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MULTIPLICITY THEOREMS FOR SEMIPOSITONE p-LAPLACIAN PROBLEMS

XUDONG SHANG

ABSTRACT. In this article, we study the existence of solutions for the semipositone p-Laplacian problems. Under a subliner behavior at infinity, using degree theoretic arguments based on the degree map for operators of type $(S)_+$, we prove the existence of at least two nontrivial solutions.

1. INTRODUCTION

In this article, we study the existence of multiple solutions for the following nonlinear elliptic boundary-value problem

$$\begin{gathered} -\Delta_p u = \lambda f(u) \quad x \in \Omega, \\ u = 0 \quad x \in \partial\Omega, \end{gathered} \tag{1.1}$$

where $-\Delta_p u = -\operatorname{div}(|\nabla u|^{p-2}\nabla u)$ is the p-Laplacian operator, $p > 1$, $\lambda > 0$, $\Omega \subseteq \mathbb{R}^n (n \geq 1)$ is a bounded open set with smooth boundary, $f : [0, +\infty) \to \mathbb{R}$ is a continuous function satisfying the condition $f(0) < 0$. Such problems are usually referred in the literature as semipositone problems comparing with the positone case of $f(0) \geq 0$.

Such semipositone problems arise in buckling of mechanical systems, design of suspension bridges, chemical reactions, astrophysics. As pointed out by Lions in [9], semipositone problems are mathematically very challenging. The semilinear semipositone problems have been studied for more than a decade. The usual approaches to such semipositone problems are through quadrature methods [5, 7], the method of sub-super-solution [4], bifurcation theory [1, 12]. We refer the reader to the survey paper [6] and references therein. See [3, 11] for related results for multiparameter semipositone problems.

Costa, Tehrani and Yang [8] studied the semipositone problems

$$\begin{gathered} -\Delta u = \lambda f(u) \quad x \in \Omega, \\ u = 0 \quad x \in \partial\Omega. \end{gathered} \tag{1.2}$$

They applied variational methods for locally Lipschitz functional and obtained positive solutions for sublinear and superlinear cases.

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Let us consider the sublinear case. It is well know that the main difficulty in proving the existence of a positive solution for (1.1) consists in finding a positive sub-solution. As a matter of fact, it can be easily seen, no positive sub-solution can exist if $f(u)$ does not assume positive values.

Our main objective in this article is to using degree theoretic arguments based on the degree map for operators of type $(S)_+$, improve the problem (1.2) to quasilinear case, we obtain two nontrivial solutions for problem (1.1) in the sublinear case.

The hypotheses on the nonlinearity f in problem (1.1) are as follows:

- (F1) $f(0) < 0$,
- (F2) $\lim_{s\to+\infty} \frac{f(s)}{s^{p-1}} = 0$,
- (F3) $F(\beta) > 0$ for some $\beta > 0$, where $F(s) = \int_0^s f(t)dt$.

Under the above assumptions, we state our main result for problem (1.1).

Theorem 1.1. *Suppose that* (F1)–(F3) *hold. Then, there exists* $\Lambda_0 > 0$ *such that* (1.1) *has at least two nontrivial solutions for all* $\lambda > \Lambda_0$.

The rest of this article is organized as follows. In section 2, we shall present some mathematical background needed in the sequel. Section 3 contains the proof of our main result.

2. Preliminaries

First, we recall some basic facts about the spectrum of $(-\Delta_p, W_0^{1,p}(\Omega))$ with weights. Let $v \in L_+^\infty(\Omega)$, $v \neq 0$, $L_+^\infty(\Omega) = \{u \in L^\infty(\Omega) : u \geq 0,\ x \in \Omega\}$. Consider the nonlinear weighted eigenvalue problem

$$\begin{gathered} -\Delta_p u = \lambda v(x)|u|^{p-2}u \quad x \in \Omega, \\ u = 0 \quad x \in \partial\Omega. \end{gathered} \tag{2.1}$$

This problem has a smallest eigenvalue denoted by $\lambda_1(v)$ which is positive, isolated, simple and admits the variational characterization

$$\lambda_1(v) = \inf\big\{ \frac{\int_\Omega |\nabla u|^p dx}{\int_\Omega v(x)|u|^p dx} : u \in W_0^{1,p}(\Omega), u \neq 0 \big\}. \tag{2.2}$$

