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EXISTENCE OF MILD SOLUTIONS FOR QUASILINEAR INTEGRODIFFERENTIAL EQUATIONS WITH IMPULSIVE CONDITIONS

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Abstract. We prove the existence and uniqueness of mild solutions of quasilinear integrodifferential equations with nonlocal and impulsive conditions in Banach spaces. The results are obtained by using a fixed point technique and semigroup theory. Examples are provided to illustrate the theory.

1. Introduction

Many evolution process are characterized by the fact that at certain moments of time they experience a change of state abruptly. These processes are subject to short-term perturbations whose duration is negligible in comparison with the duration of the process. Consequently, it is natural to assume that these perturbations act instantaneously, that is, in the form of impulses. It is known, for example, that many biological phenomena involving thresholds, bursting rhythm models in medicine and biology, optimal control model in economics, pharmacokinetics and frequency modulated systems, do exhibit impulsive effects. Thus differential equations involving impulsive effects appear as a natural description of observed evolution phenomena of several real world problems.

Existence of solutions of impulsive differential equation of the form

$$u'(t) = Au(t) + f(t, u(t)), \quad t \in (0, a] \tag{1.1}$$

$$u(0) + g(u) = u_0, \tag{1.2}$$

$$\Delta u(t_i) = I_i(u(t_i)), \quad i = 1, 2, 3, \dots, p, \ 0 < t_1 < t_2 <, \dots t_p < a \tag{1.3}$$

has been studied by Liang et al [8]. The impulsive condition is the combination of traditional initial value problem and short-term perturbations whose duration can be negligible in comparison with the duration of process. They have advantages over traditional initial value problem because they can be used to model phenomena that cannot be modelled by traditional initial value problem. Recently, the study of the impulsive differential equations has attracted a great deal of attention. The theory of impulsive differential equations is an important branch of differential equations [7, 11, 13, 15, 16].

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Several authors have studied the existence of solutions of abstract quasilinear evolution equations in Banach space [1, 2, 3, 4, 5, 14]. Bahuguna [1], Oka [9] and Oka and Tanaka [10] discussed the existence of solutions of quasilinear integrodifferential equations in Banach spaces. Kato [6] studied the nonhomogeneous evolution equations where as Chandrasekaran [4] proved the existence of mild solutions of the nonlocal Cauchy problem for a nonlinear integrodifferential equation. An equation of this type occurs in a nonlinear conversation law with memory

$$u_t(t,x) + \Psi(u(t,x))_x = \int_0^t b(t-s)\Psi(u(t,x))_x \, ds + f(t,x), \quad t \in [0,T], \tag{1.4}$$

$$u(0,x) = \phi(x), \quad x \in \mathbb{R}. \tag{1.5}$$

It is clear that if nonlocal condition (1.2) is introduced to (1.4), then it will also have better effect than the classical condition $u(0,x) = \phi(x)$.

The aim of this paper is to prove the existence and uniqueness of mild solutions of quasilinear impulsive evolution integrodifferential equation of the form

$$u'(t) + A(t,u)u(t) = f(t,u(t)) + \int_0^t g(t,s,u(s))ds, \tag{1.6}$$

$$u(0) + h(u) = u_0, \tag{1.7}$$

$$\Delta u(t_i) = I_i(u(t_i)), \quad i = 1,2,3,\dots,m, \ 0 < t_1 < t_2 <, \dots t_m < T. \tag{1.8}$$

Let $A(t,u)$ be the infinitesimal generator of a C_0-semigroup in a Banach space X. Let $PC([0,T];X)$ consist of functions u from $[0,T]$ into X, such that $u(t)$ is continuous at $t \neq t_i$ and left continuous at $t = t_i$, and the right limit $u(t_i^+)$ exists for $i = 1,2,3,\dots m$. Evidently $PC([0,T],X)$ is a Banach space with the norm

$$\|u\|_{PC} = \sup_{t\in[0,T]} \|u(t)\|.$$

Let $u_0 \in X$, $f : [0,T] \times X \to X$, $g : \Omega \times X \to X$, $h : PC([0,T] : X) \to X$ and $\Delta u(t_i) = u(t_i^+) - u(t_i^-)$ constitutes an impulsive condition. Here $[0,T] = J$ and $\Omega = \{(t,s) : 0 \le s \le t \le T\}$. The results obtained in this paper are generalizations of the results given by Balachandran and Uchiyama [3] and Pazy [12].

