Electronic Journal of Differential Equations, Vol. 2013 (2013), No. 238, pp. 1–8.
ISSN: 1072-6691. URL: http://ejde.math.txstate.edu or http://ejde.math.unt.edu
ftp ejde.math.txstate.edu

COEFFICIENTS OF SINGULARITIES FOR A SIMPLY SUPPORTED PLATE PROBLEMS IN PLANE SECTORS

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Abstract. This article represents the solution to a plate problem in a plane sector that is simple supported, as a series. By using appropriate Green's functions, we establish a biorthogonality relation between the terms of the series, which allows us to calculate the coefficients.

1. Introduction

Let S be the truncated plane sector of angle $\omega \leq 2\pi$, and radius ρ (ρ is positive and fixed) defined by:

$$S = \{(r\cos\theta, r\sin\theta) \in \mathbb{R}^2, 0 < r < \rho,\ 0 < \theta < \omega\} \tag{1.1}$$

and Σ the circular boundary part

$$\Sigma = \{(\rho\cos\theta, \rho\sin\theta) \in \mathbb{R}^2, 0 < \theta < \omega\}. \tag{1.2}$$

We are interested in the study of a function u, belonging to the Sobolev space $H^2(S)$, and being the solution of

$$\begin{gathered} \Delta^2 u = 0, \quad \text{in } S \\ u = Mu = 0, \quad \text{for } \theta = 0, \omega, \end{gathered} \tag{1.3}$$

where the operator M represents the bending moment and is defined as

$$Mu = \nu\Delta u + (1-\nu)(\partial_1^2 u n_1^2 + 2\partial_{12}^2 u n_1 n_2 + \partial_2^2 u n_2^2). \tag{1.4}$$

Here ν is a real number ($0 < \nu < 1/2$) called Poisson coefficient and $n = (n_1, n_2)$ is the unit outward normal vector to Γ_0 and Γ_1 (See Figure 1).

The boundary conditions $u = 0$ and $Mu = 0$, for $\theta = 0$, $\theta = \omega$ mean that the plate is simply supported.

This type of boundary conditions arises in problems of linear or non linear vibrations of thin imperfect plates. See for example [2, pages 5,6] and the references therein.

2000 *Mathematics Subject Classification.* 35B40, 35B65, 35C20.
Key words and phrases. Crack sector; singularity; bilaplacian; solution series.

Submitted May 13, 2013. Published October 24, 2013.

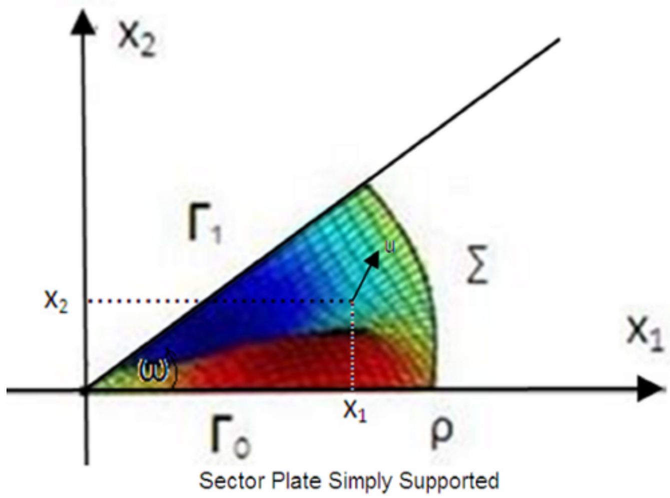

FIGURE 1.

We show that the solutions u of this problems can be written as a series of the form

$$u(r,\theta)=\sum_{\alpha\in E} c_\alpha r^\alpha \phi_\alpha(\theta). \tag{1.5}$$

Here E stands for the set of solutions of the equation in a complex variable α:

$$\sin^2(\alpha-1)\omega=\sin^2\omega, \quad \operatorname{Re}\alpha>1 \tag{1.6}$$

For further studies of the set E, see for example Blum and Rannacher [1], and Grisvard [4].

We will compute the coefficients c_α in (1.5). This sort of calculations have already been done by Tcha-Kondor [5] for the Dirichlet's boundary conditions, and by Chikouche-Aibeche [3] for the Neumann's boundary conditions. These authors have established, thanks to the Green's formula, a relation of biorthorgonality between any two functions ϕ_α, which allows them to calculate the coefficients c_α. We follow the same approach. Thus we need to write the appropriate Green formula for the domain S. Using this formula, we establish a relation of biorthorgonality between the functions ϕ_α.