In (2.2) the infimum is attained at a corresponding eigenfunction ϕ_1 taken to satisfy $\|\phi_1\|_p = 1$. If $v_1, v_2 \in L_+^\infty(\Omega) \setminus \{0\}$ are two weight functions such that $v_1 \leq v_2$ a.e. on Ω with strict inequality on a set of positive measure, then $\lambda_1(v_2) < \lambda_1(v_1)$. As usually we denote $\lambda_1 = \lambda_1(1)$. If the $u \in W_0^{1,p}(\Omega)$ is an eigenfunction corresponding to an eigenvalue $\lambda \neq \lambda_1(v)$, then u must change sign.

We extend f as $f(s) = f(0)$ for all $s < 0$. It's well know that u is a weak solution to (1.1) if $u \in W_0^{1,p}(\Omega)$ and

$$\int_\Omega |\nabla u|^{p-2}\nabla u \nabla\varphi dx - \lambda \int_\Omega f(u)\varphi dx = 0$$

for every $\varphi \in W_0^{1,p}(\Omega)$.

For each $u \in W_0^{1,p}(\Omega)$, we define $I, K : W_0^{1,p}(\Omega) \to W_0^{-1,p'}(\Omega)$ by

$$\langle I(u), \varphi\rangle = \int_\Omega |\nabla u|^{p-2}\nabla u \nabla\varphi dx, \quad \forall \varphi \in W_0^{1,p}(\Omega),$$

$$\langle K(u), \varphi\rangle = \int_\Omega f(u)\varphi dx, \quad \forall \varphi \in W_0^{1,p}(\Omega).$$

Hence, the weak solution of (1.1) are exactly the solutions of the equation $I-\lambda F = 0$.

Definition 2.1 ([14])**.** Let X be a reflexive Banach space and X^* its topological dual. We recall the mapping $A : X \to X^*$ is of type $(S)_+$, if any sequence u_n in X satisfying $u_n \rightharpoonup u_0$ in X and

$$\limsup_{n\to+\infty}\langle A(u_n), u_n - u_0\rangle \le 0$$

contains a convergent subsequence.

Now consider triples (A, Ω, x_0) such that Ω is a nonempty, bounded, open set in X, $A : \overline{\Omega} \to X^*$ is a demicontinuous mapping of type $(S)_+$ and $x_0 \notin A(\partial\Omega)$. On such triples Browder [2] defined a degree denoted by $\deg(A, \Omega, x_0)$, which has the following three basic properties:

- (i) (Normality) If $x_0 \in A(\Omega)$ then $\deg(A, \Omega, x_0) = 1$;
- (ii) (Domain additivity) If Ω_1, Ω_2 are disjoint open subsets of Ω and $x_0 \notin A(\overline{\Omega}\backslash(\Omega_1 \cup \Omega_2))$ then $\deg(A, \Omega, x_0) = \deg(A, \Omega_1, x_0) + \deg(A, \Omega_2, x_0)$;
- (iii) (Homotopy invariance) If $\{A_t\}_{t\in[0,1]}$ is a homotopy of type $(S)_+$ such that A_t is bounded for every $t \in [0, 1]$ and $x_0 : [0, 1] \to X^*$ is a continuous map such that $x_0(t) \notin A_t(\partial\Omega)$ for all $t \in [0, 1]$, then $\deg(A_t, \Omega, x_0(t))$ is independent of $t \in [0, 1]$.

Remark 2.2. The operator A is of type $(S)_+$ and B is compact implies that $A+B$ is of type $(S)_+$.

Lemma 2.3 ([10])**.** *If X is a reflexive Banach space, $U \subset X$ is open, $\psi \in C^1(U)$, ψ' is of type $(S)_+$, and there exist $x_0 \in X$ and numbers $\gamma < \mu$ and $r > 0$ such that*

- (i) $V = \{\psi < \mu\}$ *is bounded and* $\overline{V} \subset U$*;*
- (ii) $\{\psi \le \gamma\} \subseteq \overline{B_r(x_0)} \subset V$*;*
- (iii) $\psi'(x) \ne 0$ *for all* $x \in \{\gamma \le \psi \le \mu\}$*,*

then $\deg(\psi', V, 0) = 1$.