2. Preliminaries

Let X and Y be two Banach spaces such that Y is densely and continuously embedded in X. For any Banach spaces Z the norm of Z is denoted by $\|\cdot\|$ or $\|\cdot\|_Z$. The space of all bounded linear operators from X to Y is denoted by $B(X,Y)$ and $B(X,X)$ is written as $B(X)$. We recall some definitions and known facts from Pazy [12].

Definition 2.1. Let S be a linear operator in X and let Y be a subspace of X. The operator $\tilde{S}$ defined by $D(\tilde{S}) = \{x \in D(S) \cap Y : Sx \in Y\}$ and $\tilde{S}x = Sx$ for $x \in D(\tilde{S})$ is called the part of S in Y.

Definition 2.2. Let B be a subset of X and for every $0 \le t \le T$ and $b \in B$, let $A(t,b)$ be the infinitesimal generator of a C_0 semigroup $S_{t,b}(s), s \ge 0$, on X. The family of operators $\{A(t,b)\}, (t,b) \in [0,T] \times B$, is stable if there are constants

$M \geq 1$ and ω such that

$$\rho(A(t,b)) \supset (\omega, \infty) \quad \text{for } (t,b) \in [0,T] \times B,$$
$$\| \prod_{j=1}^{k} R(\lambda : A(t_j, b_j))\| \leq M(\lambda - \omega)^{-k}$$

for $\lambda > \omega$ every finite sequences $0 \leq t_1 \leq t_2 \leq \dots \leq t_k \leq T$, $b_j \in B$, $1 \leq j \leq k$. The stability of $\{A(t,b)\}, (t,b) \in [0,T] \times B$ implies (see [12]) that

$$\| \prod_{j=1}^{k} S_{t_j,b_j}(s_j)\| \leq M \exp\big\{\omega \sum_{j=1}^{k} s_j\big\}, \quad s_j \geq 0$$

and any finite sequences $0 \leq t_1 \leq t_2 \leq \dots \leq t_k \leq T$, $b_j \in B$, $1 \leq j \leq k$. $k = 1, 2, \dots$

Definition 2.3. Let $S_{t,b}(s), s \geq 0$ be the C_0-semigroup generatated by $A(t,b)$, $(t,b) \in J \times B$. A subspace Y of X is called $A(t,b)$-admissible if Y is invariant subspace of $S_{t,b}(s)$ and the restriction of $S_{t,b}(s)$ to Y is a C_0-semigroup in Y.

Let $B \subset X$ be a subset of X such that for every $(t,b) \in [0,T] \times B$, $A(t,b)$ is the infinitesimal generator of a C_0-semigroup $S_{t,b}(s), s \geq 0$ on X. We make the following assumptions:

- (H1) The family $\{A(t,b)\}, (t,b) \in [0,T] \times B$ is stable.
- (H2) Y is $A(t,b)$-admissible for $(t,b) \in [0,T] \times B$ and the family $\{\tilde{A}(t,b)\}, (t,b) \in [0,T] \times B$ of parts $\tilde{A}(t,b)$ of $A(t,b)$ in Y, is stable in Y.
- (H3) For $(t,b) \in [0,T] \times B$, $D(A(t,b)) \supset Y$, $A(t,b)$ is a bounded linear operator from Y to X and $t \to A(t,b)$ is continuous in the $B(Y,X)$ norm $\|.\|$ for every $b \in B$.
- (H4) There is a constant $L > 0$ such that
$$\|A(t,b_1) - A(t,b_2)\|_{Y \to X} \leq L\|b_1 - b_2\|_X$$
holds for every $b_1, b_2 \in B$ and $0 \leq t \leq T$.

Let B be a subset of X and $\{A(t,b)\}, (t,b) \in [0,T] \times B$ be a family of operators satisfying the conditions (H1)–(H4). If $u \in PC([0,T] : X)$ has values in B then there is a unique evolution system $U(t,s;u), 0 \leq s \leq t \leq T$, in X satisfying, (see [12, Theorem 5.3.1 and Lemma 6.4.2, pp. 135, 201-202]

- (i) $\|U(t,s;u)\| \leq Me^{\omega(t-s)}$ for $0 \leq s \leq t \leq T$. where M and ω are stability constants.
- (ii) $\frac{\partial^+}{\partial t} U(t,s;u)y = A(s,u(s))U(t,s;u)y$ for $y \in Y$, for $0 \leq s \leq t \leq T$.
- (iii) $\frac{\partial}{\partial s} U(t,s;u)y = -U(t,s;u)A(s,u(s))y$ for $y \in Y$, for $0 \leq s \leq t \leq T$.

