

## ANALYTICAL MODELING OF BARGAINING SOLUTIONS FOR MULTICAST CELLULAR SERVICES

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ABSTRACT. Nowadays, the growing demand for group-oriented services over mobile devices has led to the definition of new communication standards and multimedia applications in cellular systems. In this article we study the use of game theoretic solutions for these services to model and perform a trade-off analysis between fairness and efficiency in the resources allocation. More precisely, we model bargaining solutions for the multicast data services provisioning and introduce the analytical resolution for the proposed solutions.

### 1. INTRODUCTION AND STATEMENT OF THE PROBLEM

Game theory uses mathematics to identify some appropriate strategies in order to describe the behavior of specific situations (games) in which the success of an individual's choices is directly linked to the choices of others. Games are characterized by a number of players or decision makers who interact, possibly threaten each other and form coalitions, take actions under uncertain conditions, and finally receive some benefit or reward or possibly some punishment or monetary loss. As we outline the contents of this text, we introduce some of the key words and terminology used in game theory. First there is the number of players which will be denoted by  $n$ . Let us label the players with the integers 1 to  $n$ , and denote the set of players by  $N = \{1, 2, \dots, n\}$ . In the game a player has a winning strategy and the other players have not a winning strategy, or some of the players have a common winning strategy, or else neither player has a winning strategy in the game. In the last case the game is said to be *undetermined*.

Since the end of the previous century game theoretic solutions have been used to solve some diagonalization problems in general topology. In particular, many authors have described the behavior of some combinatorial scheme, called *selection principle* in terms of games (see [11, 1]).

In this article we study a concrete and technical problem adopting some different optimization strategies. Specifically, the effectiveness of any adopted policy for the delivery of multicast data services in modern cellular systems depends on a good trade-off between the *total utility* and the *fairness*. Game theoretic bargaining solutions are suitable solutions to be adopted to this scope [6], [7]. An interesting

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way to provide multicast services foresees to split the multicast destinations into different subgroups, according to the User Equipments' (UEs) channel quality, and implement a subgroup based Adaptive Modulation and Coding (AMC) scheme.

Let us consider a general study case where the multicast group is composed of  $N$  UEs. Let  $c$  be the channel quality indicator (CQI) associated to a generic user in the cellular system, where  $c$  can vary from 1 to a maximum value  $n_c$ , which depends on the adopted cellular standard. For a given CQI value, the attainable data throughput depends on the number of assigned network resources, which in general can vary from 1 to  $R$  and is defined by the specific standard adopted as cellular system.

Let  $\mathbf{V} = \{v_1, v_2, \dots, v_{n_c}\}$  be the vector containing the number of UEs per CQI level, where  $\sum_{c=1}^{n_c} v_c = N$ . Being  $G$  a generic bargaining solution to be adopted, the proposed framework is implemented as follows:

- Be  $\tilde{\mathcal{S}}$  the set of all the possible multicast subgroup configurations. Let  $\mathcal{S} \subseteq \tilde{\mathcal{S}}$  be the subset of all potential configurations, which ensure, according to the  $\mathbf{V}$  vector, that all UEs of the multicast group are served, that each UE supports the CQI it is associated to and that only subgroups with a CQI value reported by at least one of the UEs are considered.
- For each subgroup configuration  $s \in \mathcal{S}$ , the generic UE with CQI value  $c$  is assigned to the subgroup  $j$  associated to the closest CQI supported by the user (with  $j = 1, \dots, J^s$  and  $J^s$  representing the total number of enabled subgroups in configuration  $s$ ). Let  $V_j^s$  be the number of UEs served by the subgroup  $j$  in the configuration  $s$ ;
- For every subgroup configuration  $s \in \mathcal{S}$ , the resource allocation is performed based on a selected game theoretic bargaining solution  $G$ , which meets the constraint that all UEs in the system must be served;
- The subgroup configuration  $s_t \in \mathcal{S}$  that maximizes a performance index  $P$  is selected.  $s_t$  determines the subgroups to be activated with their respective modulation and coding schemes (MCSs) and the resources allocated to each group according to the bargaining solution  $G$ .

