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PERIODICITY OF SOLUTIONS TO DELAYED DYNAMIC EQUATIONS WITH FEEDBACK CONTROL

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Abstract. Using coincidence degree theory, the related continuation theorem, and some priori estimates, we investigate the existence of periodic solutions of a class of delayed dynamic equations with feedback on time scales. Some sufficient criteria are established for the existence solutions. In particular, when the time scale is chosen as the set of the real numbers or the integers, the existence of the periodic solutions to the corresponding continuous-time and discrete-time models follows.

1. Introduction

In real life, biological controls have been successfully and frequently implemented by nature and human beings. Therefore, control variables are introduced to the mathematical ecological models. The reasons for introducing control variables are based on two points. On one hand, ecosystems are continuously distributed by unpredictable forces which can results in changes in the biological parameters such as survival rates. A very basic and practical problem in ecology is whether or not an ecosystem can withstand those unpredictable disturbances which persist for a finite period of time. In the language of control, we call the disturbance functions as control variables, for more information, one can see [13]. On the other hand, it has been proved that under certain conditions some species are permanence and some are possible extinction in the competitive system [1]. However, in paper [20], when some control variables are imposed on the competitive system, some sufficient conditions are derived for the permanence and existence of globally asymptotically stable periodic solution in the two competitive species, which shows that the controls can save extinction of the species. Therefore, in order to search for certain schemes to ensure all the species coexist, it is necessary to introduce control variables.

It is well known that, as the effects of the environmental factors are considered, the assumption of periodicity of parameters is more realistic. Moreover, to model the oscillatory behavior of observed population densities in the field, one of typical approaches is to take into account the time delay in the population dynamics.

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Thus, a more important and realistic population model should take into both the periodicity of the environment and effects of time delay.

Probably motivated by the above mentioned and the practical problem, many authors devote themselves to studying the delayed population dynamic systems with feedback control [7, 8, 9, 10, 13, 17], Huo [15] discussed the following general nonlinear delayed differential system with feedback control

$$\begin{gathered} x'(t) = F(t, x(t-\tau_1(t)), \dots, x(t-\tau_n(t)), u(t-\delta(t))), \\ u'(t) = -a(t)u(t) + b(t)x(t-\tau(t)), \end{gathered} \tag{1.1}$$

where $x(t)$ denotes the density of species at time t and $u(t)$ is the regulator or control variable. $F(t, z_1, z_2, \dots, z_n, z_{n+1})$ is in $C(\mathbb{R}^{n+2}, \mathbb{R})$, $F(t+\omega, z_1, z_2, \dots, z_n, z_{n+1}) = F(t, z_1, z_2, \dots, z_n, z_{n+1})$, $\tau(t)$, $\tau_i(t), \delta(t)$ are in $C(\mathbb{R}, \mathbb{R})$, $1 \le i \le n$, $a(t), b(t)$ are in $C(\mathbb{R}, (0, \infty))$, all of the above functions are ω-periodic functions and $\omega > 0$ is a constant. By using the coincidence degree theory, some sufficient conditions were derived that guarantee the existence of positive periodic solutions.

Very recently, attempts have been made towards the study of population dynamic systems on time scales, for example, see [5, 6, 11, 21]. The theory of calculus on time scales was initiated by Stefan Hilger in his Ph.D Thesis in 1988 [14] in order to unify continuous and discrete analysis, and it has a tremendous potential for applications in some mathematical models of real processes and phenomena studied in physics, chemical technology, population dynamics, biotechnology, economics, neutral networks and social sciences. For more details, see the monographs of Aulbach and Hilger [2], Bohner and Peterson [4], Lakshmikantham et al. [16] and the references therein. The main advantage offered by this theory is to help us to avoid proving results twice, once for differential equations and once again for difference equations.

Up to now, to the author's best knowledge, the studies of delayed dynamic equations with feedback control on time scales are scarce. Therefore, in the present paper, by employing the coincidence degree theory, we will explore the existence of periodic solutions of a class of delayed dynamic equations with feedback control, which incorporate as special cases many species models governed by ordinary differential and difference equations when the time scale is chosen as the set of all real numbers and all integer numbers.

The remainder of the paper is comprised of three sections. In the coming section, we presented some preliminary results on the calculus on time scales and the famous Gaines and Mawhin's continuation theorem of coincidence degree theory. In section 3, by using the coincidence degree theory, we will establish some sufficient conditions for the existence of periodic solutions of a class of delayed dynamic equations with feedback control. In section 4, we present some examples to verify our theoretical findings. At last, some conclusions are given.

2. Preliminaries

In this section, we will recall some fundamental definitions and results from the calculus on time scales [2, 3, 4, 14, 16].

Definition 2.1. A time scale is an arbitrary nonempty closed subset $\mathbb{T}$ of $\mathbb{R}$, the real numbers. The set $\mathbb{T}$ inherits the standard topology of $\mathbb{R}$.

Remark 2.2. It is easy to see the set of all real numbers $\mathbb{R}$, the set of all integer numbers $\mathbb{Z}$ and $\cup_{k\in\mathbb{Z}}[2k, 2k+1]$, as well as $\cup_{k\in\mathbb{Z}} \cup_{n\in\mathbb{N}} \{k + \frac{1}{n}\}$ are such time scales.

Definition 2.3. The forward jump operator $\sigma : \mathbb{T} \to \mathbb{T}$, the backward jump operator $\rho : \mathbb{T} \to \mathbb{T}$, and the graininess $\mu : \mathbb{T} \to \mathbb{R}^+ = [0, +\infty)$ are defined, respectively, by

$$\sigma(t) = \inf\{s \in \mathbb{T} : s > t\}, \quad \rho(t) = \sup\{s \in \mathbb{T} : s < t\}, \quad \mu(t) = \sigma(t) - t \quad \text{for } t \in \mathbb{T}.$$

If $\sigma(t) = t$, then t is called right-dense (otherwise: right-scattered), and if $\rho(t) = t$, then t is called left-dense (otherwise: left-scattered).

Definition 2.4. A function $f : \mathbb{T} \to \mathbb{R}$ is said to be rd-continuous if it is continuous at right-dense points in $\mathbb{T}$ and its left-sided limits exist (finite) at left-dense point in $\mathbb{T}$. The set of rd-continuous function $f : \mathbb{T} \to \mathbb{R}$ will be denoted by $C_{rd}(\mathbb{T})$.