Further we assume that

- (H5) For every $u \in PC([0,T] : X)$ satisfying $u(t) \in B$ for $0 \leq t \leq T$, we have
$$U(t,s;u)Y \subset Y, \quad 0 \leq s \leq t \leq T$$
and $U(t,s;u)$ is strongly continuous in Y for $0 \leq s \leq t \leq T$.
- (H6) Closed bounded convex subsets of Y are closed in X.
- (H7) For every $(t,b) \in J \times B$, $f(t,b) \in Y$ and $((t,s),b) \in \Omega \times B$, $g(t,s,b) \in Y$.
- (H8) $h : PC([0,T] : B) \to Y$ is Lipschitz continuous in X and bounded in Y, that is, there exist constant $H > 0$ such that
$$\|h(u) - h(v)\|_Y \leq H\|u - v\|_{PC}, \quad u, v \in PC([0,T]; X).$$

For the conditions (H9) and (H10) let Z be taken as both X and Y.

(H9) $g : \Omega \times Z \to Z$ is continuous and there exist constants $G > 0$ and $G_1 > 0$ such that

$$\int_0^t \|g(t,s,u) - g(t,s,v)\|_Z ds \le G\|u-v\|_Z), \quad u,v \in X,$$

$$G_1 = \max\{\int_0^t \|g(t,s,0)\|_Z \ ds : (t,s) \in \Omega\}.$$

(H10) $f : [0,T]\times Z \to Z$ is continuous and there exist constants $F > 0$ and $F_1 > 0$ such that

$$\|f(t,u) - f(t,v)\|_Z \le F\|u-v\|_Z, \ u,v \in X,$$
$$F_1 = \max_{t\in[0,T]} \|f(t,0)\|_Z.$$

Let us take $M_0 = \max\{\|U(t,s;u)\|_{B(Z)}, 0 \le s \le t \le T, \ u \in B\}$.

(H11) $I_i : X \to X$ is continuous and there exist constant $l_i > 0$, $i = 1,2,3,\dots,m$ such that

$$\|I_i(u) - I_i(v)\| \quad \le \quad l_i\|u-v\|, \quad u,v \in X.$$

(H12) There exist a positive constant $r > 0$ such that

$$M_0\Big[\|u_0\|_Y + Hr + \|h(0)\| + T[r(F+G) + F_1 + G_1] + \sum_{i=1}^m (l_i r + \|I_i(0)\|)\Big] \le r \quad \text{and}$$

$$q = \Big\{KT\Big[\|u_0\|_Y + Hr + \|h(0)\| + T[r(F+G) + F_1 + G_1] + \sum_{i=1}^m (l_i r + \|I_i(0)\|)\Big] + M_0\Big[H + T(F+G) + \sum_{i=1}^m l_i\Big] < 1.$$

Definition 2.4. A function $u \in PC([0,T] : X)$ is a mild solution of equations (1.6)–(1.8) if it satisfies

$$\begin{aligned} u(t) = U(t,0;u)u_0 - U(t,0;u)h(u) + \int_0^t U(t,s;u)\Big[f(s,u(s)) \\ + \int_0^s g(s,\tau,u(\tau))d\tau\Big]ds + \sum_{0<t_i<t} U(t,t_i;u)I_i(u(t_i)), \quad 0 \le t \le T \end{aligned} \tag{2.1}$$

Definition 2.5. A function $u \in PC([0,T] : X)$ such that $u(t) \in D(A(t,u(t))$ for $t \in (0,T], u \in C^1((0,T]\backslash\{t_1,t_2,\dots,t_m\} : X)$ and satisfies (1.6)–(1.8) in X is called a classical solution of (1.6)–(1.8) on $[0,T]$,

Further there exists a constant $K > 0$ such that for every $u,v \in PC([0,T] : X)$ and every $y \in Y$ we have

$$\|U(t,s;u)y - U(t,s;v)y\| \le KT\|y\|_Y\|u-v\|_{PC}. \tag{2.2}$$

3. Existence Result

Theorem 3.1. *Let $u_0 \in Y$ and let $B = \{u \in X : \|u\|_X \le r\}$, $r > 0$. If the assumptions* (H1)–(H12) *are satisfied, then* (1.6)–(1.8) *has a unique mild solution* $u \in PC([0,T] : Y)$.