We define the  $P$  index as the *Aggregate Utility (AU)* for the system. In terms of Game Theory, we will model each subgroup as a *player* of the game; thus, the utility (or payoff) assigned to each player is computed as the total data throughput for the subgroup. In particular, the total utility for a generic player  $j$  of the generic subgroup configuration  $s$  is strictly related to the number of associated UEs and the number of assigned RBs:

$$u_j^s = V_j^s \cdot g_j, \quad (1.1)$$

where  $g_j$  represents the throughput assigned by the base station to the  $j$ -th subgroup and varies as a function of the MCS and the resources RB (i.e.,  $1 \leq RB_j \leq R$ ) assigned to the  $j$ -th subgroup by the game theoretic solution  $G$ . With this utility definition, the Aggregate Utility for the subgroup configuration  $s$  is defined as:  $AU^s = \sum_{j=1}^{J^s} u_j^s$ , where  $J^s$  is the total number of enabled subgroups.

## 2. GAME THEORETIC BARGAINING SOLUTIONS

A game is defined as  $G = \langle \mathcal{K}, A, \{u_i\} \rangle$  where  $\mathcal{K} = \{1, 2, \dots, K\}$  is the set of players which compete for the resources assignment,  $A_i$  is the set of actions available to player  $i$ ,  $A = A_1 \times \dots \times A_K$  and  $u_i$  is the objective function, also called the

utility function, player  $i$  wishes to maximize. A vector of such utilities is denoted by  $u = (u_1, \dots, u_K)$  and we define  $\mathcal{U} = \{u(a) | a \in A\}$  as the set of achievable utilities for all players [12]-[8].

Further terminology and notations for the bargaining game are:

- a *disagreement point* is an action vector  $a \in A$  expected to be the result of non-cooperative play given a failure of the bargaining process;
- an *agreement point* is an action vector  $a \in A$  that is a possible outcome of a bargaining process where the utility obtained by every player has to be at least as much as the utility obtained at the disagreement point;
- we say that  $u \in \mathcal{U}$  is *Pareto optimal* if  $\nexists u' \in \mathcal{U}$  such that  $u'_i \geq u_i, \forall i \in \mathcal{K}$  and  $u'_i > u_i$  for some index  $i$ . When the inequality is strict,  $u$  is *weak Pareto optimal*. We define  $\text{PO}(\mathcal{U})$  as the set of Pareto optimal points of  $\mathcal{U}$  and  $\text{WPO}(\mathcal{U})$  as the set of weak Pareto optimal points of  $\mathcal{U}$ , where  $\text{PO}(\mathcal{U}) \subseteq \text{WPO}(\mathcal{U})$ ;
- a vector containing a utility space and a disagreement point for a game is a *bargaining problem*;
- a *bargaining solution*  $\phi$ , defined on a class of bargaining problems  $\Sigma$ , is a map that associates with each problem  $(\mathcal{U}, u^0) \in \Sigma$  a unique point in  $\mathcal{U}$ , where  $u^0 = u(a^0)$  is the utility achieved at the disagreement point  $a^0$ .

The conditions that apply on the utility space  $\mathcal{U}$  are:  $\mathcal{U} \subset \mathbb{R}^K$  is upper-bounded, closed and convex and there exists  $u \in \mathcal{U}$  such that  $u^0 < u$ .

**2.1. Nash Bargaining Solution.** Nash proposed the following axioms:

- (1) *Individual Rationality*:  $\phi(\mathcal{U}, u^0) > u^0$ .
- (2) *Pareto Optimality*:  $\phi(\mathcal{U}, u^0) \in \text{PO}(\mathcal{U})$ .
- (3) *Invariance to affine transformations*: if  $\psi : \mathbb{R}^K \rightarrow \mathbb{R}^K, \psi(u) = u'$  with  $u'_i = c_i u_i + d_i, c_i, d_i \in \mathbb{R}, c_i > 0, \forall i \in \mathcal{K}$ , then  $\phi(\psi(\mathcal{U}), \psi(u^0)) = \psi(\phi(\mathcal{G}, u^0))$ .
- (4) *Independence of irrelevant alternatives*: if  $u' \in \mathcal{G} \subset \mathcal{U}$  and  $u' = \phi(\mathcal{U}, u^0)$  then  $\phi(\mathcal{G}, u^0) = u'$ .
- (5) *Symmetry*: if  $\mathcal{U}$  is symmetric with respect to  $i$  and  $j$ ,  $u_i^0 = u_j^0$ , and  $u' = \phi(\mathcal{U}, u^0)$ , then  $u'_i = u'_j$ .

