exp(-ML(t)) \le \epsilon.\end{aligned} \tag{3.7}$$

Now from (3.5)-(3.7), we deduce that the operator F is continuous on the set B.

Finally, taking into account all facts concerning set B and the operator $F : B \to B$ and applying the classical Schauder fixed point principle we infer that F has at least one fixed point x in the set B. Obviously, the function $x = x(t)$ is a solution of (1.1). Moreover, keeping in mind that $B \in \ker\mu$, we obtain that $x(t) = o(\exp(ML(t)))$ as $t \to \infty$. □

4. Example

Consider the following Volterra integral equation with deviating arguments

$$x(t) = \frac{1 + \arctan\big(x(t/3)\big)}{4 + t^2} + \int_0^t \big[s\cos 2t + 3s^2 e^{-2t}\ln\big(1 + |x(s/2)|\big)\big]ds, \tag{4.1}$$

where $t \ge 0$. This equation is clearly of the form (1.1) with $g(t,x) = \dfrac{1 + \arctan(x)}{4 + t^2}$ and $k(t,s,x) = s\cos(2t) + 3s^2e^{-2t}\ln\big(1 + |x|\big)$ where $m = n = 1$, $h_1(t) = t/3$ and $H_1(t) = t/2$.

It is easily seen that for (4.1), the assumptions (H1) and (H2) are satisfied with $K = 1/4$ and $a(t) = \dfrac{1}{4 + t^2}$. Let us observe that

$$|k(t,s,x)| \le s + 3s^2e^{-2t}|x|.$$

Note that the assumption (H3) is satisfied with $L_0(t) = t$, $L_1(t) = -2t$ and $q(s) = 3s^2$. On the other hand, (4.1) satisfies assumption (H5) with $b(t) = t^2/2$. Now, we get $(Kn + m/M) < 1$ for $M > 1$.

Thus, in view of Theorem 3.1 we conclude that problem (4.1) has a solution $x = x(t)$ such that

$$x(t) = o\Big(\exp\big(M\big[\frac{t^2}{2} - \frac{3e^{-2t}}{2}(t^2 + t + \frac{1}{2}) + \frac{3}{4}\big]\big)\Big)$$

as $t \to \infty$, where $M > 1$ is a constant.

References

[1] K. Balachandran and S. Illamaran; An existence theorem for a Volterra integral equation with deviating arguments, *J. Appl. Math and Stoch. Anal.*, 3 (1990), 155-162.

[2] J. Banaś; Measures of noncompactness in the space of continuous tempered functions, *Demonstratio Math.* 14 (1981), 127-133.

[3] J. Banaś; On solutions of some functional integral equations, *Bull. Inst. Math. Acad. Sinica*, 12 (1984), 23-30.

[4] J. Banaś; An existence theorem for nonlinear Volterra integral equation with deviating argument, *Rendi. Circ. Math. Palermo*, Series 2, 35 (1986), 82-89.

[5] J. Banaś and I. J. Cabrera; On existence and asymptotic behavior of solutions of a functional integral equation, *Nonlinear Anal.* 66 (2007), 2246-2254.

[6] J. Banaś and B. C. Dhage; Global asymptotic stability of solutions of a functional integral equation, *Nonlinear Anal.* (in Press)

[7] J. Banaś and K. Goebel; Measures of Noncompactness in Banach Spaces, *Marcel Dekker*, New York, 1980.

[8] J. Banaś, M. Lecko and W. G. El-Sayed; Existence theorems for some quadratic integral equations, *J. Math. Anal. Appl.* 222 (1998) 276-285.

[9] J. Banaś and B. Rzepka; An application of a measure of noncompactness in the study of asymptotic stability, *Appl. Math. Lett.* 16 (2003), 1-6.

[10] J. Banaś and B. Rzepka; On existence and asymptotic behavior of solutions of a nonlinear integral equation, *J. Math. Anal. Appl.* 284 (2003), 165-173.

[11] T. A. Burton; Volterra Integral and Differential Equations, *Academic Press*, NewYork, 1983.

[12] B. Cahlon and M. Eskin; Existence theorems for an integral equation of the Chandrasekhar H-equation with perturbation, *J. Math. Anal. Appl.* 83(1981) 159-171.

[13] M. Diana Julie and K. Balachandran; Asymptotic behavior of solutions to functional integral equation with deviating arguments, *Elec. J. Diff. Eqns.* 2008(2008), No 77, 1-9.

[14] W. G. El-Sayed; Solvability of a neutral differential equation with deviated argument, *J. Math. Anal. Appl.* 327 (2007), 342-350.

[15] S. Hu, M. Khavani and W. Zhuang; Integral equations arising in the kinetic theory of gases, *Appl. Anal.* 34(1989) 261-266.

[16] Z. Liu and S. M. Kang; Existence and asymptotic stability of solutions to a functional integral equation, *Taiwanese J. Math.* 11 (2007), 187-196.

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