Definition 2.5. Assume $f : \mathbb{T} \to \mathbb{R}$ is a function and let $t \in \mathbb{T}$. Then we define $f^\Delta(t)$ to be the number (provided it exists) with the property that given any $\varepsilon > 0$, there is a neighborhood U of t such that

$$|f(\sigma(t)) - f(s) - f^\Delta(t)(\sigma(t) - s)| \le \varepsilon|\sigma(t) - s| \quad \text{for all } s \in U.$$

In this case, $f^\Delta(t)$ is called the delta (or Hilger) derivative of f at t. Moreover, f is said to be delta or Hilger differentiable on $\mathbb{T}$ if $f^\Delta(t)$ exists for all $t \in \mathbb{T}$. The set of functions $f : \mathbb{T} \to \mathbb{R}$ that are delta-differentiable and whose delta-derivative are rd-continuous functions is denoted by $C^1_{rd} = C^1_{rd}(\mathbb{T}) = C^1_{rd}(\mathbb{T}, \mathbb{R})$.

Definition 2.6. A function $F : \mathbb{T} \to \mathbb{R}$ is called a antiderivative of $f : \mathbb{T} \to \mathbb{R}$ provided $F^\Delta(t) = f(t)$ for all $t \in \mathbb{T}$. Then we write

$$\int_r^s f(t)\Delta t = F(s) - F(r) \quad \text{for } r, s \in \mathbb{T}.$$

Throughout the paper, we need below the set $\mathbb{T}^\kappa$ is derived from the time scale $\mathbb{T}$ as follows: If $\mathbb{T}$ has a left-scattered maximum m, then $\mathbb{T}^\kappa = \mathbb{T} - \{m\}$, otherwise $\mathbb{T}^\kappa = \mathbb{T}$. In summary,

$$\mathbb{T}^\kappa = \begin{cases} \mathbb{T}\backslash(\rho(\sup \mathbb{T}), \sup \mathbb{T}] & \text{if } \sup \mathbb{T} < \infty, \\ \mathbb{T} & \text{if } \sup \mathbb{T} = \infty. \end{cases}$$

Moreover, we will assume the time scale $\mathbb{T}$ is ω-periodic, that is, $t \in \mathbb{T}$ implies $t + \omega \in \mathbb{T}$. In particular, the time scale under consideration is unbounded above and below. For simplicity, we also denote

$$\kappa = \min\{\mathbb{R}^+ \cap \mathbb{T}\}, \quad I_\omega = [\kappa, \kappa + \omega] \cap \mathbb{T}, \quad g^l = \inf_{t \in \mathbb{T}} g(t), \quad g^u = \sup_{t \in \mathbb{T}} g(t)$$

and

$$\bar{g} = \frac{1}{\omega} \int_{I_\omega} g(s)\Delta s = \frac{1}{\omega} \int_\kappa^{\kappa+\omega} g(s)\Delta s,$$

where $g \in C_{rd}(\mathbb{T})$ is an ω-periodic real function; i.e., $g(t + \omega) = g(t)$ for all $t \in \mathbb{T}$.

Lemma 2.7. *If $f : \mathbb{T} \to \mathbb{R}$ is delta differentiable at $t \in \mathbb{T}^\kappa$, then*

$$f^\sigma(t) = f(t) + \mu(t) f^\Delta(t),$$

where $f^\sigma = f \circ \sigma$ and σ, μ are as in Def. 2.3.

Lemma 2.8. *If $f \in C_{rd}$ and $t \in \mathbb{T}^\kappa$, then $\int_t^{\sigma(t)} f(s)\Delta s = \mu(t) f(t)$.*

Lemma 2.9. *If $a, b, c \in \mathbb{T}$ and $f \in C_{rd}$, then*

(i) $\int_a^b f(t)\Delta t = \int_a^c f(t)\Delta t + \int_c^b f(t)\Delta t$,

(ii) *if* $|f(t)| < g(t)$ *for all* $t \in [a, b)$*, then* $|\int_a^b f(t)\Delta t| \le \int_a^b g(t)\Delta t$*,*
(iii) *if* $f(t) \ge 0$ *for all* $a \le t < b$*, then* $\int_a^b f(t)\Delta t \ge 0$*.*

Lemma 2.10. *Every rd-continuous function has an antiderivative and every continuous function is rd-continuous.*

Definition 2.11. If $a \in \mathbb{T}$, $\sup \mathbb{T} = \infty$, and f is rd-continuous on $[a, \infty)$, then we define the improper integral by

$$\int_a^{\infty} f(t)\Delta t := \lim_{b\to\infty} \int_a^b f(t)\Delta t$$

provided this limit exists, and we say that the improper integral converges in this case. If this limit does not exist, then we say that the improper integral diverges.

Lemma 2.12. *If* $f, g : \mathbb{T} \to \mathbb{R}$ *are delta differentiable at* $t \in \mathbb{T}^{\kappa}$*, then*

$$(fg)^{\Delta}(t) = f^{\Delta}(t)g(t) + f^{\sigma}(t)g^{\Delta}(t) = f(t)g^{\Delta}(t) + f^{\Delta}(t)g^{\sigma}(t).$$

Definition 2.13. A function $r : \mathbb{T} \to \mathbb{R}$ is called regressive provided

$$1 + \mu(t)r(t) \neq 0, \quad \text{for all } t \in \mathbb{T}^{\kappa}.$$

The set of all regressive and rd-continuous functions will be denoted by $\mathcal{R}$.