Axioms 1-2 represent the binding conditions for any agreement point given by a bargaining solution, while axioms 3-5 are the so-called fairness axioms. In particular, the invariance to affine transformations states that the solution is invariant if scaled in an affine manner. The independence of irrelevant alternatives states that if the domain is reduced to a subset of the domain that contains the bargaining solution, then the solution remains the same. Finally, the symmetry axiom states that the solution does not depend on the labels, i.e., if the players have the same disagreement point and the same set of feasible utility, then they will achieve the same utility at the solution.

**Definition 2.1. Nash Bargaining Solution (NBS).**  $\phi(\mathcal{U}, u^0)$  is the NBS if and only if it satisfies all axioms 1-5.

When  $\mathcal{U}$  is convex [5], Nash showed that the unique NBS is the maximizer of the Nash Product:

$$\arg \max_{u > u^0} \prod_{i=1}^K (u_i - u_i^0). \quad (2.1)$$

The intuitive idea is that, after a minimal requirement is satisfied for all players, the remaining resources are allocated according to the conditions of each player. The presented optimization problem is equivalent to the following optimization:

$$\arg \max_{u > u^0} \sum_{i=1}^K \ln(u_i - u_i^0). \tag{2.2}$$

**2.2. Problem resolution with the NBS.** The utilities  $u_i$  for the players are defined according to equation (1.1), and the utility set  $\mathcal{U}$  collects the possible utilities assigned to each activated subgroup. In details, for any considered subgroup configuration, the number of nodes and the minimum rate per activated subgroup are actually constant values; the only variable is the number of RBs assigned to each subgroup. Thus, the set  $A$  for the bargaining problem is the set of possible RB assignments to each player. This set is actually not a convex set in  $\mathbb{R}^K$  because only integer values of RB can be assigned to each subgroup. Many real problems have a non-convex definition set, and several approaches have been proposed in the literature to extend bargaining solutions to these cases. In this paper we will approach the problem by studying it on the smallest convex set that contains  $A$ , the *convex hull* denoted by  $\text{Conv}(A)$  (consequently also the utility set  $\mathcal{U}$  becomes  $\text{Conv}(\mathcal{U})$ ). Obviously, considering the convex hull of  $A$  is a relaxation of the problem and in general this yields suboptimal solutions. In fact, a final step will be introduced to approximate the bargaining solutions, defined on the convex hull set, to integer values of RBs and to the corresponding utilities. This final step, will make us approximate the bargaining result to the closest point that can be implemented in real environments.

The problem resolution can be performed by calculating the NBS directly on the unknowns of the problem instead of the utilities  $u_i$  [4].

**Proposition 2.2.** *Consider a strategies set*

$$A' := \{a \in \mathbb{R}^K : a_i^{\min} \leq a_i \leq a_i^{\max}, \forall i \in \mathcal{K}\},$$

and a set of weights associated to the players  $\Omega = \{\omega_1, \dots, \omega_K\}$ , with  $0 < \omega_i < 1$ , for every  $i \in \mathcal{K}$ . Further, let

$$J := \{j \in \mathcal{K} : \exists a \in A \text{ s.t. } u_j > u_j^0\},$$

to be the set of players  $J$  able to achieve a performance strictly superior to their disagreement point. Then there exists a unique NBS which is the solution of the optimization problem

$$\max \sum_{j \in J} \omega_j \ln(a_j - a_j^{\min}). \tag{2.3}$$

*Proof.*  $A'$  is a convex compact subset of  $\mathbb{R}^K$ , the power functions  $(a_j - a_j^{\min})^{\omega_j}$  defined on  $A'$  are concave, upper-bounded and injective on  $A'$  and they are used as the utility functions. Substituting these function in equation (2.2) it yields:

$$\max \sum_{j \in J} \ln((a_j - a_j^{\min})^{\omega_j} - u_j^0). \tag{2.4}$$

Note that  $u_j^0 = (a_j^{\min} - a_j^{\min})^{\omega_j} = 0$ , so the solution proposed in equation (2.3) is found. □

Referring to the problem formulation presented in section 1, for any subgroup configuration  $s$ , the unknowns of the problem  $a_j$  are the number of RB allocated to subgroup  $j$ . Let us define a set  $\Omega$  of weights  $\omega_j$  [4], where both the number of users in the group and the channel quality are considered:

$$\omega_j = \frac{V_j^s \cdot b_j^s}{\sum_{t \in J} V_t^s \cdot b_t^s}, \quad (2.5)$$

where  $b_j^s$  and  $b_t^s$  are the throughput achieved with 1 RB assigned to the MCS related to the subgroups  $j$  and  $t$  respectively.