Proof. Let S be a nonempty closed subset of $PC([0,T]:X)$ defined by

$$S = \{u : u \in PC([0,T]:X), \|u(t)\|_{PC} \le r \text{ for } 0 \le t \le T\}.$$

Consider a mapping Φ on S defined by

$$\begin{aligned}(\Phi u)(t) &= U(t,0;u)u_0 - U(t,0;u)h(u) + \int_0^t U(t,s;u)\Big[f(s,u(s)) \\ &\quad + \int_0^s g(s,\tau,u(\tau))d\tau\Big]ds + \sum_{0<t_i<t} U(t,t_i;u)I_i(u(t_i)).\end{aligned} \tag{3.1}$$

We claim that Φ maps S into S. For $u \in S$, we have

$$\begin{aligned}&\|\Phi u(t)\|_Y \\ &\le \|U(t,0;u)u_0\| + \|U(t,0;u)h(u)\| \\ &\quad + \int_0^t \|U(t,s;u)\|\Big[\|f(s,u(s)) - f(s,0)\| + \|f(s,0)\| \\ &\quad + \|\int_0^s [g(s,\tau,u(\tau)) - g(s,\tau,0)]d\tau\| + \|\int_0^s g(s,\tau,0)d\tau\|\Big]ds \\ &\quad + \sum_{0<t_i<t} \|U(t,t_i;u)I_i(u(t_i))\| \\ &\le M_0\|u_0\|_Y + M_0\Big[H\|u\| + \|h(0)\|\Big] + M_0\Big[\int_0^t F\|u(s)\|ds + F_1T \\ &\quad + \int_0^t G\|u(s)\|ds + G_1T\Big] + M_0\sum_{i=1}^m \Big(l_i\|u\| + \|I_i(0)\|\Big) \\ &\le M_0\Big[\|u_0\|_Y + Hr + \|h(0)\| + T\Big[r(F+G) + F_1 + G_1\Big] + \sum_{i=1}^m \Big(l_ir + \|I_i(0)\|\Big).\end{aligned}$$

From assumption (H12), one gets $\|\Phi u(t)\|_Y \le r$. Therefore Φ maps S into itself. Moreover, if $u, v \in S$, then

$$\begin{aligned}&\|\Phi u(t) - \Phi v(t)\| \\ &\le \|U(t,0;u)u_0 - U(t,0;v)u_0\| + \|U(t,0;u)h(u) - U(t,0;v)h(v)\| \\ &\quad + \int_0^t \|U(t,s;u)\Big[f(s,u(s)) + \int_0^s g(s,\tau,u(\tau))d\tau\Big] + \sum_{0<t_i<t} U(t,t_i;u)I_i(u(t_i)) \\ &\quad - U(t,s;v)\Big[f(s,v(s)) + \int_0^s g(s,\tau,v(\tau))d\tau\Big] - \sum_{0<t_i<t} U(t,t_i;v)I_i(v(t_i))\|ds\end{aligned}$$

Using assumptions (H8)-(H12), one can get

$$\begin{aligned}&\|\Phi u(t) - \Phi v(t)\| \\ &\le KT\|u_0\|_Y\|u-v\|_{PC} + KT\Big[H\|u\| + \|h(0)\|\Big]\|u-v\|_{PC}\end{aligned}$$

$$
\begin{aligned}
&+ M_0H\|u-v\|_{PC} + KT\|u-v\|_{PC}\Big[\int_0^t (F\|u(s)\| + F_1)ds \\
&+ \int_0^t (G\|u(s)\| + G_1)ds\Big] + KT\|u-v\|_{PC}\sum_{i=1}^m \Big(l_ir + \|I_i(0)\|\Big) \\
&+ M_0\Big[\int_0^t F\|u(s)-v(s)\|ds + \int_0^t G\|u(s)-v(s)\|ds\Big] + M_0\sum_{i=1}^m l_i\|u-v\|_{PC} \\
\le{}& \Big\{KT\Big[\|u_0\|_Y + Hr + \|h(0)\| + T[r(F+G) + F_1 + G_1] \\
&+ \sum_{i=1}^m (l_ir + \|I_i(0)\|)\Big] + M_0\Big[H + T(F+G) + \sum_{i=1}^m l_i\Big]\Big\}\|u-v\|_{PC} \\
={}& q\|u-v\|_{PC}, \quad u,v \in PC([0,T];X)
\end{aligned}
$$