Definition 2.14. We define the set $\mathcal{R}^+$ of all positively regressive elements of $\mathcal{R}$ by

$$\mathcal{R}^+ = \{p \in \mathcal{R} : 1 + \mu(t)p(t) > 0, \quad \text{for all } t \in \mathbb{T}\}.$$

Definition 2.15. If $p \in \mathcal{R}$, then the delta exponential function $e_p(\cdot, s)$ is defined as the unique solution of the initial value problem

$$y^{\Delta} = p(t)y, \quad y(s) = 1,$$

where $s \in \mathbb{T}$. Furthermore, for p, $q \in \mathcal{R}$, we also define

$$p \oplus q = p + q + \mu pq, \quad p \ominus q = \frac{p - q}{1 + \mu q}.$$

Lemma 2.16. *If* p, $q \in \mathcal{R}$*, then*

$$e_p(t, t) = 1, \quad e_p(t, s) = 1/e_p(s, t), \quad e_p(t, a)e_p(a, s) = e_p(t, s),$$
$$e_p(\sigma(t), s) = (1 + \mu(t)p(t))e_p(t, s), \quad e_p(s, \sigma(t)) = \frac{e_p(s, t)}{1 + \mu(t)p(t)},$$
$$e_p^{\Delta}(\cdot, s) = pe_p(\cdot, s), \quad e_p^{\Delta}(s, \cdot) = (\ominus p)e_p(s, \cdot).$$

Lemma 2.17. *If* $p \in \mathcal{R}^+$ *and* $t_0 \in \mathbb{T}$*, then* $e_p(t, t_0) > 0$ *for all* $t \in \mathbb{T}$*.*

Lemma 2.18. *Suppose* $\mathbb{T}$ *is* ω*-periodic,* $p \in C_{rd}(\mathbb{T})$ *is* ω*-periodic, and* $a, b \in \mathbb{T}$*. Then*

$$\sigma(t + \omega) = \sigma(t) + \omega, \quad \rho(t + \omega) = \rho(t) + \omega, \quad \mu(t + \omega) = \mu,$$
$$\int_{a+\omega}^{b+\omega} p(t)\Delta t = \int_a^b p(t)\Delta t, \quad e_p(b, a) = e_p(b + \omega, a + \omega), \quad k_p = e_p(t + \omega, t) - 1$$

are independent of $t \in \mathbb{T}$ *whenever* $p \in \mathcal{R}$*.*

Lemma 2.19. *Let $\mathbb{T}$ be ω-periodic and suppose $f : \mathbb{T} \times \mathbb{T} \to \mathbb{R}$ satisfies the assumptions of* [4, Theorem 1.117]. *Define $g(t) = \int_t^{t+\omega} f(t,s)\Delta s$. If $f^{\Delta}(t,s)$ denotes the derivative of f with respect to t, then*

$$g^{\Delta}(t) = \int_t^{t+\omega} f^{\Delta}(t,s)\Delta s + f(\sigma(t), t+\omega) - f(\sigma(t), t).$$

Next, we introduce the famous Gaines and Mawhin's continuation theorem of coincidence degree theory [12], which will come into play later on.

Let X, Z be normed vector spaces, $L : DomL \subset X \to Z$ a linear mapping, $N : X \to Z$ is a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $dim \ker L = codimImL < +\infty$ and ImL is closed in Z. If L is a Fredholm mapping of index zero there exist continuous projectors $P : X \to X$ and $Q : Z \to Z$ such that $ImP = \ker L, ImL = KerQ = Im(I - Q)$. It follows that $L|DomL \cap KerP : (I - P)X \to ImL$ is invertible. We denote the inverse of that map by K_P. If Ω be an open bounded subset of X, the mapping N will be called L-compact on $\bar{\Omega}$ if $QN(\bar{\Omega})$ is bounded and $K_P(I-Q)N : \bar{\Omega} \to X$ is compact. Since ImQ is isomorphic to $\ker L$, there exists an isomorphism $J : \operatorname{Im} Q \to \ker L$.

Lemma 2.20 (Continuation Theorem)**.** *Let L be a Fredholm mapping of index zero and let N be L-compact on $\bar{\Omega}$. Suppose*

(i) *For each $\lambda \in (0,1)$, every solution x of $Lx = \lambda Nx$ is such that $x \notin \partial\Omega$;*
(ii) *$QNx \neq 0$ for each $x \in \partial\Omega \cap \ker L$ and*

$$\deg\{JQN, \Omega \cap \ker L, 0\} \neq 0.$$

Then the equation $Lx = Nx$ has at least one solution lying in $DomL \cap \bar{\Omega}$.

3. Existence of periodic solutions

In this section, we utilize the continuation theorem of coincidence degree theory introduced in the previous section to establish some sufficient criteria for the existence of periodic solutions.

Consider the following more general delayed dynamic equation on a time scale

$$\begin{aligned} x^{\Delta}(t) = F\Big(&t, \exp\{x(g_1(t))\}, \dots, \exp\{x(g_n(t))\}, \\ &\int_{-\infty}^{t} c(t,s)\exp\{x(s)\}\Delta s, \exp\{u(t-\delta(t))\}\Big), \\ [\exp\{u(t)\}]^{\Delta} &= -a(t)\exp\{u(\sigma(t))\} + b(t)\exp\{x(t-\tau(t))\}. \end{aligned} \tag{3.1}$$

To obtain our main results, we assume the following hypotheses:

(H1) $F : \mathbb{T} \times \mathbb{R}^{n+2} \to \mathbb{R}$, $F(t,\cdot)$ is continuous on $\mathbb{R}^{n+2}$ for all $t \in \mathbb{T}$ and is ω-periodic with respect to the first variable; i.e., $F(t+\omega, v_1, v_2, \dots, v_{n+2}) = f(t, v_1, v_2, \dots, v_{n+2})$,
(H2) $g_i : \mathbb{T} \to \mathbb{T}$ is ω-periodic and satisfies $g_i(t) \leq t$, $\int_{-\infty}^{t} c(t,s)\Delta s$ is rd-continuous in $t \in \mathbb{T}$, $c(t+\omega, s+\omega) = c(t,s)$,
(H3) $a(t), b(t) \in C_{rd}(\mathbb{T}, (0,\infty))$ are ω-periodic, $\delta(t), \tau(t) \in C_{rd}(\mathbb{T}, \mathbb{R})$ are ω-periodic, $\sigma(t)$ is the forward jump operator defined in Definition 2.3.

