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EXISTENCE AND UNIQUENESS OF POSITIVE SOLUTIONS FOR A NONLOCAL DISPERSAL POPULATION MODEL

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ABSTRACT. In this article, we study the solutions of a nonlocal dispersal equation with a spatial weight representing competitions and aggregation. To overcome the limitations of comparison principles, we introduce new definitions of upper-lower solutions. The proof of existence and uniqueness of positive solutions is based on the method of monotone iteration sequences.

1. INTRODUCTION

Let $J : \mathbb{R}^N \to \mathbb{R}$ be a non-negative, continuous function such that $\int_{\mathbb{R}^N} J(x)dx = 1$. With this function, we define the nonlocal dispersal operator

$$D[u] = \int_\Omega J(x-y)u(y)dy - b(x)u,$$

where $\Omega \subset \mathbb{R}^N$ and $b(x) \in C(\Omega)$. This operator and variations of it have been widely used for modeling dispersal processes in material science, phase transitions, and genetics. In particular, the studies of the integro-differential equation

$$u_t(x,t) = \int_\Omega J(x-y)u(y,t)dy - b(x)u(x,t) + f(x,u) \tag{1.1}$$

have attracted much attention; see, among other references, [2, 4, 5, 6, 13, 14, 20]. As stated in [12], if $u(x,t)$ is thought as a density at position x at time t and the probability distribution that individuals jump from y to x is given by $J(x-y)$, then the rate of dispersal is the difference in the rate at which individuals are arriving to position x from all other places, or $\int_{\mathbb{R}^N} J(x-y)u(y,t)dy$ and the rate at which they are leaving position x to all other places, or $-u(x,t) = \int_{\mathbb{R}^N} J(y-x)u(x,t)dy$. This also suggests that the asymptotic behavior for a linear nonlocal problem may be fractional, see [2, 9]. The nonlocal dispersal equation (1.1) also represents a model for solid phase transitions and peri-dynamic heat conduction [10]. However, the dispersal kernel J might take negative values in physical situations.

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In this article, we consider the nonlocal dispersal equation

$$u_t = d\Big[\int_{\mathbb{R}^N} J(x-y)u(y,t)dy - u\Big] + u(1+\alpha u - \beta u^2 - [1+\alpha-\beta]G*u) \quad (1.2)$$

for $(x,t) \in \mathbb{R}^N \times (0,\infty)$, subject to the initial condition

$$u(x,0) = \psi(x) \text{ in } \mathbb{R}^N, \quad (1.3)$$

where

$$G*u(x,t) = \int_{\mathbb{R}^N} G(x-y)u(y,t)dy\,.$$

It is assumed that d, α, β are non-negative constants and $1+\alpha-\beta>0$. Here d is the dispersal rate, the term αu in (1.2) represents an advantage to the population in local aggregation or grouping, by making available different food success or protecting measure against predation [7, 8, 11]. The term $-\beta u^2$ represents competition for space. The integral term $G*u$ in (1.2) represents intraspecific competition for food resources with non-negative weight function G.

In the limit, the most localized version ($G(x)=\delta(x)$) and $\beta=0$, (1.2) reduces to the nonlocal Logistic equation

$$u_t = d(J*u-u) + u(1-u), \quad (1.4)$$

which is studied in [3, 18, 19]. It is important to point out that in [7], Britton first posed a mathematical model of aggregation and nonlocal competition effects in a singe species. The reader is referred to [8, 15] for a detailed background to such models.

In this article, we focus mainly on the existence and uniqueness of solutions to problem (1.2)-(1.3). It is well-known from [1, 16, 17, 21] that the monotone iteration method is effective for the study of existence and uniqueness of solutions of the reaction-diffusion equation. Recently, Deng [11] and Tian and Zhu [21] extend the monotone iteration method to the reaction-diffusion equations with nonlocal effects and reaction-diffusion systems with mixed quasimonotone nonlinearities. In this paper, we consider the nonlocal dispersal equation (1.2). Since the comparison principle is not valid for (1.2)-(1.3) ([13]), we cannot use the classical nonlocal upper-lower solutions method [2]. To overcome the limitations of the comparison principles, we introduce new definitions of upper-lower solutions. The main approach is based on the construction of a monotone approximation. First, we give two definitions of upper-lower solutions and establish that the upper-lower solutions are ordered. Then the iteration sequences are obtained by the corresponding characterization of coupled upper-lower solutions. The use of aggregation and spatial averages competition is discussed. We show that there may not exist stable steady states or time-dependent spatial uniform solution. Under some additional assumptions on α , J and G, we find that the dynamic behavior of (1.2) is quite different from the one in the limit equation (1.4), that is to say the non-zero steady state may be unstable under the spatial perturbations.