3. Proof of main results

In this section, first several technical results will be established.

Lemma 3.1. *The mapping $I : W_0^{1,p}(\Omega) \to W_0^{-1,p'}(\Omega)$ is of type $(S)_+$.*

Proof. Assume that $u_n \rightharpoonup u$ in $W_0^{1,p}(\Omega)$ and

$$\limsup_{n\to+\infty}\langle I(u_n), u_n - u\rangle \le 0.$$

Then we obtain

$$\limsup_{n\to+\infty}\langle I(u_n) - I(u), u_n - u\rangle \le 0.$$

By the monotonicity property of I we have

$$\lim_{n\to+\infty}\langle I(u_n) - I(u), u_n - u\rangle = 0;$$

i.e.,

$$\lim_{n\to+\infty}\int_\Omega(|\nabla u_n|^{p-2}\nabla u_n - |\nabla u|^{p-2}\nabla u)(\nabla u_n - \nabla u)dx = 0. \tag{3.1}$$

Observe that for all $x, y \in \mathbb{R}^n$,

$$|x-y|^p \le \begin{cases} (|x|^{p-2}x - |y|^{p-2}y)(x-y) & \text{if } p \ge 2,\\ [(|x|^{p-2}x - |y|^{p-2}y)(x-y)]^{p/2}(|x|+|y|)^{(2-p)p/2} & \text{if } 1 < p < 2. \end{cases}$$

Substituting x and y by ∇u_n and ∇u respectively and integrating over Ω, we obtain

$$\int_\Omega |\nabla u_n - \nabla u|^p dx \le \int_\Omega (|\nabla u_n|^{p-2}\nabla u_n - |\nabla u|^{p-2}\nabla u)(\nabla u_n - \nabla u)dx, \tag{3.2}$$

Using (3.1), passing to the limit in (3.2), we obtain

$$\lim_{n\to+\infty} \int_\Omega |\nabla u_n - \nabla u|^p dx = 0.$$

Thus $\nabla u_n \to \nabla u$ in $L^p(\Omega)$. Also $u_n \rightharpoonup u$ in $W_0^{1,p}(\Omega)$, which implies that $u_n \to u$ in $L^p(\Omega)$. Hence,

$$u_n \to u \quad \text{in } W_0^{1,p}(\Omega).$$

The proof is complete. ☐

Lemma 3.2. *The mapping $K : W_0^{1,p}(\Omega) \to W_0^{-1,p'}(\Omega)$ is compact.*

Proof. According to the hypotheses (F2) and the compactness of the embedding of $W_0^{1,p}(\Omega)$ into $L^p(\Omega)$, K is compactness as a map from $W_0^{1,p}(\Omega)$ to $W_0^{-1,p'}(\Omega)$. ☐

Using Remark 2.2 we have the following result.

Lemma 3.3. *The mapping $J = I - \lambda K$ is type of $(S)_+$.*

Defining $B_R = \{u \in W_0^{1,p}(\Omega), \|u\| < R\}$ with any $R > 0$, we now calculate the $\deg(J, B_R, 0)$.

Lemma 3.4. *Under hypotheses* (F2)*, there exists $R_0 > 0$ such that*

$$\deg(J, B_R, 0) = 0 \quad \textit{for all } R \ge R_0. \tag{3.3}$$

Proof. Let

$$\langle T(u), \varphi\rangle = \int_\Omega (u^+)^{p-1}\varphi dx, \quad \forall \varphi \in W_0^{1,p}(\Omega). \tag{3.4}$$

where the $u^+ = \max\{u, 0\}$, $u^- = \max\{-u, 0\}$. Since T is a completely continuous operator, the homotopy $H_1(t,u) : [0,1] \times W_0^{1,p}(\Omega) \to W^{-1,p'}(\Omega)$ defined by

$$\langle H_1(t,u), \varphi\rangle = \int_\Omega |\nabla u|^{p-2}\nabla u \nabla\varphi dx - (1-t)\int_\Omega \lambda f(u)\varphi dx - t\int_\Omega k(x)(u^+)^{p-1}\varphi dx \tag{3.5}$$

for all $u, \varphi \in W_0^{1,p}(\Omega)$, $t \in [0,1]$, $k(x) \in L_+^\infty(\Omega) \setminus \{0\}$ and $k(x) < \lambda_1$. Clearly $H_1(t,u)$ is of type $(S)_+$. We claim that there exists $R_0 > 0$ such that

$$H_1(t,u) \ne 0 \quad \text{for all } t \in [0,1],\ u \in \partial B_R,\ R \ge R_0. \tag{3.6}$$