Remark 3.1. Let $\hat{x} = \exp\{x(t)\}$, $\hat{u}(t) = \exp\{u(t)\}$. If $\mathbb{T} = \mathbb{R}$, then (3.1) reduces to the following delayed differential system with feedback control,

$$\begin{aligned} \hat{x}'(t) &= \hat{x}(t)F(t, \hat{x}(g_1(t)), \dots, \hat{x}(g_n(t)), \int_{-\infty}^{t} c(t,s)\hat{x}(s)ds, \hat{u}(t-\delta(t))), \\ \hat{u}'(t) &= -a(t)\hat{u}(t) + b(t)\hat{x}(t-\tau(t)), \end{aligned} \tag{3.2}$$

while if $\mathbb{T} = \mathbb{Z}$, then (3.1) is reformulated as the difference equation with feedback control

$$\begin{aligned} \hat{x}(t+1) &= \hat{x}(t)\exp\Big\{F\Big(t, \hat{x}(g_1(t)), \dots, \hat{x}(g_n(t)), \\ &\qquad \sum_{s=-\infty}^{t-1} c(t,s)\hat{x}(s), \hat{u}(t-\delta(t))\Big)\Big\}, \\ \hat{u}(t+1) - \hat{u}(t) &= -a(t)\hat{u}(t+1) + b(t)\hat{x}(t-\tau(t)). \end{aligned} \tag{3.3}$$

Lemma 3.2. *The function $(x(t), u(t))^T$ is an ω-periodic solution of (3.1) if and only if it is also an ω-periodic solution of the system*

$$\begin{aligned} x^{\Delta}(t) &= F\Big(t, \exp\{x(g_1(t))\}, \dots, \exp\{x(g_n(t))\}, \\ &\qquad \int_{-\infty}^{t} c(t,s)\exp\{x(s)\}\Delta s, \exp\{u(t-\delta(t))\}\Big), \\ u(t) &= \ln\big\{\frac{1}{k_a}\int_t^{t+\omega} b(s)\exp\{x(s-\tau(s))\}e_a(s,t)\Delta s\big\} := (\varphi x)(t). \end{aligned} \tag{3.4}$$

Here, $e_a(s,t)$ is defined in Definition 2.15 and $k_a = e_a(t+\omega, t) - 1$.

Proof. First, we assume $(x(t), u(t))^T$ is an ω-periodic solution of (3.1). For convenience, denote $f(t) = b(t)\exp\{x(t-\tau(t))\}$ and let $t_0 \in \mathbb{T}$. Using Lemma 2.17, for $s \in [t, t+\omega]$, $e_a(s,t) > 0$, and thus $u(t)$ is well-defined. By considering Lemma 2.18, we have

$$\begin{aligned} u(t+\omega) &= \ln\big\{\frac{1}{k_a}\int_{t+\omega}^{t+2\omega} f(s)e_a(s, t+\omega)\Delta s\big\} \\ &= \ln\big\{\frac{1}{k_a}\int_{t}^{t+\omega} f(s+\omega)e_a(s+\omega, t+\omega)\Delta s\big\} \\ &= \ln\big\{\frac{1}{k_a}\int_{t}^{t+\omega} f(s)e_a(s, t)\Delta s\big\} = u(t), \end{aligned}$$

so that $u(t)$ is ω-periodic.

By Lemma 2.12 and Lemma 2.16, we have

$$\begin{aligned} [\exp\{u(t)\}e_a(t,t_0)]^{\Delta} &= [\exp\{u(t)\}]^{\Delta}e_a(t,t_0) + [\exp\{u(t)\}]^{\sigma}e_a^{\Delta}(t,t_0) \\ &= [\exp\{u(t)\}]^{\Delta}e_a(t,t_0) + [\exp\{u(t)\}]^{\sigma}a(t)e_a(t,t_0) \\ &= e_a(t,t_0)\{[\exp\{u(t)\}]^{\Delta} + a(t)[\exp\{u(t)\}]^{\sigma}\} = e_a(t,t_0)f(t). \end{aligned}$$

Integrating both sides of this equation from t to $t+\omega$ leads to

$$\begin{aligned} \int_t^{t+\omega} e_a(s,t_0)f(s)\Delta s &= \exp\{u(t+\omega)\}e_a(t+\omega,t_0) - u(t)e_a(t,t_0) \\ &= \exp\{u(t)\}[e_a(t+\omega,t_0) - e_a(t,t_0)] = \exp\{u(t)\}e_a(t,t_0)k_a. \end{aligned}$$

This proves one part of the lemma.

Next, let $(x(t), u(t))^T$ be an ω-periodic solution of (3.4). Then by Lemma 2.19, Lemma 2.12 and Lemma 2.16, we obtain

$$\begin{aligned}
&[\exp\{u(t)\}]^{\Delta} \\
&= \frac{1}{k_a}\Big[\int_t^{t+\omega}[f(s)e_a(s,t)]^{\Delta}\Delta s + f(t+\omega)e_a(t+\omega,\sigma(t)) - f(t)e_a(t,\sigma(t))\Big] \\
&= \frac{1}{k_a}\Big[\int_t^{t+\omega} f(s)(\ominus a)(t)e_a(s,t)\Delta s + f(t+\omega)e_a(t+\omega,\sigma(t)) - f(t)e_a(t,\sigma(t))\Big] \\
&= \frac{1}{k_a}\Big[\int_t^{t+\omega} f(s)\frac{-a(t)}{1+\mu(t)a(t)}e_a(s,t)\Delta s + f(t+\omega)e_a(t+\omega,\sigma(t)) \\
&\quad - f(t)e_a(t,\sigma(t))\Big] \\
&= \frac{1}{k_a}\Big[\int_t^{t+\omega} f(s)\frac{-a(t)}{(1+\mu(t)a(t))e_a(t,s)}\Delta s + f(t+\omega)e_a(t+\omega,\sigma(t)) \\
&\quad - f(t)e_a(t,\sigma(t))\Big] \\
&= \frac{1}{k_a}\Big[\int_t^{t+\omega} f(s)a(t)e_a(s,\sigma(t))\Delta s + f(t)e_a(t+\omega,\sigma(t)) - f(t)e_a(t,\sigma(t))\Big].
\end{aligned}$$

Moreover by Lemma 2.8, Lemma 2.9, Lemma 2.16 and Lemma 2.18, we have

$$\begin{aligned}
a(t)\exp\{u(\sigma(t))\} &= \frac{a(t)}{k_a}\int_{\sigma(t)}^{\sigma(t)+\omega} f(s)e_a(s,\sigma(t))\Delta s \\
&= \frac{a(t)}{k_a}\Big[\int_t^{t+\omega} f(s)e_a(s,\sigma(t))\Delta s - \int_t^{\sigma(t)} f(s)e_a(s,\sigma(t))\Delta s \\
&\quad + \int_{t+\omega}^{\sigma(t)+\omega} f(s)e_a(s,\sigma(t))\Delta s\Big] \\
&= \frac{a(t)}{k_a}\Big[\int_t^{t+\omega} f(s)e_a(s,\sigma(t))\Delta s - \mu(t)f(t)e_a(t,\sigma(t)) \\
&\quad + \mu(t+\omega)f(t+\omega)e_a(t+\omega,\sigma(t))\Big] \\
&= \frac{a(t)}{k_a}\Big[\int_t^{t+\omega} f(s)e_a(s,\sigma(t))\Delta s - \mu(t)f(t)e_a(t,\sigma(t)) \\
&\quad + \mu(t)f(t)e_a(t+\omega,\sigma(t))\Big].
\end{aligned}$$