For the reaction-diffusion equation with aggregations and nonlocal competitions as considered in [7], it could be transformed into a system by using a special form of function G. Then the nonlocal term which contains a spatial average is transformed into local term. So the linear stability of uniform state and some bifurcation phenomena of the local problem are well studied. It is not the case for nonlocal problems, as the dispersal operator D is nonlocal and there is a deficiency of regularization [9]. We shall investigate further the effects of aggregation and traveling fronts of (1.2) in a forthcoming work.

In Section 2, we give the definitions of two coupled upper-lower solutions and establish the existence and uniqueness of non-negative solutions to (1.2)-(1.3). The main method is based on two iteration sequences. We also discuss the effects of aggregation on the dynamic of (1.2)-(1.3).

2. Existence and Uniqueness

We first give the basic assumptions:

(A1) $J \in C(\mathbb{R}^N)$ verifies $J > 0$ in B_1 (the unit ball), $J = 0$ in $\mathbb{R}^N \setminus B_1$ with $\int_{\mathbb{R}^N} J(x)dx = 1$ and $J(x) = J(-x)$.

(A2) G is continuous with $G \geq 0$ and $G * 1 = 1$.

(A3) ψ is continuous, non-negative and $\psi \in L^1(\mathbb{R}^N) \cap L^\infty(\mathbb{R}^N)$.

Note that the monotone iteration sequence method is non-unique due to different upper-lower solutions. In this section, we give two different iteration sequences to obtain the existence of solutions of (1.2)-(1.3). Throughout the rest of paper, we assume that (A1)–(A3) hold.

2.1. **Classical iteration sequence.**

In this subsection, we define a pair of coupled upper-lower solutions. Then we obtain the existence of solutions to our nonlocal problem. To begin, let us give the basic definition.

Definition 2.1. A pair of functions $\omega(x,t)$ and $v(x,t)$ are called an upper and a lower solution of (1.2)-(1.3) of type I, if all of the following hold:

(i) $\omega, v \in C^1([0,T); L^1(\mathbb{R}^N) \bigcap L^\infty(\mathbb{R}^N))$ and $\omega(\cdot, x), v(\cdot, x) \in L^\infty([0,T))$.

(ii) $\omega(x,0) \geq \psi(x) \geq v(x,0)$ in $\mathbb{R}^N$.

(iii) For $(x,t) \in \mathbb{R}^N \times [0,T)$,

$$\omega_t \geq d[J * \omega - \omega] + \omega[1 + \alpha\omega] - \beta v^3 - (1 + \alpha - \beta) v G * v, \tag{2.1}$$

$$v_t \leq d[J * v - v] + v[1 + \alpha v] - \beta\omega^3 - (1 + \alpha - \beta)\omega G * \omega. \tag{2.2}$$

We can show that all the upper-lower solutions of type I are ordered. In fact, we have the following result, whose proof is given at the end of this subsection.

Theorem 2.2. *Let ω (respectively v) be an upper solution (respectively a lower solution) of (1.2)-(1.3) of type I. Then*

$$v(x,t) \leq \omega(x,t) \ ((x,t) \in \mathbb{R}^N \times [0,T)).$$

Theorem 2.3. *Suppose that ω and v are a pair of non-negative upper-lower solutions of type I to (1.2)-(1.3). Then (1.2)-(1.3) admit a unique solution $u(x,t)$ in $\mathbb{R}^N \times [0,T)$ which satisfies the relation*

$$v(x,t) \leq u(x,t) \leq \omega(x,t) \ ((x,t) \in \mathbb{R}^N \times [0,T)).$$

Proof. We give the main proof in the following steps.