Suppose that is not true. Then we can find sequences $\{t_n\} \subset [0,1]$ and $\{u_n\} \subset W_0^{1,p}(\Omega)$ such that $t_n \to t \in [0,1]$, $\|u_n\| \to \infty$ and

$$\int_\Omega |\nabla u_n|^{p-2}\nabla u_n \nabla\varphi dx = (1-t_n)\int_\Omega \lambda f(u_n)\varphi dx + t_n\int_\Omega k(u_n^+)^{p-1}\varphi dx \tag{3.7}$$

for all $\varphi \in W_0^{1,p}(\Omega)$. Let $h_n = \frac{u_n}{\|u_n\|}$, we may assume that there exists $h \in W_0^{1,p}(\Omega)$ satisfying

$$h_n \rightharpoonup h \quad \text{in } W_0^{1,p}(\Omega), \quad h_n \to h \quad \text{in } L^p(\Omega), \quad h_n(x) \to h(x) \quad \text{a.e. on } \Omega.$$

Acting with the test function $h_n - h \in W_0^{1,p}(\Omega)$ in (3.7) we find

$$\begin{aligned} &\int_\Omega |\nabla h_n|^{p-2}\nabla h_n \nabla(h_n - h)dx \\ &= (1-t_n)\lambda \int_\Omega \frac{f(u_n)}{\|u_n\|^{p-1}}(h_n - h)dx + t_n \int_\Omega k(h_n^+)^{p-1}(h_n - h)dx. \end{aligned} \tag{3.8}$$

We are already show that

$$\begin{gathered} (1-t_n)\lambda \int_\Omega \frac{f(u_n)}{\|u_n\|^{p-1}}(h_n - h)dx \to 0 \quad n \to \infty, \\ t_n \int_\Omega k(h_n^+)^{p-1}(h_n - h)dx \to 0 \quad n \to \infty. \end{gathered}$$

Using this and (3.8), we obtain

$$\lim_{n\to+\infty} \int_\Omega |\nabla h_n|^{p-2}\nabla h_n \nabla(h_n - h)dx = 0,$$

i.e., $\lim_{n\to+\infty}\langle I(h_n), h_n - h\rangle = 0$. By Lemma 3.1 we obtain $h_n \to h$ in $W_0^{1,p}(\Omega)$ as $n \to \infty$ and $\|h\| = 1$. This shows that $h \neq 0$. Acting with the test function $h \in W_0^{1,p}(\Omega)$ in (3.8), we have

$$\int_\Omega |\nabla h_n|^{p-2}\nabla h_n \nabla h dx = (1-t_n)\lambda \int_\Omega \frac{f(u_n)}{\|u_n\|^{p-1}} h dx + t_n \int_\Omega k(h_n^+)^{p-1} h dx. \tag{3.9}$$

Passing to the limit in (3.9) as $n \to \infty$, using hypothesis (F2) we find

$$\int_\Omega |\nabla h|^p dx = \int_\Omega tk(h^+)^p dx. \tag{3.10}$$

Acting with the test function $h^- \in W_0^{1,p}(\Omega)$ we obtain $h \geq 0$. Hence

$$\int_\Omega |\nabla h|^p dx = \int_\Omega tkh^p dx. \tag{3.11}$$

If $t = 0$, then $h = 0$, a contradiction. So assume $t \in (0,1]$, exploiting the monotonicity of the principal eigenvalue on the weight function, we obtain

$$1 = \lambda_1(\lambda_1) < \lambda_1(k) \leq \lambda_1(tk). \tag{3.12}$$

We infer that $h = 0$, which contradicts to the fact that $h \neq 0$. This contradiction shows the claim stated in (3.6).