For an easier notation we will hereafter focus on a generic subgroup configuration  $s$  with  $J$  being the set of players (i.e. the subgroups considered in configuration  $s$ ). From Proposition 1, we can find the NBS by resolving the following constrained optimization problem:

$$\begin{aligned} \max \quad & \sum_{j \in J} \omega_j \ln(a_j - a_j^{\min}) \\ & a_j > a_j^{\min}, \forall j \in J \\ & a_j \leq a_j^{\max}, \forall j \in J \\ & \sum_{j \in J} a_j = R. \end{aligned} \quad (2.6)$$

The properties of the logarithmic function guarantee the function to be strictly concave and, under the assumption of studying the problem on the convex hull, the problem has a unique solution. Being  $a_j^{\max} = R$ , for all  $j \in J$ , the conditions on the minimum value coupled with the condition on the sum of values, make actually  $a_j \leq a_j^{\max}$  a superfluous condition for the problem. We can thus map the problem to a combinatorial problem with linear constraints. The Lagrangian for the problem is denoted as follows:

$$\mathcal{L}(a, \lambda) = \sum_{j \in J} \omega_j \ln(a_j - a_j^{\min}) + \lambda \sum_{j \in J} (a_j - R). \quad (2.7)$$

Taking the derivative of the Lagrangian function, we obtain the Karush-Kuhn-Tucker (KKT) [2] sufficient and necessary conditions:

$$a_j \left( \frac{\omega_j}{a_j - a_j^{\min}} + \lambda \right) = 0, \quad \forall j \in J, \quad (2.8)$$

$$\lambda \sum_{j \in J} (a_j - R) = 0, \quad \forall j \in J. \quad (2.9)$$

From (2.8), being  $a_j > 0$  we obtain

$$\frac{\omega_j}{a_j - a_j^{\min}} + \lambda = 0. \quad (2.10)$$

Hence, we can obtain  $a_j$  as a function of  $\lambda$ :

$$a_j = a_j^{\min} - \frac{\omega_j}{\lambda}. \quad (2.11)$$

By (2.9), considering  $\lambda > 0$ , we obtain the original condition for the problem

$$\sum_{j \in J} a_j = R. \quad (2.12)$$

Substituting the value of  $a_j$  we obtain

$$\sum_{j \in J} \left( a_j^{\min} - \frac{\omega_j}{\lambda} \right) = R. \quad (2.13)$$

Reorganizing the function, we can isolate  $\lambda$  being equal to:

$$\lambda = \frac{\sum_{j \in J} \omega_j}{\sum_{j \in J} a_j^{\min} - R}. \quad (2.14)$$

Substituting the obtained formulation of  $\lambda$  in expression (2.11) we obtain

$$a_j = 1 + \frac{\omega_j}{\sum_{j \in J} \omega_j} \left( R - \sum_{j \in J} a_j^{\min} \right). \quad (2.15)$$

With equation (2.15) we can compute the exact RB association to each of the activated CQI levels (subgroups). Note that since the problem is studied on the convex hull for the problem, the found  $a_j$  values will likely be real values. In order to use the found solutions in a real environment we need to introduce a further step such that only integer values are adopted. The simple solution we adopt, hereafter called *integer procedure*, is to use the *floor* function to take only the integer part of all  $a_j$  values. The remaining resources will then be distributed among the players following a descending order of the  $(a_j - \text{floor}(a_j))$  difference. One RB is associated to each player according to the mentioned order until all the RB are associated and  $\sum_{j \in J} a_j = R$ . This procedure, which will be equally applied for any of the considered bargaining solutions, permits to minimize the distance from the solutions found on the convex hull and the solutions that can be applied in practice.

For the specific problem we will consider a disagreement point  $u^0$  assigning the minimum of one RB to each single player in order to meet the service requirement of serving all UEs, thus  $a_j^{\min} = 1$ . Nevertheless, the formulation is valid for any value of  $a_j^{\min}$ .

