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AN ITERATION METHOD FOR CONTROLLABILITY OF SEMILINEAR PARABOLIC EQUATIONS

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Abstract. We present a method based on Picard's idea to construct a sequence of controls and a sequence of solutions of linearized systems such that their limits form a solution to the control problem. By doing this, we simplified the works in the references, and deduced the controllability for semilinear coupled parabolic systems.

1. Introduction

The controllability for semilinear parabolic equations has been intensively studied in previous decades; see for example [2, 14, 3, 5, 6, 7]. Some new systems were studied recently, such as nonlinear heat equation with memory effects, Lavanya [9], and some semilinear parabolic equations arising in finance, Sakthivel [11]. Tang and Zhang [12] studied null controllability for forward and backward linear stochastic parabolic equations. It may not be easy to generalize their results to nonlinear cases.

Generally speaking, all these works are based on the same idea: the methods of approximating controllability for linear systems are based on the unique continuation of solutions from an open set, while the exact null-controllability depends on the inverse estimates of dual observed systems. Furthermore, the controllability are generalized to semilinear systems by Schauder fixed-point arguments. All the arguments are based on the compact embedding of H_0^1 into L^2. The Schauder fixed-point arguments are good for semilinear parabolic systems with globally Lipschitz nonlinearity, but difficult for those with superlinear terms. In this paper, we try to do it in other ways, and give a method based on iteration idea, by which we simplified some works concerning the controllability of semilinear parabolic systems in literatures. Moreover, we deduced the controllability of coupled parabolic systems, which seems impossible to be obtained by previous arguments.

This paper is organized as follows. In Section 2 we recall two lemmas which are necessary for our analysis. In Section 3 we prove the exact null-controllability of semilinear parabolic systems with mobile distributed control, to illustrate our method. We discuss the controllability of coupled parabolic systems in Section 4.

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2. Preliminaries

For convenience, we recall the following two propositions, from [10].

Proposition 2.1. *Let X, H, Y be Banach spaces with $X \subset\subset H \subseteq Y$ and X reflexive. Suppose that $\{u_n\}_{n=1}^{\infty}$ is a sequence that is bounded in $L^2(0,T;X)$, and that $\{\frac{du_n}{dt}\}_{n=1}^{\infty}$ is bounded in $L^p(0,T;Y)$, for some $p>1$. Then there is a subsequence of $\{u_n\}_{n=1}^{\infty}$ that converges strongly in $L^2(0;T;H)$.*

Proposition 2.2. *Let $\mathcal{O}$ be a bounded open set in $\mathbb{R}^m$, and let $\{g_j\}_{j=1}^{\infty}$ be a sequence of functions in $L^p(\mathcal{O})$ with*
$$\|g_j\|_{L^p(\mathcal{O})} \le C \quad \text{for all } j \in \mathbb{Z}^+.$$
If $g_j \to g \in L^p(\mathcal{O})$ almost everywhere, then $g_j \rightharpoonup g$ in $L^p(\mathcal{O})$.

3. Exact null-controllability for semilinear systems with mobile distributed control

Consider the controllability for the Dirichlet problem
$$\frac{\partial u}{\partial t} = \Delta u + g(u) + f(x,t)\chi_{\omega(t)}(x) \quad \text{in } Q_T, \tag{3.1}$$
$$u = 0 \quad \text{in } \Sigma_T, \quad u|_{t=0} = u_0, \quad u_0 \in L^2(\Omega), \tag{3.2}$$
where Ω is an open bounded domain in $\mathbb{R}^n$ with boundary $\partial\Omega$, $Q_T = \Omega \times (0,T)$, $\Sigma_T = \partial\Omega \times (0,T)$, f is a distributed control in $L^2(Q_T)$, $\omega(\cdot)$ is a mobile support, and $\chi_{\omega(\cdot)}$denotes its indication function.

We assume growth conditions as follows:
$$|g(u)| \le c_1 + c_2|u|^p, \quad p = 1 + 4/n, \quad \forall u \in R, \tag{3.3}$$
$$g(u)u \le c_3 u^2, \quad \forall u \in R, \tag{3.4}$$
$$g(0) = 0, \quad \lim_{u\to 0} \frac{g(u)}{u} = g'(0). \tag{3.5}$$
Conditions (3.3)-(3.5) ensure the existence of at least one generalized solution of (3.1)-(3.2) from $C([0,T];L^2(\Omega)) \bigcap H_0^{1,0}(Q_T)$ [8, pp. 466-467]. Khapalov proved that (3.1)-(3.5) is exactly null-controllable [6], but his generalization to nonlinear systems is somewhat complicated. So in this paper we put forward some different methods.