Step 1. Denote $v^0(x,t) = v(x,t)$ and $\omega^0(x,t) = \omega(x,t)$, we construct sequences $\{v^k\}$ and $\{\omega^k\}$ from classical process in $\mathbb{R}^N \times (0,T)$

$$v_t^k - d[J * v^k - v^k] = v^{k-1}[1 + \alpha v^{k-1}] - \beta[\omega^{k-1}]^3 - (1 + \alpha - \beta)\omega^{k-1} G * \omega^{k-1}, \tag{2.3}$$

$$\omega_t^k - d[J*\omega^k - \omega^k] = \omega^{k-1}[1+\alpha\omega^{k-1}] - \beta[v^{k-1}]^3 - (1+\alpha-\beta)v^{k-1}G*v^{k-1}, \tag{2.4}$$

with initial conditions

$$v^k(x,0) = \psi(x), \quad \omega^k(x,0) = \psi(x).$$

Since (2.3) and (2.4) are linear nonlocal dispersal equations, we know that for each $k \geq 1$, the sequences $\{v^k\}$ and $\{\omega^k\}$ are well defined by the nonlocal semigroup theory [2].

Step 2. We show that the sequences defined above satisfy

$$v(x,t) \leq v^l(x,t) \leq v^{l+1}(x,t) \leq \omega^l(x,t) \leq \omega^{l+1}(x,t) \leq \omega(x,t) \tag{2.5}$$

for $l = 1, 2, \dots$ and $(x,t) \in \mathbb{R}^N \times (0,T)$.

Let us begin to show that (2.5) holds if $l = 1$. Take $z(x,t) = v(x,t) - v^1(x,t)$, it follows from (2.2) and (2.3) that

$$\begin{gathered} z_t - d[J*z - z] \leq 0 \quad \text{in } \mathbb{R}^N \times (0,T), \\ z(x,0) \leq 0 \quad \text{in } \mathbb{R}^N. \end{gathered}$$

Thus we know that $z(x,t) \leq 0$ in $\mathbb{R}^N \times (0,T)$ by the comparison principle of nonlocal equation [12]. A similar discussion gives that $\omega^1(x,t) \leq \omega(x,t)$ in $\mathbb{R}^N \times (0,T)$.

Denote $z^1(x,t) = v^1(x,t) - \omega^1(x,t)$. Since $v(x,t) \leq \omega(x,t)$, it follows from (2.3)-(2.4) that

$$\begin{gathered} z_t^1 - d[J*z^1 - z^1] \leq 0 \quad \text{in } \mathbb{R}^N \times (0,T), \\ z^1(x,0) \leq 0 \quad \text{in } \mathbb{R}^N. \end{gathered}$$

Thus we know that $v^1(x,t) \leq \omega^1(x,t)$ in $\mathbb{R}^N \times (0,T)$.

Now we show that $v^1(x,t)$ and $\omega^1(x,t)$ are a pair of lower-upper solutions of type I. Since $v(x,t) \leq v^1(x,t)$ and $\omega^1(x,t) \leq \omega(x,t)$, we have

$$\begin{aligned} &v_t^1 - d[J*v^1 - v^1] - v^1 - \alpha[v^1]^2 + \beta[\omega^1]^3 + (1+\alpha-\beta)\omega^1 G*\omega^1 \\ &= (v - v^1) + \alpha([v]^2 - [v^1]^2) + \beta([\omega^1]^3 - [\omega]^3) + (1+\alpha-\beta)(\omega^1 G*\omega^1 - \omega G*\omega) \\ &\leq 0 \end{aligned}$$

and

$$\begin{aligned} &\omega_t^1 - d[J*\omega^1 - \omega^1] - \omega^1 - \alpha[\omega^1]^2 + \beta[v^1]^3 + (1+\alpha-\beta)v^1 G*v^1 \\ &= (\omega - \omega^1) + \alpha([\omega]^2 - [\omega^1]^2) + \beta([v^1]^3 - [v]^3) + (1+\alpha-\beta)(v^1 G*v^1 - vG*v) \\ &\geq 0. \end{aligned}$$

Next, we use a simple induction method. By choosing v^1 and ω^1 as the ordered upper-lower solutions, after the similar above argument, we have

$$v^1(x,t) \leq v^2(x,t) \leq \omega^2(x,t) \leq \omega^1(x,t) \quad \text{in } \mathbb{R}^N \times (0,T).$$

Also $v^2(x,t)$ and $\omega^2(x,t)$ are ordered lower-upper solutions of (1.1) of type I. The conclusion in (2.5) follows from the induction principle.