where $0 < q < 1$. From this inequality it follows that for any $t \in [0,T]$,

$$\|\Phi u(t) - \Phi v(t)\| \le q\|u-v\|_{PC},$$

so that Φ is a contraction on S. From the contraction mapping theorem it follows that Φ has a unique fixed point $u \in S$ which is the mild solution of (1.6)–(1.8) on $[0,T]$. Note that $u(t)$ is in $PC([0,T]:Y)$ by (H6) see [12, pp. 135, 201-202 lemma 7.4]. In fact, $u(t)$ is weakly continuous as a Y-valued function. This implies that $u(t)$ is separably valued in Y, hence it is strongly measurable. Then $\|u(t)\|_{PC}$ is bounded and measurable function in t. Using the relation $u(t) = \Phi u(t)$, we conclude that $u(t)$ is in $PC([0,T]:Y)$. $\square$

Remark. Using the additional assumption $A(t,b)u_0, b \in B$ is bounded in Y one can establish a unique local classical solution for the equations (1.6)–(1.8).

4. Quasilinear Delay Integrodifferential Equation

Next we consider the following quasilinear delay integrodifferential equation with impulsive nonlocal conditions (1.7) and (1.8)

$$u'(t) + A(t,u)u(t) = f(t,u(\alpha(t))) + \int_0^t g(t,s,u(\beta(s)))ds, \quad t \in [0,T], \tag{4.1}$$

where A, f and h are as before. Assume the following additional conditions:

- (H13) α, β: $[0,T] \to [0,T]$ are absolutely continuous and there exists constants $\delta_1, \delta_2 > 0$ and such that $\alpha'(t) \ge \delta_1$ and $\beta'(t) \ge \delta_2$ for $0 < t \le T$.
- (H14) There exist a positive constant $k > 0$ such that

$$
\begin{aligned}
M_0\Big[\|u_0\|_Y + Hk + \|h(0)\| + T\Big[k/\delta_1\delta_2(F\delta_2 + G\delta_1) + F_1 + G_1\Big] \\
+ \sum_{i=1}^m \Big(l_ik + \|I_i(0)\|\Big) \le k
\end{aligned}
$$

and

$$
\begin{aligned}
p = \{KT\Big[\|u_0\|_Y + Hk + \|h(0)\| + T[k/\delta_1\delta_2(F\delta_2 + G\delta_1) + F_1 + G_1] \\
+ \sum_{i=1}^m (l_ik + \|I_i(0)\|)\Big] + M_0\Big[H + T/\delta_1\delta_2(F\delta_2 + G\delta_1) + \sum_{i=1}^m l_i\Big]\Big\} < 1.
\end{aligned}
$$

For a mild solution of the equation (4.1) and (1.7)-(1.8) we mean a function $u \in PC([0,T]:X)$ and $u_0 \in X$ satisfying the integral equation

$$\begin{aligned} u(t) = U(t,0;u)u_0 - U(t,0;u)h(u) + \int_0^t U(t,s;u)\Big[f(s,u(\alpha(s))) \\ + \int_0^s g(s,\tau,u(\beta(\tau)))d\tau\Big]ds + \sum_{0<t_i<t} U(t,t_i;u)I_i(u(t_i)), \quad 0 \le t \le T. \end{aligned} \tag{4.2}$$

Theorem 4.1. *If the assumptions* (H1)–(H11) *and* (H13)–(H14) *are satisfied, then the equation* (4.1) *with nonlocal and impulsive conditions* (1.7)-(1.8) *has a unique mild solution* $u \in PC([0,T]:Y)$.

Proof. Let S be a nonempty closed subset of $PC([0,T]:X)$ defined by $S = \{u : u \in PC([0,T]:X), \|u(t)\|_{PC} \le k \text{ for } 0 \le t \le T\}$.