The corresponding linear system is as follows:
$$\frac{\partial w}{\partial t} = \Delta w + a(x,t)w + f(x,t)\chi_{\omega(t)}(x) \quad \text{in } Q_T, \tag{3.6}$$
$$w = 0 \quad \text{in} \quad \Sigma_T, \quad w|_{t=0} = w_0 \in L^2(\Omega). \tag{3.7}$$
The counterparts to (3.3)-(3.4) are
$$a(x,t) \le c_3, \quad a \in L^{r_2}(0,T;L^{r_1}(\Omega)), \tag{3.8}$$
$$\frac{1}{r_2} + \frac{n}{2r_1} = 1, \tag{3.9}$$
$$r_1 \in (\frac{n}{2}, \infty], \quad r_2 \in [1,\infty), \quad \text{for } n \ge 2, \tag{3.10}$$
$$r_1 \in [1,\infty], \quad r_2 \in [1,2], \quad \text{for } n = 1. \tag{3.11}$$

Khapalov proved that (3.6)-(3.11) is exactly null-controllable by a mobile internal control, while the support may be chosen to be arbitrarily small. Moreover, he gives the following estimates:

$$\|w\|_{L^q(Q_T)}, \|w\|_{\mathcal{B}} \le c(T)(\|w_0\|_{L^2(\Omega)} + \|v\|_{L^2(\omega(\cdot))}), \tag{3.12}$$
$$\|f\|_{L^2(\omega(\cdot))} \le M\|w_0\|_{L^2(\Omega)}, \tag{3.13}$$

where $q = 2 + 4/n$,

$$\|w\|_{\mathcal{B}} = \|w\|_{C([0,T];L^2(\Omega))} + \Big(\int_0^T \int_\Omega \|\nabla w\|_{R^n}^2 \mathrm{d}x\mathrm{d}t\Big)^{1/2},$$

$c(T)$ and M do not depend on the choice of $a(x,t)$. Then he deduced the exact null-controllability for (3.1)-(3.2) by Schauder fixed-point argument, with $p \le 1 + 4/n$. Now we do in a different way.

As usual, we introduce the non-linearity

$$h(s) = \begin{cases} g(s)/s, & s \neq 0, \\ g'(0), & s = 0, \end{cases}$$

and construct a "linearized" system

$$\frac{\partial u}{\partial t} = \Delta u + h(z)u + f(x,t)\chi_{\omega(t)}(x) \quad \text{in } Q_T \tag{3.14}$$

with the same initial and boundary conditions as (3.7).

We construct a sequence in $L^2(Q_T)$ as follows: Take any $z_1 \in L^2(Q_T)$, substitute it for z in (3.14), then there exists a control $f_1 \in L^2(Q_T)$ such that the corresponding solution u_1 satisfies that $u_1(T) = 0$; Take $z_2 = u_1$, substitute it for z in (3.14), then there exists a control $f_2 \in L^2(Q_T)$ such that the corresponding solution u_2 satisfies that $u_2(T) = 0$; Repeating this process yields two sequences $\{z_n\}_{n=1}^\infty$(or $\{u_n\}_{n=1}^\infty$) and $\{f_n\}_{n=1}^\infty$ in $L^2(Q_T)$. This process can be illustrated as follows:

$$z_1 \Rightarrow f_1 \Rightarrow u_1 = z_2 \Rightarrow f_2 \Rightarrow u_2 = \dots,$$
$$\frac{\partial u_n}{\partial t} = \Delta u_n + h(z_n)u_n + f_n\chi_{\omega(t)}. \tag{3.15}$$

Now we prove that they converge in some topology, their limits solve (3.1)-(3.2), and the limit satisfies that $u(T) = 0$. To simplify notation, we denote $\int_{Q_T} \cdot \mathrm{d}x\mathrm{d}t$ by $\int_{Q_T} \cdot$.