Step 3. We show the existence of solutions to (1.2)-(1.3). Since the sequences $\{v^k\}_{k=1}^\infty$ and $\{\omega^k\}_{k=1}^\infty$ are monotone and bounded, there exist two function $\bar{v}$ and $\bar{\omega}$ such that

$$\lim_{k\to\infty} v^k(x,t) = \bar{v}(x,t) \quad \text{and} \quad \lim_{k\to\infty} \omega^k(x,t) = \bar{\omega}(x,t)$$

pointwise in $\mathbb{R}^N \times (0,T)$. It is trivial to see that $\bar{v} \le \bar{\omega}$ and

$$\bar{v}_t - d[J * \bar{v} - \bar{v}] = \bar{v}[1 + \alpha\bar{v}] - \beta[\bar{\omega}]^3 - (1+\alpha-\beta)\bar{\omega}G * \bar{\omega},$$
$$\bar{\omega}_t - d[J * \bar{\omega} - \bar{\omega}] = \bar{\omega}[1 + \alpha\bar{\omega}] - \beta[\bar{v}]^3 - (1+\alpha-\beta)\bar{v}G * \bar{v}.$$

Meanwhile, we can treat $\bar{v}$ and $\bar{\omega}$ as upper-lower solutions to (1.2)-(1.3) of type I, respectively. Thus we have $\bar{v} \ge \bar{\omega}$. Hence $\bar{v} = \bar{\omega}$ and $\bar{v}$ is a solution to (1.2)-(1.3).

Step 4. Inspired by [11], we give the uniqueness by some nonlocal estimates and the Gronwall's inequality. Assume that $u_1(x,t)$ and $u_2(x,t)$ are two solutions to (1.2)-(1.3) in $\mathbb{R}^N \times (0,T)$. Let $\omega_1(x,t) = u_1(x,t) - u_2(x,t)$. Our main estimates are based on the solution to the following nonlocal Dirichlet problem

$$\begin{gathered}
\gamma_s(x,s) = d[J * \gamma(x,s)dy - \gamma(x,s)] - h(x,s)\gamma(x,s) \quad \text{in } B(0,r) \times (0,t),\\
\gamma(x,s) = 0 \quad \text{in } \mathbb{R}^N \setminus B(0,r) \times [0,t),\\
\gamma(x,0) = \chi(x) \quad \text{in } B(0,r).
\end{gathered} \tag{2.6}$$

Here $h(x,t)$ is a bounded and continuous function, $B(0,r) = \{x : |x| \le r\}$ for some $r > 0$. The initial value function $\chi(x) \in C_c^\infty(B(0,r))$, $0 \le \chi \le 1$ in $B(0,r)$. The global existence and uniqueness of the non-negative solution $\gamma(x,s)$ of (2.6) is well studied, see [18]. Now let $\tau = t - s$, by a simple translation, we have

$$\begin{gathered}
\gamma_\tau(x,\tau) = d[\gamma(x,\tau) - J * \gamma(x,\tau)dy] - h(x,\tau)\gamma(x,\tau) \quad \text{in } B(0,r) \times (0,t),\\
\gamma(x,\tau) = 0 \quad \text{in } \mathbb{R}^N \setminus B(0,r) \times [0,t),\\
\gamma(x,t) = \chi(x) \quad \text{in } B(0,r).
\end{gathered} \tag{2.7}$$

Since u_1 and u_2 are two solutions to (1.2)-(1.3), then we have

$$\begin{aligned}
&\int_{\mathbb{R}^N} \gamma(x,t)\omega_1(x,t)dx\\
&= \int_0^t \int_{\mathbb{R}^N} [\gamma_s(x,s) + d(J * \gamma(x,s) - \gamma(x,s)) - h_1(x,s)]\,\omega_1(x,s)\,dx\,ds\\
&\quad + (1+\alpha-\beta)\int_0^t \int_{\mathbb{R}^N} G * \omega_1(x,s)u_2(x,s)\gamma(x,s)\,dx\,ds,
\end{aligned}$$

where $h_1(x,s) = 1 + 2\alpha\theta(x,s) - 3\beta\theta^2(x,s) + (1+\alpha-\beta)G * u_1(x,s)$ for some θ between u_1 and u_2.