In the case of a crack domain ($\omega=2\pi$) this relation reduces to the simple one obtained by Tcha-Kondor. This enables us, in this particular situation, to find an explicit formula for the coefficients c_β.

2. Separation of variables

Replacing u by $r^\alpha\phi_\alpha(\theta)$ in problem (1.3) leads us to the boundary value problem

$$\phi_\alpha^{(4)}(\theta)+(2\alpha^2-4\alpha+4)\phi_\alpha^{(2)}(\theta)+\alpha^2(\alpha-2)^2\phi_\alpha(\theta)=0, \tag{2.1}$$

$$\phi_\alpha^{(2)}(\theta)+[\nu\alpha^2+(1-\nu)\alpha]\phi_\alpha(\theta)=0, \quad \theta=0, \quad \theta=\omega \tag{2.2}$$

$$\phi_\alpha(\theta)=0, \quad \theta=0, \quad \theta=\omega \tag{2.3}$$

A relation similar to the orthogonality obtained for the biharmonic operator is given, in the following theorem.

Theorem 2.1. *Let ϕ_α and ϕ_β be solutions of* (2.1) *with α and β solutions of* (1.6). *Then, for $\alpha \neq \beta$, one has*

$$\begin{aligned}[\phi_\alpha, \phi_\beta] = \int_0^\omega & \big[(\alpha^2 - 2\alpha)\phi_\alpha - \frac{\nu(\alpha+\overline{\beta}) + (3-\nu) - 2\alpha}{\alpha - \overline{\beta}}\phi_\alpha''\big]\overline{\phi_\beta} \\ & + \big[(\overline{\beta}^2 - 2\overline{\beta})\overline{\phi_\beta} - \frac{\nu(\alpha+\overline{\beta}) + (3-\nu) - 2\overline{\beta}}{\alpha - \overline{\beta}}\overline{\phi_\beta''}\big]\phi_\alpha \, d\theta = 0.\end{aligned} \tag{2.4}$$

Proof. We use the Green formula

$$\int_S (v\Delta^2 u - u\Delta^2 v)dx = \int_\Gamma \big[(uNv + \frac{\partial u}{\partial n}Mv) - (vNu + \frac{\partial v}{\partial n}Mu)\big]\, d\sigma, \tag{2.5}$$

where

$$Nu = -\frac{\partial \Delta u}{\partial n} + (1-\nu)(\partial_1^2 u n_1 n_2 - \partial_{12}^2 u(n_1^2 - n_2^2) + \partial_2^2 u n_1 n_2),$$

and Γ is the boundary of S. For two functions u, v which are solutions of (1.3), using the Green's formula we obtain

$$\int_\Sigma \big[(uNv + \frac{\partial u}{\partial n}Mv) - (vNu + \frac{\partial v}{\partial n}Mu)d\sigma\big] = 0 \tag{2.6}$$

On Σ, for the function $u_\alpha = r^\alpha \phi_\alpha$, we have

$$\begin{gathered}\frac{\partial u_\alpha}{\partial n} = \frac{\partial u_\alpha}{\partial r} = \alpha r^{\alpha-1}\phi_\alpha, \\ M u_\alpha = r^{\alpha-2}\big\{[\alpha^2 - (1-\nu)\alpha]\phi_\alpha + \nu\phi_\alpha''\big\}, \\ N u_\alpha = r^{\alpha-3}\big\{-\alpha^2(\alpha-2)\phi_\alpha + [(\nu-2)\alpha + (3-\nu)]\phi_\alpha''\big\}.\end{gathered} \tag{2.7}$$

The results follow from the application of formula (2.6) to the biharmonic functions $u_\alpha = r^\alpha\phi_\alpha$ and $u_\beta = r^{\overline{\beta}}\overline{\phi_\beta}$, and by using relations (2.7). $\square$

Remark 2.2. The relation (2.4) between the functions ϕ_α and ϕ_β is similar to the relation of biorthorgonality obtained when the functions ϕ_α and ϕ_β satisfying (2.1) with the Dirichlet boundary conditions $\phi_\alpha = \phi_\alpha' = \phi_\beta = \phi_\beta' = 0$ for $\theta = 0$ and $\theta = \omega$. In this latter case, the relation is given by

$$\int_0^\omega \phi_\alpha \phi_\beta'' d\theta = \int_0^\omega \phi_\alpha'' \phi_\beta d\theta \tag{2.8}$$

which is obtained by a double integration by parts:

$$\int_0^\omega \phi_\alpha \phi_\beta'' d\theta = \int_0^\omega \phi_\alpha'' \phi_\beta d\theta + [\phi_\alpha, \phi_\beta']_0^\omega - [\phi_\alpha', \phi_\beta]_0^\omega, \tag{2.9}$$

and using the Dirichlet's boundary conditions.