Condition (3.5) implies that there is $r > 0$ such that

$$|h(z)| \le c_4 = |g'(0)| + 1$$

when $|z| < r$. On the other hand, it follows from (3.3) that

$$\begin{aligned} |h(z)| &\le \frac{c_1}{|z|} + c_2|z|^{p-1} \\ &\le \frac{c_1}{|z|^p}|u|^{p-1} + c_2|z|^{p-1} \\ &\le \frac{c_1}{r^p}|z|^{p-1} + c_2|z|^{p-1} \\ &\le c_5|z|^{p-1} \end{aligned}$$

when $|z| \geq r$. Therefore,

$$\begin{aligned}
&\int_{Q_T} |h(z_n)u_n|^{2n/(n+2)} \\
&= \int_{|z_n|<r} |h(z_n)u_n|^{2n/(n+2)} + \int_{|z_n|\geq r} |h(z_n)u_n|^{2n/(n+2)} \\
&\leq c_4 \int_{Q_T} |u_n|^{2n/(n+2)} + c_5 \int_{Q_T} |z_n|^{\frac{8}{n+2}} |u_n|^{2n/(n+2)}.
\end{aligned} \tag{3.16}$$

It follows from the continuous embedding of $L^{2+4/n}(Q_T)$ into $L^{2n/(n+2)}(Q_T)$ that

$$\int_{Q_T} |u_n|^{2n/(n+2)} \leq c_4 \int_{Q_T} |u_n|^{2+4/n}. \tag{3.17}$$

By Hölder inequality we have

$$\begin{aligned}
&\int_{Q_T} |z_n|^{\frac{8}{n+2}} |u_n|^{2n/(n+2)} \\
&\leq c_5 \Big(\int_{Q_T} |z_n|^{2+4/n} \Big)^{4(n+1)/(n+2)^2} \Big(\int_{Q_T} |u_n|^{2+4/n} \Big)^{n^2/(n+2)^2}.
\end{aligned} \tag{3.18}$$

Combining (3.16), (3.17) and (3.18) yields

$$\begin{aligned}
&\int_{Q_T} |h(z_n)u_n|^{2n/(n+2)} \\
&\leq c_4 \int_{Q_T} |u_n|^{2+4/n} + c_5 \Big(\int_{Q_T} |z_n|^{2+4/n} \Big)^{4(n+1)/(n+2)^2} \Big(\int_{Q_T} |u_n|^{2+4/n} \Big)^{n^2/(n+2)^2}.
\end{aligned}$$

Due to (3.12) and (3.13), the sequence $\{u_n\}_{n=1}^{\infty}$ or he sequence $\{z_n\}_{n=1}^{\infty}$ is bounded in $L^{2+4/n}(Q_T)$. So $\{h(z_n)u_n\}_{n=1}^{\infty}$ is bounded in $L^{2n/(n+2)}(Q_T)$, and it has a weak convergent subsequence. Furthermore, it follows from (3.12) and (3.13) that $\{u_n\}_{n=1}^{\infty}$ is bounded in $L^2(0,T;H_0^1(\Omega))$, and $\{f_n\}_{n=1}^{\infty}$ is bounded in $L^2(\omega(\cdot))$. By extracting subsequences (that we denote by the index j to simplify the notation) we have

$$\begin{gathered}
u_j \rightharpoonup u \quad \text{in } L^2(0,T;H_0^1(\Omega)), \\
\Delta u_j \rightharpoonup \Delta u \quad \text{in } L^2(0,T;H^{-1}(\Omega)), \\
f_j \rightharpoonup f \quad \text{in } L^2(\omega(\cdot)).
\end{gathered}$$

Moreover, it follows from the continuous embedding of $H_0^1(\Omega)$ into $L^{2n/(n-2)}$ that $L^{2n/(n+2)}(\Omega)$ is continuously embedded in $H^{-1}(\Omega)$. Therefore, $L^{2n/(n+2)}(Q_T)$ is continuously embedded in $L^{2n/(n+2)}(0,T;H^{-1}(\Omega))$, and thus $\{h(z_n)u_n\}_{n=1}^{\infty}$ is bounded in $L^{2n/(n+2)}(0,T;H^{-1}(\Omega))$. These facts imply that $\frac{\partial u_n}{\partial t}$ is bounded in $L^{2n/(n+2)}(0,T;H^{-1}(\Omega))$, so it has subsequence which converges weakly to $\frac{\partial u}{\partial t}$. Next we will show that the subsequence of $\{h(z_n)u_n\}_{n=1}^{\infty}$ converges weakly to $h(u)u$ in $L^{2n/(n+2)}(0,T;H^{-1}(\Omega))$.