Now by taking $h = h_1$ in (2.7), we know that

$$\begin{aligned}
\int_{B(0,r)} \chi(x)\omega_1(x,t)dx &= (1+\alpha-\beta)\int_0^t \int_{B(0,r)} G * \omega_1(x,s)u_2(x,s)\gamma(x,s)\,dx\,ds\\
&\le (1+\alpha-\beta)M\int_0^t \int_{B(0,r)} |\omega_1(x,s)|\,dx\,ds,
\end{aligned}$$

here $M = \max_{[0,T]\times\mathbb{R}^N} |u_2\gamma|$. By the arbitrary of $\chi(x)$, without loss of generality, we assume that

$$\chi(x) = \begin{cases} 1 & \text{if } \omega_1(x,t) \ge 0,\\ 0 & \text{if } \omega_1(x,t) = 0,\\ -1 & \text{if } \omega_1(x,t) \le 0. \end{cases}$$

So we have

$$\int_{\mathbb{R}^N} |\omega_1(x,t)|dx \le (1+\alpha-\beta)\int_0^t \int_{\mathbb{R}^N} |\omega_1(x,s)|\,dx\,ds.$$

Then the Gronwall's inequality implies $|\omega_1(x,t)| = 0$ and we complete the proof. □

Remark 2.4. The iteration sequences in (2.3) and (2.4) are classical in the sense that the right sides are only related to the previous step. We give another define of iteration sequences whose right sides are related to the current step in the following subsection. From Theorem 2.3, we obtain a unique bounded solution $u(x,t)$ to (1.2)-(1.3).

Proof of Theorem 2.2. Let $\gamma(x,t)$ be a non-negative function, since u and v satisfy (2.1)-(2.2), an easy calculation gives

$$\begin{aligned}
&\int_{\mathbb{R}^N} \gamma(x,t)v(x,t)dx \\
&\leq \int_0^t \int_{\mathbb{R}^N} [\gamma_s(x,s) + d(J*\gamma(x,s) - \gamma(x,s))]v(x,s)\,dx\,ds \\
&\quad - (1+\alpha-\beta)\int_0^t \int_{\mathbb{R}^N} G*u(x,s)u(x,s)\gamma(x,s)\,dx\,ds \\
&\quad + \int_0^t \int_{\mathbb{R}^N} [(1+\alpha v)v - \beta u^3]\gamma(x,s)\,dx\,ds + \int_{\mathbb{R}^N} v(x,0)v(x,0)dx
\end{aligned}$$

and

$$\begin{aligned}
&\int_{\mathbb{R}^N} \gamma(x,t)u(x,t)dx \\
&\geq \int_0^t \int_{\mathbb{R}^N} [\gamma_s(x,s) + d(J*\gamma(x,s) - \gamma(x,s))]u(x,s)\,dx\,ds \\
&\quad - (1+\alpha-\beta)\int_0^t \int_{\mathbb{R}^N} G*v(x,s)v(x,s)\gamma(x,s)\,dx\,ds \\
&\quad + \int_0^t \int_{\mathbb{R}^N} [(1+\alpha u)u - \beta v^3]\gamma(x,s)\,dx\,ds + \int_{\mathbb{R}^N} u(x,0)v(x,0)dx.
\end{aligned}$$

Let $\theta(x,t) = v(x,t) - u(x,t)$, then we have $\theta(x,0) = v(x,0) - u(x,0) \leq 0$. Accordingly,

$$\begin{aligned}
&\int_{\mathbb{R}^N} \gamma(x,t)\theta(x,t)dx \\
&\leq \int_0^t \int_{\mathbb{R}^N} [\gamma_s(x,s) + d(J*\gamma(x,s) - \gamma(x,s)) + h(x,s)]\,\theta(x,s)\,dx\,ds \\
&\quad + (1+\alpha-\beta)\int_0^t \int_{\mathbb{R}^N} G*\theta(x,s)u(x,s)\gamma(x,s)\,dx\,ds.
\end{aligned}$$

Take $\gamma(x,s)$ the solution of (2.7) with $h(x,\tau) = (1+\alpha-\beta)G*v + 1 + 2\alpha\theta'(x,s) - 3\beta\theta'^2(x,s)$ for some θ' between v and u. Then we know that

$$\int_{B(0,r)} \chi(x)\theta(x,t)dx \leq (1+\alpha-\beta)C\int_0^t \int_{B(0,r)} |\theta(x,s)|\,dx\,ds,$$

where $C > 0$ is a constant. Hence we complete our proof by a similar way as in the proof of Theorem 2.3. □

2.2. **Another iteration sequence.** In this subsection, we give a new definition of upper-lower solutions and then obtain the existence and uniqueness solution to (1.2)-(1.3). We also show that the solution is global at the end of this subsection.