The following corollary is an immediate consequence of remark 2.2.

Corollary 2.3. *Let ϕ_α and ϕ_β be solutions of* (2.1) *with α and β solutions of* (2.6). *Suppose in addition that*

$$[\phi_\alpha, \phi_\beta']_0^\omega - [\phi_\alpha', \phi_\beta]_0^\omega = 0, \tag{2.10}$$

and $\alpha \neq \beta$, then

$$[\phi_\alpha, \phi_\beta] = \int_0^\omega \left\{[(\alpha^2 - 2\alpha)\phi_\alpha + \phi''_\alpha]\overline{\phi_\beta} + [(\overline{\beta}^2 - 2\overline{\beta})\overline{\phi_\beta} + \overline{\phi''_\beta}]\phi_\alpha\right\} d\theta = 0 \tag{2.11}$$

Remark 2.4. For $u_\alpha = r^\alpha \phi_\alpha$ we have

$$\Delta u_\alpha - \frac{2}{r}\frac{\partial u_\alpha}{\partial r} = r^{\alpha-2}[(\alpha^2 - 2\alpha)\phi_\alpha + \phi''_\alpha]. \tag{2.12}$$

Let P be the operator $P = \Delta - \frac{2}{r}\frac{\partial}{\partial r}$. From the corollary 2.3 and Remark 2.4, we deduce the following result.

Corollary 2.5. *Under the hypotheses of corollary 2.3, if $\alpha \neq \beta$, we have*

$$\int_\Sigma (Pu_\alpha \cdot \overline{u_\beta} + u_\alpha \cdot P\overline{u_\beta}) d\sigma = 0. \tag{2.13}$$

3. Formula for the coefficients in the crack case

The crack case ($\omega = 2\pi$) is an important one, among singular domains, in the applications. Moreover in this case the solutions of (2.6) are explicitly known and we have

$$E = \{\frac{k}{2}, k \in \mathbb{N},\, k > 2\}$$

and these roots are of multiplicity 2.

In this framework we assume that the solution u admits the representation

$$\begin{gathered} u = \sum_{\alpha \in E}(c_\alpha u_\alpha + d_\alpha v_\alpha), \quad E = \{\frac{k}{2},\, k \in \mathbb{N},\, k > 2\}, \\ u_\alpha = r^\alpha \phi_\alpha(\theta), \quad v_\alpha = r^\alpha \psi_\alpha(\theta) \end{gathered} \tag{3.1}$$

the solutions ϕ_α and ψ_α, in terms of θ, are the odd functions:

$$\phi_\alpha(\theta) = \sin(\alpha - 2)\theta, \tag{3.2}$$

$$\psi_\alpha(\theta) = \sin \alpha\theta. \tag{3.3}$$

and since $\alpha = k/2$, we obtain

$$\phi_\alpha(0) = \phi_\alpha(\omega) = \psi_\alpha(0) = \psi_\alpha(\omega) = 0 \tag{3.4}$$

and thus

$$\begin{gathered} [\phi_\alpha, \phi'_\beta]_0^\omega = [\phi'_\alpha, \phi_\beta]_0^\omega = 0, \quad [\psi_\alpha, \psi'_\beta]_0^\omega = [\psi'_\alpha, \psi_\beta]_0^\omega = 0, \\ [\phi_\alpha, \psi'_\beta]_0^\omega = [\phi'_\alpha, \psi_\beta]_0^\omega = 0. \end{gathered}$$

From here comes the idea of decomposing the solution u of (1.3) into two parts as follows:

$$\begin{gathered} u = w_1 + w_2, \\ w_i = \sum_{\alpha \in E_i}(c_\alpha u_\alpha + d_\alpha v_\alpha), \quad i = 1, 2, \\ E_1 = \{2m,\, m > 1\}, \quad E_2 = \{2m + 1,\, 2m > 1\}. \end{gathered} \tag{3.5}$$