Remark 3.1. The constants c_4 and c_5 above are generic constants.

Substituting $H_0^1(\Omega)$ for X, $L^2(\Omega)$ for H, and $H^{-1}(\Omega)$ for Y in Proposition 2.1, by extracting subsequence we have that $\{u_j\}_{j=1}^{\infty}$ converges to u strongly in $L^2(Q_T)$. A classical result in real analysis tells that another subsequence $\{u_j\}_{j=1}^{\infty}$ converges to u almost everywhere in Q_T. It follows, using the continuity of g, that $h(z_j)$

converges to $h(u)$ almost everywhere. Combining this fact with Proposition 2.2 leads to that $\{h(z_j)u_j\}_{j=1}^{\infty}$ converges to $h(u)u$ weakly in $L^{2n/(n+2)}(Q_T)$, and thus in $L^{2n/(n+2)}(0,T;H^{-1}(\Omega))$. Taking limit of (3.15) in $L^{2n/(n+2)}(0,T;H^{-1}(\Omega))$, we see that u and v solve (3.1)-(3.2).

It remains to show that $u(T)=0$. The basic idea is that $\{u_j\}_{j=1}^{\infty}$ converges to u strongly in $L^2(0,T;L^2(\Omega))$ implies that a subsequence $\{u_j(t)\}_{j=1}^{\infty}$ converges to $u(t)$ strongly in $L^2(\Omega)$, for almost every t in $[0,T]$. So we may think that $\{u_j(T)\}_{j=1}^{\infty}$ converges to $u(T)$ strongly in $L^2(\Omega)$. It follows from $u_n(T)=0$ that $u(T)=0$.

However, the a.e. convergence of $\{u_n\}$ to u and that $u_n(T)=0$ do not always imply that $u(T)=0$, e.g., for a sequence $\{t^n\}$, $t\in(0,1)$, $T=1$. To avoid the strange case with steep curve near $t=T$, we require that each control system as (3.15) vanishes in advance, e.g.,

$$u_n(t)\equiv 0$$

for $t\in(\frac{T}{2},T)$ or $(\frac{3T}{4},T)$. Then it follows from the a.e. convergence of u_n to u that $u(t)=0$ in $(\frac{T}{2},T)$ or $(\frac{3T}{4},T)$. On the other hand, it follows from the regularity of parabolic systems that

$$u\in C([0,T];L^2(\Omega)).$$

So $u(T)=0$. We summarize the analysis above as follows:

Theorem 3.2. *Suppose $n\ge 3$ and that* (3.3), (3.4), (3.5) *hold. Then* (3.1)-(3.2) *is exactly null-controllable by a mobile internal control, and that the support can be chosen arbitrarily small.*

Remark 3.3. Schauder fixed-point argument requires that map $N: z\mapsto u$ is continuous and compact, from a bounded closed convex set into itself. These are difficult to verify for systems with superlinear terms. Our method just requires that the iteration generates bounded sequences of solutions and controls.

Remark 3.4. Of course, our method is also fit for those systems with globally Lipschitz nonlinear terms or point controls. We take a system with superlinear term and a mobile internal control just to show our method and its advantage.

4. Controllability of coupled parabolic systems

Consider a coupled parabolic system

$$u_t-\Delta u=\Phi(u,v)+f\chi_{\omega_1}\quad\text{in } Q_T, \tag{4.1}$$
$$v_t-\Delta v=\Psi(u,v)+g\chi_{\omega_2}\quad\text{in } Q_T, \tag{4.2}$$
$$u(x,t)=v(x,t)=0\quad\text{on } \Sigma_T, \tag{4.3}$$
$$u|_{t=0}=u_0,\quad v|_{t=0}=v_0,\quad u_0,v_0\in L^2(\Omega), \tag{4.4}$$

where ω_1 and ω_2 are proper subsets of Ω.

The system above is a widely used mathematical modelling for many chemical, physical, biological or ecological phenomena. Many papers are devoted to study the existence and uniqueness of local solution, global existence and blow-up of solutions (see [1, 4]), but less is known about its controllability. So we consider this problem. Suppose that Φ and Ψ are continuous functions, Φ is globally Lipschitz continuous and differentiable with respect to u, so is Ψ with respect to v, and

$$\Phi(0,v)=\Psi(u,0)=0,\quad \forall u,v\in\mathbb{R}.$$

The solvability will be implied in our arguments on controllability.