Due to (3.6) we are allowed to use the homotopy invariance of the degree map, which through the homotopy $H_1(t,u)$ yields

$$\deg(J, K_R, 0) = \deg(H_1(1,u), B_R, 0) \quad \text{for all } R \geq R_0. \tag{3.13}$$

Due to (3.13), the problem reduces to computing $\deg(H_1(1,u), B_R, 0)$. To this end let the homotopy $H_2(t,u) : [0,1] \times W_0^{1,p}(\Omega) \to W^{-1,p'}(\Omega)$ be defined by

$$\langle H_2(t,u), \varphi\rangle = \int_\Omega |\nabla u|^{p-2}\nabla u \nabla\varphi dx + t\int_\Omega m(x)(u^+)^{p-1}\varphi dx - \int_\Omega k(x)(u^+)^{p-1}\varphi dx$$

for all $u, \varphi \in W_0^{1,p}(\Omega)$, $t \in [0,1]$, $m(x) \in L_+^\infty(\Omega)$ and $m(x) > \lambda_1$. Clearly, $H_2(t,u)$ it is a homotopy of type $(S)_+$. Let us check that $H_2(t,u) \neq 0$ for all $t \in [0,1]$ and

$u \in \partial B_R$. Arguing by contradiction, assume that there exist $u \in W_0^{1,p}(\Omega)$ with $\|u\| = R$ and $t \in [0,1]$ such that

$$\int_\Omega |\nabla u|^{p-2}\nabla u \nabla \varphi dx = -t\int_\Omega m(x)(u^+)^{p-1}\varphi dx + \int_\Omega k(x)(u^+)^{p-1}\varphi dx \tag{3.14}$$

for all $\varphi \in W_0^{1,p}(\Omega)$. Acting with the test function $u^- \in W_0^{1,p}(\Omega)$, we obtain $u \geq 0$. So

$$\int_\Omega |\nabla u|^{p-2}\nabla u \nabla \varphi dx = -t\int_\Omega m(x)u^{p-1}\varphi dx + \int_\Omega k(x)u^{p-1}\varphi dx \tag{3.15}$$

Acting with the test function u in (3.15), we have

$$\int_\Omega |\nabla u|^p dx = \int_\Omega (k(x) - tm(x))u^p dx < (1-t)\lambda_1 \int_\Omega u^p dx. \tag{3.16}$$

From this inequality, we conclude that

$$\lambda_1 \leq \frac{\int_\Omega |\nabla u|^p dx}{\int_\Omega |u|^p dx} < (1-t)\lambda_1. \tag{3.17}$$

The contradiction obtained justifies the desired conclusion. By the homotopy invariance of the degree map, we have

$$\deg(H_1(1,u), B_R, 0) = \deg(H_2(1,u), B_R, 0) \quad \text{for all } R \geq R_0. \tag{3.18}$$

We choose $\|m(x)\|_{L^\infty}$ sufficiently large such that

$$\int_\Omega |\nabla u|^{p-2}\nabla u \nabla \varphi dx - \int_\Omega k(x)u^{p-1}\varphi dx \neq -\int_\Omega m(x)u^{p-1}\varphi dx \tag{3.19}$$

for all $u \in B_R$. Then we obtain

$$\deg(H_2(1,u), B_R, 0) = 0 \quad \text{for all } R \geq R_0.$$

Hence

$$\deg(J, B_R, 0) = 0 \quad \text{for all } R \geq R_0.$$

The proof is complete. □

Now we can give the proof of our main result.

Proof of Theorem 1.1. From the assumption of f, we see that for all $\epsilon > 0$, there exists $\theta > 0$ such that

$$|f(s)| \leq \epsilon|s|^{p-1} + \theta, \quad \text{for all } x \in \Omega,\ s \in \mathbb{R} \tag{3.20}$$

Define $\phi : W_0^{1,p}(\Omega) \to \mathbb{R}$ as

$$\phi(u) = \frac{1}{p}\int_\Omega |\nabla u|^p dx - \lambda \int_\Omega F(u)dx.$$