Calculation of c_β and d_β. From the expressions of ϕ_α, ψ_α one easily sees that:

$$\begin{gathered}\text{if } \alpha \in E_1, \text{ then } \phi'_\alpha(0) = \phi'_\alpha(\omega) \text{ and } \psi'_\alpha(0) = \psi'_\alpha(\omega),\\ \text{if } \alpha \in E_2, \text{ then } \phi'_\alpha(0) = -\phi'_\alpha(\omega) \text{ and } \psi'_\alpha(0) = -\psi'_\alpha(\omega).\end{gathered} \tag{3.6}$$

Equations (3.4) and (3.6) allow us to apply corollary 2.5 to functions u_α and u_β (resp. u_α, v_β and v_α, v_β) and get the relations:

$$\begin{gathered}\int_\sigma (Pw_i \cdot u_\beta + w_i \cdot Pu_\beta) d\sigma = 2c_\beta \int_\sigma (u_\beta \cdot Pu_\beta) d\sigma + d_\beta \int_\sigma (Pv_\beta . u_\beta + v_\beta \cdot Pu_\beta) d\sigma,\\ \int_\sigma (Pw_i \cdot v_\beta + w_i \cdot Pv_\beta) d\sigma = c_\beta \int_\sigma (Pu_\beta \cdot v_\beta + u_\beta \cdot Pv_\beta) d\sigma + 2d_\beta \int_\sigma (Pv_\beta \cdot v_\beta) d\sigma.\end{gathered} \tag{3.7}$$

By direct calculations we obtain

$$\begin{gathered}\int_\sigma (Pu_\beta \cdot v_\beta + u_\beta \cdot Pv_\beta) d\sigma = 0,\\ \int_\sigma (u_\beta \cdot Pu_\beta) d\sigma = (\beta - 2)\omega\rho^{2\beta-1}\\ \int_\sigma (Pv_\beta . v_\beta) d\sigma = -\beta\omega\rho^{2\beta-1}\end{gathered} \tag{3.8}$$

and from this we get our main the result.

Theorem 3.1. *Let u be a the solution of* (1.3) *written in the form*

$$u = w_1 + w_2 \tag{3.9}$$

where

$$w_i = \sum_{\alpha \in E_i} (c_\alpha u_\alpha + d_\alpha v_\alpha), i = 1, 2 \tag{3.10}$$

Suppose that the series that gives w_i is uniformly convergent in S. Then for any $\alpha \in E_i$, $i = 1, 2$ we have

$$\begin{gathered}c_\alpha = \frac{\rho^{1-2\alpha}}{2(\alpha - 2)\omega} \int_\sigma (Pw_i \cdot u_\alpha + w_i \cdot Pu_\alpha) d\sigma\\ d_\alpha = \frac{-\rho^{1-2\alpha}}{2\alpha\omega} \int_\Sigma (Pu_i \cdot v_\alpha + w_i \cdot Pv_\alpha) d\sigma\end{gathered} \tag{3.11}$$

Remark 3.2. Let $\zeta \in H^{3/2}(\Sigma) \cap H^1_0(\Sigma)$ be the trace of u on Σ and $\chi \in H^{-1/2}(\Sigma)$ the trace of Pu on Σ.

If u is regular in order that $\zeta \in H^4(]0, 2\pi[)$ and $\chi \in H^2(]0, 2\pi[)$, then we have a uniform convergence of the series in $\overline{S_{\rho_0}}$ for all $\rho_0 < \rho$, see [5].

3.1. **Independence of the coefficients.**

Proposition 3.3. *The coefficients c_β (resp d_β) are independent of ρ.*

Proof. Let us prove that the derivative of c_β with respect to ρ is zero. Observing the expression of c_β in Theorem 3.1, we just have to prove that the derivative, with respect to ρ, of

$$\gamma_\beta = \rho^{1-2\beta} \int_\sigma (Pw_i \cdot u_\beta + w_i \cdot Pu_\beta) d\sigma. \tag{3.12}$$

vanishes. By derivation with respect to r we have

$$\begin{aligned}\gamma'_\beta = \int_0^\omega \Big\{ &\frac{\partial}{\partial r}(\Delta w_i) r^{2-\beta}\phi_\beta + \big[(2-\beta)\Delta w_i - 2\frac{\partial^2 w_i}{\partial r^2} + (\beta^2-2)\frac{1}{r}\frac{\partial w_i}{\partial r}\big] r^{1-\beta}\phi_\beta \\ &+ \frac{\partial w_i}{\partial r} r^{-\beta}\phi''_\beta - \beta w_i r^{-1-\beta}\big[(\beta^2-2\beta)\phi_\beta + \phi''_\beta\big]\Big\}\, d\theta.\end{aligned} \tag{3.13}$$