The globally approximate controllability and finite dimensional exact controllability for (4.1)-(4.4) is formulated as follows (see [14] for single parabolic equation): Given u_T, v_T in $L^2(\Omega)$, $\varepsilon > 0$ and finite dimensional subspaces E_1, E_2 of $L^2(\Omega)$, there exist controls f and g in $L^2(Q_T)$ such that

$$\|u(T) - u_T\|_{L^2(\Omega)} \leq \varepsilon, \quad \Pi_1 u(T) = \Pi_1 u_T, \tag{4.5}$$
$$|v(T) - v_T|_{L^2(\Omega)} \leq \varepsilon, \quad \Pi_2 v(T) = \Pi_2 v_T, \tag{4.6}$$

where Π_1 and Π_2 are the orthogonal projections from $L^2(\Omega)$ into E_1 and E_2 respectively.

It seems difficult or impossible to construct a linearized systems for (4.1)-(4.4), as (3.14) for (3.1)-(3.2). We will deduce the controllability without linearized system. At first, we construct sequences of solutions and controls as follows: Take any $v_1 \in L^2(Q_T)$, substitute it for v in (4.1), then it follows from the controllability results on single parabolic systems (see [14]) that there exists control $f_1 \in L^2(Q_T)$ such that the corresponding solution u_1 of (4.1) satisfies (4.5). Substitute u_1 for u in (4.2), there exists control $g_1 \in L^2(Q_T)$ such that the corresponding solution v_2 satisfies (4.6). Then substitute v_2 for v in (4.1), and get f_2 and u_2. Repeating this process yields sequences $\{u_n\}_{n=1}^{\infty}$, $\{v_n\}_{n=1}^{\infty}$, $\{f_n\}_{n=1}^{\infty}$ and $\{g_n\}_{n=1}^{\infty}$, which satisfy (4.5) and (4.6). This process can be illustrated as follows:

$$v_1 \Rightarrow f_1 \Rightarrow u_1 \Rightarrow g_1 \Rightarrow v_2 \Rightarrow f_2 \Rightarrow u_2 \Rightarrow \dots,$$
$$\frac{\partial u_n}{\partial t} - \Delta u_n = \Phi(u_n, v_n) + f_n \chi_{\omega_1}, \tag{4.7}$$
$$\frac{v_{n+1}}{\partial t} - \Delta v_{n+1} = \Psi(u_n, v_{n+1}) + g_n \chi_{\omega_2}. \tag{4.8}$$

We will prove that these sequences converge in some topology, and that their limits solve (4.1)-(4.4), which satisfy (4.5) and (4.6).

Due to the global Lipschitz continuity of Φ and Ψ, sequences $\{f_n\}_{n=1}^{\infty}$, $\{g_n\}_{n=1}^{\infty}$, $\{u_n\}_{n=1}^{\infty}$ and $\{v_n\}_{n=1}^{\infty}$ are bounded in $L^2(Q_T)$. So they have subsequences which converge weakly. Let $u_j \rightharpoonup u$, $v_j \rightharpoonup v$, $f_j \rightharpoonup f$ and $g_j \rightharpoonup g$ in $L^2(Q_T)$. By similar arguments to those in last section, we have that

$$\Phi(u_n, v_n) \rightharpoonup \Phi(u, v),$$
$$\Psi(u_n, v_{n+1}) \rightharpoonup \Psi(u, v)$$

in $L^2(Q_T)$. Taking limits of (4.7) and (4.8), one can easily verify that u, v, f and g solve (4.1)-(4.4), and satisfy (4.5)-(4.6). We summarize our analysis as follows:

Theorem 4.1. *Suppose that Φ and Ψ are continuous functions, Φ is globally Lipschitz continuous and differentiable with respect to u, so is Ψ with respect to v, and*

$$\Phi(0, v) = \Psi(u, 0) = 0, \quad \forall u, v \in \mathbb{R}.$$

Then (4.1)-(4.4) *is globally approximately controllable and finite-dimensional exactly controllable.*

Remark 4.2. By our method, all results concerning controllability of semilinear parabolic systems can be extended to coupled parabolic systems. For example, We may consider exact null-controllability of semilinear coupled parabolic systems with superlinear terms, mobile internal controls or point-controls.