**2.3. Kalai-Smorodinsky Solution.** Axiom 4 proposed by Nash has suffered some criticism [14] because it is not taking into account how much other players have given up. For this reason, another interesting fairness criterion was introduced by modifying this axiom, namely the Kalai-Smorodinsky solution, whereby utilities are proportional to their maximum possible values. In particular, axiom 4 proposed by Nash was replaced by the Restricted Monotonicity axiom:

*Restricted Monotonicity:* if  $\mathcal{G} \subset \mathcal{U}$  and  $h(\mathcal{U}, u^0) = h(\mathcal{G}, u^0)$  then  $\phi(\mathcal{U}, u^0) \geq \phi(\mathcal{G}, u^0)$ , where  $h(\mathcal{U}, u^0)$  is the so called *utopia point* and is defined as:

$$h(\mathcal{U}, u^0) = (\max_{u > u^0} u_1(u), \dots, \max_{u > u^0} u_K(u)).$$

For the reference problem, the *utopia point* is considered as the point where all  $R$  available RBs are assigned to one player, that is to a single multicast subgroup.

**Definition 2.3** (Kalai-Smorodinsky Solution (KSS)). We say that  $\phi(\mathcal{U}, u^0)$  is the KSS if the Individual Rationality, the Pareto Optimality, the Invariance to affine transformations, the Symmetry and the Restricted Monotonicity axioms are satisfied.

Let  $\Lambda$  be set of points in the line containing  $u^0$  and  $h(\mathcal{U}, u^0)$ . The unique KSS is  $\text{WPO}(\mathcal{U}) \cap \Lambda$ , which can be expressed as [12]:

$$\max \{u > u^0 \mid \frac{1}{\theta_i}(u_i - u_i^0) = \frac{1}{\theta_j}(u_j - u_j^0), \forall i, j \in \mathcal{K}\},$$

where  $\theta_i = h_i(\mathcal{U}, u_i^0) - u_i^0$ . Max is computed with respect to the partial order of  $\mathbb{R}^K$ .

The restricted monotonicity property implies that if in a subgroups configuration  $s \in \mathcal{S}$  the worst performing player improves the CQI level, its utility would be increased without any reduction for the other players. In particular, the KSS assigns as the bargaining solution the point in the Pareto set intersecting the line connecting the disagreement point and the utopia point.

An alternative formulation for the KSS can be given in the form of a weighted max-min fairness solution [9]:

$$\phi(\mathcal{U}, u^0) = \arg \max_{u \in \mathcal{U}} \min \left( \frac{u_1 - u_1^0}{h_1(\mathcal{U}, u_1^0) - u_1^0}, \dots, \frac{u_K - u_K^0}{h_K(\mathcal{U}, u_K^0) - u_K^0} \right). \quad (2.16)$$

For the exact analytical computation of the KSS the approach we follow is based on the bisection search method [3]. This method enables us to converge to the intersection point of the feasibility set and the segment connecting the disagreement point to the utopia point. In this paper, as in [10], we can apply this method by mapping the problem onto a one variable problem. Knowing the upper and lower bound for the utility of the players, rearranging the condition for the KSS definition, we can define the utility of a generic player  $j$  as a function of utility of the first player, player 1:

$$u_j = \left( \frac{u_1 - u_1^0}{h_1(\mathcal{U}, u_1^0) - u_1^0} \right) (h_j(\mathcal{U}, u_j^0) - u_j^0) + u_j^0, \forall j \in \mathcal{K}. \quad (2.17)$$

In each iteration of the bisection method any solution is to be tested for feasibility. The set of utilities for the  $K$  players  $(u_1, u_2, \dots, u_K)$  is linearly dependent on the allocated resources. If  $(a_1, a_2, \dots, a_K)$  is the set of resources allocated to the players, the iterative bisection search method should test every solution according to the problem feasibility condition:  $\sum_{j=1}^K a_j \leq R$ , with  $R$  the maximum resources available in the system. Starting from a lower bound equal to the disagreement point and an upper bound equal to the utopia point, the bisection method is applied as given in Algorithm (1).