Consider a mapping Ψ on S defined by

$$\begin{aligned} (\Psi u)(t) = U(t,0;u)u_0 - U(t,0;u)h(u) + \int_0^t U(t,s;u)\Big[f(s,u(\alpha(s))) \\ + \int_0^s g(s,\tau,u(\beta(\tau)))d\tau\Big]ds + \sum_{0<t_i<t} U(t,t_i;u)I_i(u(t_i)). \end{aligned}$$

Obviously Ψ maps S into S, by (H14) and

$$\|\Psi u(t) - \Psi v(t)\| \le p\|u-v\|_{PC}.$$

Since $p<1$, Ψ is a contraction on S and so Ψ has a unique fixed point $u \in S$ which is the mild solution of the problem (4.1) and (1.7)-(1.8) on $[0,T]$. □

Remark. Using the additional assumption $A(t,b)u_0, b \in B$ is bounded in Y a unique local classical solution for the equations (4.1), (1.7), (1.8) can be established.

5. Examples

In this section we shall give two examples to illustrate the theorems.

Example 5.1. Consider the nonlinear partial integrodifferential equation

$$\begin{aligned} &\frac{\partial}{\partial t}z(t,y) + \frac{\partial^3}{\partial y^3}z(t,y) + z(t,y)\frac{\partial}{\partial y}z(t,y) \\ &= k_0(y)\sin z(t,y) + k_1\int_0^t e^{-z(s,y)}ds, \end{aligned} \tag{5.1}$$

$$z(0,y) + \sum_{i=1}^m c_i z(t_i,y) = z_0(y), \quad y \in \mathbb{R}, \tag{5.2}$$

$$\Delta z|_{t=t_i} = I_i(z(y)) = (\alpha_i|z(y)| + t_i)^{-1}, \quad 1 \le i \le m \tag{5.3}$$

where the constants c_i and α_i are small and $k_0(y)$ is continuous on $\mathbb{R}$, and $k_1 > 0$.

Let H^s be the Hilbert space introduced in [12]. Take $X = L^2(R) = H^0(R)$ and $Y = H^s(R)$, $s \ge 3$. Define an operator A_0 by $D(A_0) = H^3(R)$ and $A_0 z = D^3 z$ for $z \in D(A_0)$ where $D = d/dy$. Then A_0 is the infinitesimal generator of a C_0-group of isometries on X. Next we define for every $v \in Y$ an operator $A_1(v)$ by $D(A_1(v)) = H^1(R)$ and $z \in D(A_1(v)), A_1(v)z = vDz$. Then we have for every $v \in Y$ the operator $A(v) = A_0 + A_1(v)$ is the infinitesimal generator of C_0

semigroup $U(t,0;v)$ on X satisfying $\|U(t,0;v)\| \le e^{\beta t}$ for every $\beta \ge c_0\|v\|_s$ where c_0 is a constant independent of $v \in Y$. Let B_r be the ball of radius $r > 0$ in Y and it is proved that the family of operators $A(v), v \in B_r$ satisfies the conditions (H1)–(H7) (see [12]).

Put $u(t) = z(t,\cdot)$, $h(u) = \sum_{i=1}^{m} c_i z(t_i,\cdot)$ and

$$f(t,u) = k_0(\cdot)\sin z(t,\cdot), \quad g(t,s,u) = k_1 e^{-z(s,\cdot)}.$$

With this choice of $A(u)$, I_i, f, g, h we see that the equation (5.1)–(5.3) is an abstract formulation of (1.6)–(1.8).

Further other conditions (H8)–(H11) are obviously satisfied and it is possible to choose c_i, α_i, k_0, k_1 in such a way that the constant $q < 1$. Hence by Theorem 3.1 the equation (5.1)–(5.3) has a unique mild solution on J.

Example 5.2. Consider the delay partial integrodifferential equation

$$\begin{aligned} &\frac{\partial}{\partial t} z(t,y) + \frac{\partial^3}{\partial y^3} z(t,y) + z(t,y)\frac{\partial}{\partial y} z(t,y) \\ &= k_0(y)\arctan z(\sin t, y) + k_1 \int_0^t e^{-z(\sin s, y)} ds, \end{aligned} \tag{5.4}$$

with the same impulsive and nonlocal conditions as in Example 5.1. Here $f(t,u) = k_0(\cdot)\arctan z(\sin t,\cdot)$ and $\alpha(t) = \beta(t) = \sin t$. With the same $A(u), I_i, g, h$ we see that the equations (5.4) with (5.2)–(5.3) is an abstract formulation of (4.1) with (1.7)–(1.8). Note that (H1)–(H11) are already satisfied and it is possible to choose the constants so that the conditions (H13) and (H14) are also satisfied. Now by Theorem 4.1 the equation (5.4) has a unique mild solution on J.

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