From the point view of application, it is reasonable to consider coupled parabolic system with a single control as follows:

$$u_t - \Delta u = \Phi(u, v) + f\chi_\omega \quad \text{in } Q_T, \tag{4.9}$$

$$v_t - \Delta v = \Psi(u, v) \quad \text{in } Q_T, \tag{4.10}$$

$$u(x, t) = v(x, t) = 0 \quad \text{on } \Sigma_T, \tag{4.11}$$

$$u|_{t=0} = u_0, \quad v|_{t=0} = v_0, \quad u_0, v_0 \in L^2(\Omega). \tag{4.12}$$

Theorem 4.3. *Under the same conditions as in Theorem 4.1, system* (4.9)-(4.12) *is globally approximately controllable and finite-dimensional exactly controllable.*

The proof is similar to that of Theorem 4.1, but there is some difference. Let us give a sketch: Given $u_T \in L^2(\Omega)$, $\varepsilon > 0$ and finite-dimensional subspace E of $L^2(\Omega)$. Take any $v_1 \in L^2(Q_T)$, substitute it for v in (4.9), then there exist control f_1 and the corresponding solution u_1 such that $\|u_1(T) - u_T\|_{L^2(\Omega)} < \varepsilon$, and $\Pi_E u_1(T) = \Pi_E u_T$. Substitute u_1 for u in (4.10), then there is an unique solution v_2. Substitute v_2 for v in (4.9), there exist control f_2 and the corresponding solution u_2, which satisfy that $\|u_2(T) - u_T\|_{L^2(\Omega)} < \varepsilon$, and $\Pi_E u_2(T) = \Pi_E u_T$. Repeating this process yields sequences $\{u_n\}_{n=1}^\infty$, $\{v_n\}_{n=1}^\infty$ and $\{f_n\}_{n=1}^\infty$. Their limits solve (4.9)-(4.12), and satisfy

$$\|u(T) - u_T\|_{L^2(\Omega)} < \varepsilon,$$
$$\Pi_E u(T) = \Pi_E u_T,$$

where Π_E represents the orthogonal projection from $L^2(\Omega)$ into E.

Theorem 4.1 and Theorem 4.3 can be generalized to multi-coupled parabolic systems as follows:

$$\begin{gathered}
\frac{\partial u_1}{\partial t} - \Delta u_1 = \Phi_1(u_1, u_2, \dots, u_m) + f_1\chi_{\omega_1} \quad \text{in } Q_T, \\
\frac{\partial u_2}{\partial t} - \Delta u_2 = \Phi_2(u_1, u_2, \dots, u_m) + f_2\chi_{\omega_2} \quad \text{in } Q_T, \\
\dots \\
\frac{\partial u_m}{\partial t} - \Delta u_m = \Phi_m(u_1, u_2, \dots, u_m) + f_m\chi_{\omega_m} \quad \text{in } Q_T, \\
u_i|_{\Sigma_T} = 0, \quad u_i|_{t=0} = u_{i0}, \quad u_{i0} \in L^2(\Omega), \quad i = 1, 2, \dots, m.
\end{gathered}$$

5. Examples

The following example, from combustion theory [13], illustrates our results.

$$\frac{\partial u_1}{\partial t} - d_1\Delta u_1 - (u_2)^p h(u_1) = f\chi_{\omega_1} \quad \text{in } Q_T, \tag{5.1}$$

$$\frac{\partial u_2}{\partial t} - d_2\Delta u_2 - (u_2)^p h(u_1) = g\chi_{\omega_2} \quad \text{in } Q_T, \tag{5.2}$$

$$u_1(x, t) = u_2(x, t) = 0 \quad \text{on} \quad \Sigma_T, \tag{5.3}$$

where $h(s) = |s|^\gamma \exp(-\alpha/|s|)$, and p, α, γ are positive constants. u_1 represents a temperature, while u_2 represents a concentration. The nonlinearity vanishes at $u_1 = 0$ and $u_2 = 0$. It is Lipschitz continuous, provided that $p = \gamma = 1$. It follows from section 4 that the coupled system (5.1)-(5.3) is globally approximately controllable and exactly null-controllable.

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