On Σ, we have

$$\frac{\partial}{\partial r}(\Delta w_i) = -N w_i + (1-\upsilon)\big[\frac{1}{r^3}\frac{\partial^2 w_i}{\partial \theta^2} - \frac{1}{r^2}\frac{\partial^3 w_i}{\partial r \partial \theta^2}\big],$$

and

$$(2-\beta)\Delta w_i - 2\frac{\partial^2 w_i}{\partial r^2} = -\beta M w_i + [2-(1-\upsilon)\beta][\frac{1}{r}\frac{\partial w_i}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w_i}{\partial \theta^2}]. \tag{3.14}$$

Using these formulas in the expression of γ'_β we obtain

$$\begin{aligned}&\gamma'_\beta \\ &= -\int_0^\omega (\beta M w_i r^{1-\beta}\phi_\beta + N w_i r^{2-\beta}\phi_\beta) d\theta \\ &\quad + \int_0^\omega \big\{[(\beta^2-(1-\upsilon)\beta)\phi_\beta + \phi''_\beta]\frac{\partial u_i}{\partial r} - (1-\upsilon)\frac{\partial^3 u_i}{\partial r\partial\theta^2}\phi_\beta\big\} r^{-\beta} d\theta \\ &\quad + \int_0^\omega \big\{[2-(1-\upsilon)(\beta-1)]\frac{\partial^2 w_i}{\partial\theta^2}\phi_\beta - \beta w_i[(\beta^2-2\beta)\phi_\beta + \phi''_\beta]\big\} r^{-1-\beta} d\theta.\end{aligned} \tag{3.15}$$

By a double integration by parts, we verify that

$$\int_0^\omega \frac{\partial^2 w_i}{\partial \theta^2}\phi_\beta d\theta = \int_0^\omega w_i \phi''_\beta d\theta \tag{3.16}$$

$$\int_0^\omega \frac{\partial^3 w_i}{\partial r\partial \theta^2}\phi_\beta d\theta = \int_0^\omega \frac{\partial w_i}{\partial r}\phi''_\beta d\theta \tag{3.17}$$

Using (3.15)–(3.17) in the expression of γ'_β and putting the $\rho^{1-2\beta}$, we obtain

$$\begin{aligned}\gamma'_\beta &= -\rho^{1-2\beta}\int_0 \omega(\beta M w_i r^{\beta-1}\phi_\beta + N w_i r^\beta \phi_\beta)\rho d\theta \\ &\quad + \rho^{1-2\beta}\int_0^\omega [(\beta^2-(1-\upsilon)\beta)\phi_\beta + \upsilon\phi''_\beta] r^{\beta-2}\frac{\partial w_i}{\partial r}\rho d\theta \\ &\quad + \rho^{1-2\beta}\int_0^\omega \big\{[-\beta^2(\beta-2)]\phi_\beta + [-(2-\upsilon)\beta + (3-\upsilon)]\phi''_\beta\big\} r^{\beta-3} w_i \rho d\theta.\end{aligned} \tag{3.18}$$

Taking into account of (2.7), whose expressions appear explicitly in γ'_β, we obtain

$$\gamma'_\beta = \rho^{1-2\beta}\int_\Sigma -\big[\big(u_\beta N w_i + \frac{\partial u_\beta}{\partial n} M w_i\big) - \big(u_i N u_\beta + \frac{\partial w_i}{\partial n} M u_\beta\big)\big]\, d\sigma = 0. \tag{3.19}$$

We follow the same analysis to prove the independence of d_β with respect to ρ. $\square$

3.2. **Convergence of the series.** We first write c_α and d_α in the form

$$c_\alpha = I_i \rho^{-\alpha}, \quad d_\alpha = J_i \rho^{-\alpha} \tag{3.20}$$

with

$$\begin{aligned} I_i &= \frac{\rho}{2\omega(\alpha-2)} \int_\sigma (Pw_i\phi_\alpha + w_i\rho^{-2}[(\alpha^2-2\alpha)\phi_\alpha + \phi''_\alpha])d\sigma, \\ J_i &= \frac{-\rho}{2\omega\alpha} \int_\sigma (Pw_i\psi_\alpha + w_i\rho^{-2}[(\alpha^2-2\alpha)\psi_\alpha + \psi''_\alpha])d\sigma. \end{aligned} \tag{3.21}$$

The solution u of (1.3) is then written as

$$u = w_1 + w_2 \tag{3.22}$$

$$w_i = \sum_{\alpha\in E_i} [(\frac{r}{\rho})^\alpha I_i\phi_\alpha + (\frac{r}{\rho})^\alpha J_i\psi_\alpha], \quad i = 1,2 \tag{3.23}$$

and we have the following result.