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#### Algorithm 1 Bisection Method

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**Require:** lower bound  $l := u_1^0$ , upper bound  $u := h_1(\mathcal{U}, u_1^0)$ , tolerance  $\epsilon > 0$

**while**  $(u - l \leq \epsilon)$  **do**

- (1) Set  $u_1$  at the mid-point of  $l$  and  $u$ ;  $u_1 = (l + u)/2$
- (2) Obtain  $u_j$  with  $(j = 2, \dots, K)$  based on  $u_1$  according to equation (2.17)
- (3) Check feasibility of the proposed utilities based on the constraint on the correspondent allocated resources:  $\sum_{j=1}^K a_j \leq R$
- (4) If solution is feasible  $l := u_1$ , else  $u := h_1(\mathcal{U}, u_1^0)$

**end while**

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Similarly to the NBS, a further final step is required to map the found solutions to real implementable solutions, through the so-called *integer procedure*.

**2.4. Egalitarian Solution.** The third bargaining solution considered in this work is the Egalitarian solution, which assigns the point in the feasible set where all players achieve maximal equal increase in utility with respect to the disagreement point. To define this solution, we need to introduce the following axioms:

*Strong monotonicity:* if  $\mathcal{G} \subset \mathcal{U}$  then  $\phi(\mathcal{G}, u^0) \geq \phi(\mathcal{G}, u^0)$ ;

*Weak Pareto optimality:*  $\phi(\mathcal{U}, u^0) \in \text{WPO}(\mathcal{U})$ .

**Definition 2.4** (Egalitarian Solution (ES)).  $\phi(\mathcal{U}, u^0)$  is the ES if the Individual Rationality, the Weak Pareto Optimality, the Symmetry and the Strong Monotonicity axioms are satisfied.

Let  $u'$  be such that  $\forall i, u'_i = u_i^0 + b, b \in \mathbb{R}$  and  $\Lambda$  be set of points in the line containing  $u^0$  and  $u'$ . The unique ES is  $\text{WPO}(\mathcal{U}) \cap \Lambda$ , which can be expressed as [12]:

$$\max \{u > u^0 \mid u_i - u_i^0 = u_j - u_j^0, \forall i, j \in \mathcal{K}\}.$$

The strong monotonicity property implies that if in a subgroup configuration  $s \in \mathcal{S}$  the worst performing player improves the CQI level, then all the players would improve their utility. For the ES, an alternative formulation can be given in the form of a strict max-min fairness:

$$\phi(\mathcal{U}, u^0) = \arg \max_{u \in \mathcal{U}} \min(u_1 - u_1^0, \dots, u_K - u_K^0). \tag{2.18}$$

Also for the ES we follow the approach based on a bisection method search. Similarly to the KSS, we can define the utilities for the players as a function of the utility of the player 1. Based on the condition given in the ES definition, the utility for a generic player  $j$  can be written as:

$$u_j = ((u_1 - u_1^0) + u_1^0), \quad \forall j \in \mathcal{K}. \tag{2.19}$$

For the ES computation, the bisection method as introduced for KSS can be equally applied by defining the utilities in step 2 of Algorithm 1 according to equation (2.19).

**2.5. Utilitarian Solution.** The last bargaining solution considered in this work is the Utilitarian solution. This solution maximizes the sum of the utilities and is not always unique.

**Definition 2.5** (Utilitarian Solution (US)).  $\phi(\mathcal{U}, u^0)$  is a US if the Individual Rationality, the Pareto Optimality, the Symmetry and the Linearity axioms are satisfied [13].

The Linearity axiom states that the players are indifferent between solving problems separately or in a single problem:

*Linearity:* For any  $S, T \in \Sigma, \mathcal{U}(\lambda S + (1 - \lambda)T) = \lambda \mathcal{U}(S) + (1 - \lambda) \mathcal{U}(T)$ , for every  $\lambda \in [0, 1]$ .

The Utilitarian solution maximizes the sum of the utilities and is equal to:

$$\phi(\mathcal{U}, u^0) = \arg \max_{u \in \mathcal{U}} \sum_{j=1}^K u_j. \tag{2.20}$$

Considering the problem formulation presented in section 1 and the utility definition in equation (1.1), for any subgroup configuration  $s$ , the number of RB allocated to a subgroup  $j, a_j$ , can be easily computed. In particular, after having assigned the minimum required resources to each of the involved subgroups, the sum of the



utilities is maximized if the remaining resources are all assigned to the player with the highest value of the  $V_j^s \cdot b_j^s$  product.

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