Therefore,

$$\begin{aligned}
&k_a\{[\exp\{u(t)\}]^{\Delta} + a(t)\exp\{u(\sigma(t))\}\} \\
&= f(t)[e_a(t+\omega,\sigma(t)) - e_a(t,\sigma(t)) - \mu(t)a(t)e_a(t,\sigma(t)) + \mu(t)a(t)e_a(t+\omega,\sigma(t))] \\
&= f(t)[(1+\mu(t)a(t))e_a(t+\omega,\sigma(t)) - (1+\mu(t)a(t))e_a(t,\sigma(t))] \\
&= f(t)[e_a(t+\omega,t) - e_a(t,t)] = k_a f(t).
\end{aligned}$$

This completes the proof. $\square$

By Lemma 3.2, to show the existence of periodic solutions of (3.1), we only need to show the existence of periodic solutions of (3.4). Now, (3.4) can be written as

$$\begin{aligned} x^{\Delta}(t) = F\Big(&t, \exp\{x(g_1(t))\}, \dots, \exp\{x(g_n(t))\}, \\ &\int_{-\infty}^{t} c(t,s)\exp\{x(s)\}\Delta s, \exp\{(\varphi x)(t-\delta(t))\}\Big). \end{aligned} \tag{3.5}$$

The following lemma will be used in the proof of our main results. The proof can be found in [5].

Lemma 3.3. *Let $t_1, t_2 \in I_\omega$ and $t \in \mathbb{T}$. If $g : \mathbb{T} \to \mathbb{R}$ is ω-periodic, then*

$$g(t) \le g(t_1) + \int_{\kappa}^{\kappa+\omega} |g^{\Delta}(s)|\Delta s \quad \text{and} \quad g(t) \ge g(t_2) - \int_{\kappa}^{\kappa+\omega} |g^{\Delta}(s)|\Delta s. \tag{3.6}$$

Theorem 3.4. *Let* (H1)–(H3) *hold. In addition, assume:*

(H4) *there exists a constant $M > 0$ such that for any ω-periodic function $x : \mathbb{T} \to \mathbb{R}$, if*

$$\begin{aligned} &\int_{\kappa}^{\kappa+\omega} F\Big(t, \exp\{x(g_1(t))\}, \exp\{x(g_2(t))\}, \dots, \exp\{x(g_n(t))\}, \\ &\int_{-\infty}^{t} c(t,s)\exp\{x(s)\}\Delta s, \exp\{(\varphi x)(t-\delta(t))\}\Big)\Delta t = 0, \end{aligned}$$

$$\begin{aligned} &\int_{\kappa}^{\kappa+\omega} \Big|F\Big(t, \exp\{x(g_1(t))\}, \exp\{x(g_2(t))\}, \dots, \exp\{x(g_n(t))\}, \\ &\int_{-\infty}^{t} c(t,s)\exp\{x(s)\}\Delta s, \exp\{(\varphi x)(t-\delta(t))\}\Big)\Big|\Delta t \le M, \end{aligned}$$

(H5) *there exist constants $A_2 > A_1 > 0$ such that if $v_i \ge A_2$ for all $1 \le i \le n+2$, then*

$$\int_{\kappa}^{\kappa+\omega} F\Big(t, v_1, v_2, \dots, v_n, \int_{-\infty}^{t} c(t,s)v_{n+1}\Delta s, v_{n+2}\Big)\Delta t < 0,$$

and if $0 < v_i \le A_1$ for all $1 \le i \le n+2$, then

$$\int_{\kappa}^{\kappa+\omega} F\Big(t, v_1, v_2, \dots, v_n, \int_{-\infty}^{t} c(t,s)v_{n+1}\Delta s, v_{n+2}\Big)\Delta t > 0.$$

Then system (3.1) *has at least one ω-periodic solution.*

Proof. By the above discussion, it suffices to show (3.5) has at least one ω-periodic solution. In order to apply Lemma 2.20 to system (3.5), we take

$$X = Z = \{x \in C_{rd}(\mathbb{T}, \mathbb{R}) | x(t+\omega) = x(t), \quad \text{for all } t \in \mathbb{T}\},$$

and denote

$$\|x\| = \max_{t \in I_\omega} |x(t)|, \quad x \in X \quad (\text{or } Z).$$

It is not difficult to show that X and Z are Banach spaces equipped with the norm $\|\cdot\|$. Set

$$L : \operatorname{Dom} L \cap X, \quad Lx = x^{\Delta}(t), \quad x \in X,$$

where $\operatorname{Dom} L = \{x(t) \in X | x(t) \in C^1_{rd}\}$. For $x(t) \in X$, we define $N : X \to X$ as follows

$$\begin{aligned} Nx(t) = F\Big(&t, \exp\{x(g_1(t))\}, \dots, \exp\{x(g_n(t))\}, \\ &\int_{-\infty}^{t} c(t,s)\exp\{x(s)\}\Delta s, \exp\{(\varphi x)(t-\delta(t))\}\Big). \end{aligned}$$

Furthermore, let us define two projectors P and Q by $Px = Qx = \bar{x}$. Then it follows that

$$\begin{gathered} \ker L = \{x \in X | x(t) \equiv h \in \mathbb{R} \text{ for } t \in \mathbb{T}\}, \\ \operatorname{Im} L = \{z \in Z | \bar{z} = 0\}, \end{gathered}$$

and

$$\dim \ker L = 1 = \operatorname{codim} \operatorname{Im} L.$$

Since $\operatorname{Im} L$ is closed in Z, then L is a Fredholm operator of index zero. Clearly, P, Q are continuous projectors by the above definition such that

$$ImP = \ker L, \quad \operatorname{Im} L = KerQ = Im(I-Q).$$

It follows that the mapping $L_{\operatorname{Dom} L \cap KerP} : (I-P)X \to \operatorname{Im} L$ is invertible. We define the inverse of the mapping by K_P, then K_P has the form

$$K_P x = \int_{\kappa}^{t} x(s)\Delta s - \frac{1}{\omega}\int_{\kappa}^{\kappa+\omega}\int_{\kappa}^{t} x(s)\Delta s\Delta t.$$