Theorem 3.4. *The series* (3.23) *converges uniformly in* $\overline{S_{\rho_0}}$ *for all* $\rho_0 < \rho$.

Proof. Set

$$\begin{aligned} H_{i,\alpha} &= \int_0^\omega (Pu_i\phi_\alpha + u_i\rho^{-2}[(\alpha^2-2\alpha)\phi_\alpha + \phi''_\alpha])d\theta \\ &= \int_0^\omega Pu_i\phi_\alpha d\theta + (\alpha^2-2\alpha)\rho^{-2}\int_0^\omega u_i\phi_\alpha d\theta + \rho^{-2}\int_0^\omega u_i\phi''_\alpha d\theta \end{aligned} \tag{3.24}$$

We show that $H_{i,\alpha}$ is $1/\alpha$ times by bounded term, for α large enough. According to (3.17), we have

$$\int_0^\omega u_i\phi''_\alpha d\theta = \int_0^\omega u''_i\phi_\alpha d\theta \tag{3.25}$$

Replacing ϕ_α by its expression and integrating by parts we obtain

$$\int_0^\omega u''_i\phi_\alpha d\theta = \frac{1}{\alpha}\Big[\frac{-\alpha}{(\alpha-2)}\int_0^\omega u'''_i \cos(\alpha-2)\theta d\theta\Big] \tag{3.26}$$

On the other hand, by a triple integration by parts, we have

$$(\alpha^2-2\alpha)\int_0^\omega u_i\phi_\alpha d\theta = \frac{1}{\alpha}\Big[\frac{\alpha^2}{(\alpha-2)^2}\int_0^\omega u'''_i \cos(\alpha-2)\theta d\theta\Big] \tag{3.27}$$

Also, integrating by parts, we obtain

$$\int_0^\omega (\Delta u_i - \frac{2}{r}\frac{\partial u_i}{\partial r})\phi_\alpha d\theta = \frac{1}{\alpha}\Big[\frac{-\alpha}{(\alpha-2)}\ int_0^\omega(\frac{\partial}{\partial\theta}(\Delta u_i) - \frac{2}{r}\frac{\partial^2 u_i}{\partial r\partial\theta})\cos(\alpha-2)\theta d\theta\Big] \tag{3.28}$$

Then, we deduce the existence of a constant C_0 such that:

$$|H_{i,\alpha}| \leq \frac{C_0}{\alpha} \tag{3.29}$$

Using this last inequality and the fact that ϕ_α is bounded as well as the term $1/(2\omega(\alpha-2))$ for large α we deduce the existence of a constant C such that

$$|\sum_{\alpha\in E_i} c_\alpha r^\alpha\phi_\alpha| \leq \sum_{\alpha\in E_i}\frac{C}{\alpha}(\frac{r}{\rho})^\alpha \tag{3.30}$$

which converges uniformly in $\overline{S_{\rho_0}}$ for $\rho_0 < \rho$. Convergence of $\sum_{\alpha\in E_i} d_\alpha r^\alpha\psi_\alpha$ is proved by the same way. ∎

References

[1] H. Blum, R. Rannacher; *On the boundary value problem of the biharmonic operator on domains with angular corners*, Maths. Methods Appl. Sci. 2 (1980), no. 4, 556-581.

[2] Cédric Camier; *Modélisation et étude numérique de vibrations non-linéaire de plaques circulaires minces imparfaites, Application aux cymbales.* Thèse Présentée et soutenue publiquement le 2 février 2009 pour l'otention du Docteur de l'Ecole Polytechnique.

[3] W. Chikouche, A. Aibeche; *Coefficients of singularities of the biharmonic problem of Neumann type: case of the crack.* IJMMS 2003: 5, 305-313, Hindawi Publishing Corp.

[4] P. Grisvard; *Singularities in boundary values problems*, Recherches en Mathématiques Appliquées, Vol. 22, Masson, Paris, 1992 (in French).

[5] O. Tcha-Kondor; *Nouvelles séries trigonométriques adapt ées à l'étude de problèmes aux limites pour l'équation biharmonique.Étude du cas de la fissure* [New trigonometric series adapted to the study of boundary value problems for the biharmonic equation. The cas of the crack], C. R. Acad. Sci. Paris sér. I, Math. 315(1992), no. 5, 541-544 (in French).

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