It is well know that under (3.20), ϕ is well defined on $W_0^{1,p}(\Omega)$, weakly lower semicontinuous and coercive. So, we can find $u_1 \in W_0^{1,p}(\Omega)$ such that

$$\phi(u_1) = \inf_{W_0^{1,p}(\Omega)} \phi(u). \tag{3.21}$$

By the assumption (F3), we letting $\Omega_\varepsilon = \{x \in \Omega : dist(x, \partial\Omega) > \varepsilon\}$,

$$u_0(x) = \beta \quad \text{for all } x \in \Omega_\varepsilon,$$
$$0 \leq u_0(x) \leq \beta \quad \text{for all } x \in \Omega \setminus \Omega_\varepsilon.$$

Then

$$\begin{aligned}\phi(u_0) &= \frac{1}{p}\int_\Omega |\nabla u_0|^p dx - \lambda\Big(\int_{\Omega_\varepsilon} F(u_0)dx + \int_{\Omega\setminus\Omega_\varepsilon} F(u_0)dx\Big)\\ &\le \frac{1}{p}\|u_0\|^p - \lambda(F(\beta)|\Omega_\varepsilon| - c(1+\beta^p)|\Omega\setminus\Omega_\varepsilon|),\end{aligned}$$

when $\epsilon > 0$ sufficiently small, there exists $\Lambda_0 > 0$ such that $\phi(u_0) < 0$ for all $\lambda > \Lambda_0$. So, $\phi(u_1) < \phi(u_0) < 0$, which shows $u_1 \neq 0$. (3.21) implies

$$\int_\Omega |\nabla u_1|^{p-2}\nabla u_1 \nabla\varphi dx = \lambda\int_\Omega f(u_1)\varphi dx \tag{3.22}$$

for all $\varphi \in W_0^{1,p}(\Omega)$. So u_1 is a nontrivial solution of (1.1).

Since u_1 is a global minimizer of ϕ, without loss of generality, we can choose $r_1 > 0$ such that

$$\phi(u_1) < \phi(u), \quad \phi'(u) \neq 0 \quad \text{for all } u \in \overline{B_{r_1}(u_1)}\backslash\{u_1\}, \tag{3.23}$$

and for all $r \in (0, r_1)$ there holds

$$\mu = \inf\{\phi(u) : u \in \overline{B_{r_1}(u_1)} \setminus B_r(u_1)\} - \phi(u_1) > 0.$$

Define the set

$$V = \{u \in B_{\frac{r}{2}}(u_1) : \phi(u) - \phi(u_1) < \mu\}$$

which is an open and bounded neighborhood of u_1. Furthermore, find a number $r_0 \in (0, \frac{r}{2})$ with $\overline{B_{r_0}(u_1)} \subset V$ and γ such that

$$0 < \gamma < \inf\{\phi(u) : u \in \overline{B_{r_1}(u_1)} \setminus B_{r_0}(u_1)\} - \phi(u_1).$$

Let $U = B_{r_1}(u_1)$ and $\psi = \phi|_{B_{r_1}(u_1)} - \phi(u_1)$. By the (3.23) we know that $0 \notin \phi'(\overline{V}\backslash B_r(u_1))$, using Lemma 2.3, we conclude that

$$\deg(J, B_r(u_1), 0) = \deg(J, V, 0) = 1. \tag{3.24}$$

From Lemma 3.4, we can find number R_0 such that

$$\deg(J, B_R, 0) = 0 \quad \text{if } R \ge R_0. \tag{3.25}$$

Now fix R_0 in (3.25) sufficiently large such that $B_r(u_1) \subset B_R$. Since the domain additivity of type $(S)_+$

$$\deg(J, B_R, 0) = \deg(J, B_r(u_1), 0) + \deg(J, B_R\backslash B_r(u_1), 0).$$

we obtain

$$\deg(J, B_R\backslash B_r(u_1), 0) = -1.$$

Hence, there exists $u_2 \in B_R\backslash B_r(u_1)$ solving the problem (1.1). According to the (F1) we have that $u_2 \neq 0$.

Hence, semipositone problem(1.1) has two nontrivial weak solutions u_1 and u_2 for all $\lambda > \Lambda_0$. □

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Xudong Shang

School of Mathematics, Nanjing Normal University, Taizhou College, 225300, Jiangsu, China

E-mail address: xudong-shang@163.com