Thus,

$$\begin{aligned} QNx = \frac{1}{\omega}\int_{\kappa}^{\kappa+\omega} F\Big(&t, \exp\{x(g_1(t))\}, \dots, \exp\{x(g_n(t))\}, \\ &\int_{-\infty}^{t} c(t,s)\exp\{x(s)\}\Delta s, \exp\{(\varphi x)(t-\delta(t))\}\Big)\Delta t \end{aligned}$$

and

$$\begin{aligned} K_p(I-Q)Nx = &\int_{\kappa}^{t}(Nx)(s)\Delta s - \frac{1}{\omega}\int_{\kappa}^{\kappa+\omega}\int_{\kappa}^{t}(Nx)(s)\Delta s\Delta t \\ &- \Big(t-\kappa-\frac{1}{\omega}\int_{\kappa}^{\kappa+\omega}(t-\kappa)\Delta t\Big)\overline{Nx}. \end{aligned}$$

Obviously, QN and $K_P(I-Q)N$ are continuous. Since X is a Banach space, by using Arzelà-Ascoli theorem, it is not difficult to show that $\overline{K_P(I-Q)N(\bar{\Omega})}$ is compact for any open bounded set $\Omega \subset X$. Moreover, $QN(\bar{\Omega})$ is bounded. Thus, N is L-compact on $\bar{\Omega}$ for any open bounded set $\Omega \subset X$.

Now, we reach the point where we search for appropriate open bounded subsets Ω for the application of the continuation theorem. For $\lambda \in (0,1)$, we consider the operator equation $Lx = \lambda Nx$; that is,

$$\begin{aligned} x^{\Delta}(t) = \lambda\int_{\kappa}^{\kappa+\omega} F\Big(&t, \exp\{x(g_1(t))\}, \dots, \exp\{x(g_n(t))\}, \\ &\int_{-\infty}^{t} c(t,s)\exp\{x(s)\}\Delta s, \exp\{(\varphi x)(t-\delta(t))\}\Big). \end{aligned} \tag{3.7}$$

Suppose that $x \in X$ is an arbitrary ω-periodic solution of system (3.7) for some $\lambda \in (0,1)$. Integrating both sides of (3.7) over the interval $[\kappa, \kappa+\omega]$, we obtain

$$\begin{aligned}&\int_{\kappa}^{\kappa+\omega} F\Big(t, \exp\{x(g_1(t))\}, \dots, \exp\{x(g_n(t))\},\\ &\int_{-\infty}^{t} c(t,s)\exp\{x(s)\}\Delta s, \exp\{(\varphi x)(t-\delta(t))\}\Big) = 0.\end{aligned} \tag{3.8}$$

Combining (H4) with (3.8), leads to

$$\begin{aligned}\int_{\kappa}^{\kappa+\omega} |x^{\Delta}(t)|\Delta t = \lambda \int_{\kappa}^{\kappa+\omega} \Big| F\Big(t, \exp\{x(g_1(t))\}, \exp\{x(g_2(t))\} \dots, \exp\{x(g_n(t))\},\\ \int_{-\infty}^{t} c(t,s)\exp\{x(s)\}\Delta s, \exp\{(\varphi x)(t-\delta(t))\}\Big)\Big| \Delta t \le M.\end{aligned} \tag{3.9}$$

Moreover, in view of (3.8) and (H5), it is easy to see that there exist an $i_0 \in \{1, \dots, n+1\}$, a point t' and a constant $A_2 > 0$, such that

$$x(g_{i_0}(t')) < \ln(A_2),\ x(t') < \ln(A_2) \quad \text{and} \quad (\varphi x)(t'-\delta(t')) < \ln(A_2). \tag{3.10}$$

Otherwise, for any $A_2 > 0$ and any $t \in I_\omega$, one has

$$x(g_{i_0}(t')) \ge \ln(A_2),\ x(t') \ge \ln(A_2) \quad \text{and} \quad (\varphi x)(t'-\delta(t')) \ge \ln(A_2).$$

In view of (H5), we see that this contradicts (3.8). Hence, (3.10) holds.

Note that since $x \in X$, there exist ξ, $\eta \in I_\omega$, such that

$$x(\xi) = \min_{t\in I_\omega}\{x(t)\}, \quad x(\eta) = \max_{t\in I_\omega}\{x(t)\}. \tag{3.11}$$

Then by (3.10), we have $x(\xi) < \ln(A_2)$. This together with the first inequality of (3.6) implies

$$x(t) \le x(\xi) + \int_{\kappa}^{\kappa+\omega} |x^{\Delta}(t)|\Delta t < \ln(A_2) + M. \tag{3.12}$$

In a similar way, it is easy to see there exists a constant $A_1 > 0$ such that $x(\eta) > \ln(A_1)$, which together with the second inequality of (3.6) produces

$$x(t) \ge x(\eta) + \int_{\kappa}^{\kappa+\omega} |x^{\Delta}(t)|\Delta t > \ln(A_1) - M. \tag{3.13}$$

Therefore, it follows from (3.12) and (3.13) that

$$\max_{t\in I_\omega} |x(t)| \le \max\{|\ln(A_2) + M|, |\ln(A_1) - M|\} := A_3. \tag{3.14}$$

Clearly, A_3 is independent of λ.

Now we define $\Omega = \{x \in X : \|x\| < A\}$, where $A = \max\{A_3, |\ln(A_1)|, |\ln(A_2)|\}$. Then it is clear that Ω satisfies the requirement (i) of Lemma 2.20.

When $x \in \partial\Omega \cap \ker L = \partial\Omega \cap \mathbb{R}$ and x is a constant vector in $\mathbb{R}$, then by (H5),

$$\begin{aligned}QNx = \frac{1}{\omega}\int_{\kappa}^{\kappa+\omega} F\Big(t, \exp\{x(g_1(t))\}, \exp\{x(g_2(t))\}, \dots, \exp\{x(g_n(t))\},\\ \int_{-\infty}^{t} c(t,s)\exp\{x(s)\}\Delta s, \exp\{(\varphi x)(t-\delta(t))\}\Big)\Delta t \ne 0.\end{aligned}$$

Moreover, note that $J = I$ since $ImQ = \ker L$. In order to compute the Brouwer degree, let us consider the homotopy

$$\psi(\nu, x) = \nu x + (1-\nu)QNx \quad \text{for } \nu \in [0,1].$$

For any $x \in \partial\Omega \cap \ker L$, $\nu \in [0,1]$, we have $x\psi(\nu, x) > 0$, so $\psi(\nu, x) \neq 0$. Thus, the homotopy invariance of the degree produces

$$\deg(JQN, \Omega \cap \ker L, 0) = \deg(QN, \Omega \cap \ker L, 0) = \deg(x, \Omega \cap \ker L, 0) \neq 0,$$

where $\deg(\cdot)$ is the Brouwer degree. By now we have verified that Ω fulfills all requirements of Lemma 2.20. Therefore, system (3.5) has at least one ω-periodic solution in $\operatorname{Dom} L \cap \bar{\Omega}$, which in turn implies that (3.1) has at least one ω-periodic solution in $\operatorname{Dom} L \cap \bar{\Omega}$. This completes the proof. $\square$

Similarly, we can prove the following two results.

Theorem 3.5. *Let* (H1)–(H4) *hold. Moreover, assume*

(H6) *there exist constants $A_2 > A_1 > 0$ such that if $v_i \geq A_2$ for all $1 \leq i \leq n+2$, then*

$$\int_{\kappa}^{\kappa+\omega} F(t, v_1, \dots, v_n, \int_{-\infty}^{t} c(t,s)v_{n+1}\Delta s, v_{n+2})\Delta t > 0,$$

and if $0 < v_i \leq A_1$ for all $1 \leq i \leq n+2$, then

$$\int_{\kappa}^{\kappa+\omega} F(t, v_1, \dots, v_n, \int_{-\infty}^{t} c(t,s)v_{n+1}\Delta s, v_{n+2})\Delta t < 0.$$

Then system (3.1) *has at least one ω-periodic solution.*

Corollary 3.6. *Let* (H1)–(H4) *hold. Moreover, assume that one of the following two conditions is valid*

(H7) *there exist a constant $A > 0$ such that if $v_i \geq A$ for all $1 \leq i \leq n+2$, then for any $t \in I_\omega$, we always have*

$$F\Big(t, e^{v_1}, \dots, e^{v_n}, \int_{-\infty}^{t} c(t,s)e^{v_{n+1}}\Delta s, e^{v_{n+2}}\Big) > 0,$$
$$F\Big(t, e^{-v_1}, \dots, e^{-v_n}, \int_{-\infty}^{t} c(t,s)e^{-v_{n+1}}\Delta s, e^{-v_{n+2}}\Big) < 0,$$

(H8) *there exist a constant $A > 0$ such that if $v_i \geq A$ for all $1 \leq i \leq n+2$, then for any $t \in I_\omega$, we always have*

$$F\Big(t, e^{v_1}, \dots, e^{v_n}, \int_{-\infty}^{t} c(t,s)e^{v_{n+1}}\Delta s, e^{v_{n+2}}\Big) < 0,$$
$$F\Big(t, e^{-v_1}, \dots, e^{-v_n}, \int_{-\infty}^{t} c(t,s)e^{-v_{n+1}}\Delta s, e^{-v_{n+2}}\Big) > 0.$$

Then system (3.1) *has at least one ω-periodic solution.*

Corollary 3.7. *Assume that (H1)–(H4) and one of (H5)–(H8) hold, then system* (3.2) *and* (3.3) *have at least one positive ω-periodic solution.*

4. Applications

In this section, we aim to apply the results obtained in the previous section to establish sufficient conditions for the existence of periodic solutions in some specific delayed dynamic equations with feedback control.

Example 4.1. Consider the delayed dynamic equation with feedback control

$$
\begin{aligned}
x^{\Delta}(t) &= r(t) - \sum_{i=1}^{n} a_i(t)\exp\{x(g_i(t))\} \\
&\quad - \int_{-\infty}^{t} c(t,s)\exp\{x(s)\}\Delta s - d(t)\exp\{u(t-\delta(t))\}, \\
[\exp\{u(t)\}]^{\Delta} &= -a(t)\exp\{u(\sigma(t))\} + b(t)\exp\{x(t-\tau(t))\},
\end{aligned} \tag{4.1}
$$

where $r(t), a_i(t), d(t), a(t), b(t) \in C_{rd}(\mathbb{T},(0,\infty))$, $g_i(t) \in C_{rd}(\mathbb{T},\mathbb{T})$ satisfies $g_i(t) \le t$, $\delta(t), \tau(t) \in C_{rd}(\mathbb{T},\mathbb{R})$, $c \in C_{rd}(\mathbb{T}\times\mathbb{T},\mathbb{R}^+)$ satisfies $c(t+\omega, s+\omega) = c(t,s)$ and $\int_{-\infty}^{t} c(t,s)\Delta s$ is rd-continuous, all the functions are ω-periodic functions and $\omega > 0$ is a constant.

Theorem 4.2. *System* (4.1) *has at least one ω-periodic solution.*

Proof. Let $x(t)$ be an ω-periodic solution and satisfy

$$
\begin{aligned}
&\int_{\kappa}^{\kappa+\omega} \Big[r(t) - \sum_{i=1}^{n} a_i(t)\exp\{x(g_i(t))\} \\
&- \int_{-\infty}^{t} c(t,s)\exp\{x(s)\}\Delta s - d(t)\exp\{(\varphi x)(t-\delta(t))\}\Big]\Delta t = 0,
\end{aligned}
$$

where $(\varphi x)(t)$ is the same as that in (3.4). Then

$$
\begin{aligned}
\int_{\kappa}^{\kappa+\omega} r(t)\Delta t &= \int_{\kappa}^{\kappa+\omega} \Big[\sum_{i=1}^{n} a_i(t)\exp\{x(g_i(t))\} \\
&\quad + \int_{-\infty}^{t} c(t,s)\exp\{x(s)\}\Delta s + d(t)\exp\{(\varphi x)(t-\delta(t))\}\Big]\Delta t.
\end{aligned}
$$

Hence,

$$
\begin{aligned}
&\int_{\kappa}^{\kappa+\omega} \Big| r(t) - \sum_{i=1}^{n} a_i(t)\exp\{x(g_i(t))\} \\
&- \int_{-\infty}^{t} c(t,s)\exp\{x(s)\}\Delta s - d(t)\exp\{(\varphi x)(t-\delta(t))\}\Big|\Delta t \\
&\le 2\int_{\kappa}^{\kappa+\omega} r(t)\Delta t = 2\bar{r}\omega.
\end{aligned}
$$

Furthermore, since

$$
\lim_{(v_1,\dots,v_{n+2})\to\infty} \Big(r(t) - \sum_{i=1}^{n} a_i(t)v_i - \int_{-\infty}^{t} c(t,s)v_{n+1}\Delta s - d(t)v_{n+2}\Big) = -\infty
$$

and

$$
\lim_{(v_1,\dots,v_{n+2})\to 0} \Big(r(t) - \sum_{i=1}^{n} a_i(t)v_i - \int_{-\infty}^{t} c(t,s)v_{n+1}\Delta s - d(t)v_{n+2}\Big) = r(t) > 0.
$$

By Theorem 3.4, we see that system (4.1) has at least one ω-periodic solution. $\square$

Example 4.3. Consider the delayed dynamic equation with feedback control

$$
\begin{aligned}
x^{\Delta}(t) &= r(t) - \prod_{i=1}^{n} a_i(t)\exp\{x(g_i(t))\} - d(t)\exp\{u(t-\delta(t))\},\\
[\exp\{u(t)\}]^{\Delta} &= -a(t)\exp\{u(\sigma(t))\} + b(t)\exp\{x(t-\tau(t))\},
\end{aligned}
\tag{4.2}
$$

where $r(t), a_i(t), d(t), a(t), b(t) \in C_{rd}(\mathbb{T}, (0,\infty))$, $g_i(t) \in C_{rd}(\mathbb{T}, \mathbb{T})$ satisfies $g_i(t) \le t$, $\delta(t), \tau(t) \in C_{rd}(\mathbb{T}, \mathbb{R})$, all the functions are ω-periodic functions and $\omega > 0$ is a constant.

Theorem 4.4. *System* (4.2) *has at least one ω-periodic solution.*

The proof of the above theorem is the same as that of Theorem 4.2, we omit it.

Example 4.5. Consider the delayed dynamic equations with feedback control

$$
\begin{aligned}
x^{\Delta}(t) &= r(t)\frac{K(t) - \exp\{x(g(t))\}}{K(t) + c(t)\exp\{x(g(t))\}} - d(t)\exp\{u(t-\delta(t))\},\\
[\exp\{u(t)\}]^{\Delta} &= -a(t)\exp\{u(\sigma(t))\} + b(t)\exp\{x(t-\tau(t))\},
\end{aligned}
\tag{4.3}
$$

$$
\begin{aligned}
x^{\Delta}(t) &= r(t) - \sum_{i=1}^{n} \frac{a_i(t)\exp\{x(g_i(t))\}}{1 + c_i(t)\exp\{x(g_i(t))\}} - d(t)\exp\{u(t-\delta(t))\},\\
[\exp\{u(t)\}]^{\Delta} &= -a(t)\exp\{u(\sigma(t))\} + b(t)\exp\{x(t-\tau(t))\},
\end{aligned}
\tag{4.4}
$$

$$
\begin{aligned}
x^{\Delta}(t) &= r(t) + m(t)\exp\{px(g(t))\} - c(t)\exp\{qx(g(t))\} - d(t)\exp\{u(t-\delta(t))\},\\
[\exp\{u(t)\}]^{\Delta} &= -a(t)\exp\{u(\sigma(t))\} + b(t)\exp\{x(t-\tau(t))\},
\end{aligned}
\tag{4.5}
$$

$$
\begin{aligned}
x^{\Delta}(t) &= r(t) - \frac{\exp\{\theta x(g(t))\}}{K(t)^{\theta}} - d(t)\exp\{u(t-\delta(t))\},\\
[\exp\{u(t)\}]^{\Delta} &= -a(t)\exp\{u(\sigma(t))\} + b(t)\exp\{x(t-\tau(t))\},
\end{aligned}
\tag{4.6}
$$

where $r(t), a_i(t), d(t), a(t), b(t), c_i(t), K(t), m(t), c(t)$ are in $C_{rd}(\mathbb{T}, (0,\infty))$; $g(t)$ and $g_i(t)$ are in $C_{rd}(\mathbb{T}, \mathbb{T})$ and satisfy $g(t) \le t$, $g_i(t) \le t$; $\delta(t), \tau(t)$ are in $C_{rd}(\mathbb{T}, \mathbb{R})$; p, q, θ are positive constants with $q > p$; all the functions are ω-periodic functions and $\omega > 0$ is a constant.

By Theorems 3.4 and 3.5, one can easily reach the following result.

Theorem 4.6. *Each of* (4.3)-(4.6) *has at least one ω-periodic solution.*

Remark 4.7. Let $\mathbb{T} = \mathbb{R}$ or $\mathbb{T} = \mathbb{Z}$ and $\hat{x}(t) = \exp\{x(t)\}$, $\hat{u}(t) = \exp\{u(t)\}$. Then the dynamic equations (4.1)-(4.6) with feedback control reduce to the well-known continuous or discrete time nonautonomous logistic equation with several deviating argument and feedback control [15, 17], multiplicative logistic type equation with several deviating argument and feedback control [9, 15], generalized food-limited population model with deviating arguments and feedback control, Michalis-Menton type single species growth model with deviating arguments and feedback control [15], Lotka-Volterra type single species growth model with deviating arguments and feedback control, nonautonomous Gilpin-Ayala single species model with feedback control, respectively, which have been studied extensively in the literature.

Conclusion. In this paper, with the help of continuation theorem based on Gaines and Mawhin's coincidence degree theory, we study the existence of periodic solutions for a class of delayed dynamic equations with feedback control. The system under consideration is more general, including many specific dynamic equations. We explore the periodicity on time scales. Specially, when the time scale $\mathbb{T}$ is chosen as $\mathbb{R}$ or $\mathbb{Z}$, the existence of the periodic solutions of many well-known continuous or discrete time population models follows.

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