Definition 2.5. A pair of functions $\hat{\omega}(x,t)$ and $\hat{v}(x,t)$ are called an upper and a lower solution of (1.2)-(1.3) of type II, if all of the following hold:

(i) $\hat{\omega}, \hat{v} \in C^1([0,T); L^1(\mathbb{R}^N))$.
(ii) $\hat{\omega}(x,0) \geq u_0(x) \geq \hat{v}(x,0)$ in $\mathbb{R}^N$.
(iii) For $(x,t) \in \mathbb{R}^N \times [0,T)$,

$$\hat{\omega}_t \geq d[J * \hat{\omega} - \hat{\omega}] + \hat{\omega}[1 + \alpha\hat{\omega}] - \beta\hat{v}^3 - (1+\alpha-\beta)\hat{\omega}G * \hat{v}], \tag{2.8}$$

$$\hat{v}_t \leq d[J * \hat{v} - \hat{v}] + \hat{v}[1 + \alpha\hat{v}] - \beta\omega^3 - (1+\alpha-\beta)\hat{v}G * \hat{\omega}]. \tag{2.9}$$

The following theorem is similar to Theorem 2.2, so we omit its proof.

Theorem 2.6. *Let $\hat{u} \in C^1([0,T); L^1(\mathbb{R}^N))$ (respectively $\hat{v}$) be an upper solution (respectively a lower solution) of (1.2)-(1.3) of type II. Then*

$$\hat{v}(x,t) \leq \hat{u}(x,t) \quad ((x,t) \in \mathbb{R}^N \times [0,T)).$$

Denote $\hat{v}^0(x,t) = \hat{v}(x,t)$ and $\hat{\omega}^0(x,t) = \hat{\omega}(x,t)$, we construct sequences $\{\hat{v}^k\}$ and $\{\hat{\omega}^k\}$ in $\mathbb{R}^N \times (0,T)$ as follows:

$$\begin{aligned} &\hat{v}^k_t - d[J * \hat{v}^k - \hat{v}^k] + M\hat{v}^k \\ &= \hat{v}^{k-1}[1 + \alpha\hat{v}^{k-1}] - \beta[\hat{\omega}^{k-1}]^3 - (1+\alpha-\beta)\hat{v}^k G * \hat{\omega}^{k-1}] + M\hat{v}^{k-1}, \end{aligned} \tag{2.10}$$

$$\begin{aligned} &\hat{\omega}^k_t - d[J * \hat{\omega}^k - \hat{\omega}^k] + M\hat{\omega}^k \\ &= \hat{\omega}^{k-1}[1 + \alpha\hat{\omega}^{k-1}] - \beta[\hat{v}^{k-1}]^3 - (1+\alpha-\beta)\hat{\omega}^k G * \hat{v}^{k-1}] + M\hat{\omega}^{k-1}, \end{aligned} \tag{2.11}$$

with initial conditions

$$\hat{v}^k(x,0) = \psi(x), \quad \hat{\omega}^k(x,0) = \psi(x).$$

Here M is a positive constant satisfying $M > \max\{(1+\alpha-\beta)G*\hat{\omega}, (1+\alpha-\beta)G*\hat{v}\}$.

We can show that $\hat{\omega}^k$ and $\hat{v}^k$ are upper-lower solutions of type II and

$$\hat{v}(x,t) \leq \hat{v}^k(x,t) \leq \hat{v}^{k+1}(x,t) \leq \hat{\omega}^k(x,t) \leq \hat{\omega}^{k+1}(x,t) \leq \hat{\omega}(x,t) \quad \text{for } k \geq 1.$$

Then the existence and uniqueness are similar to the proof of Theorem 2.3. In summary, we have the following result.

Theorem 2.7. *Suppose that $\hat{\omega}$ and $\hat{v}$ are a pair of ordered upper-lower solutions to (1.2)-(1.3) of type II. Then (1.2) admits a unique solution $u(x,t)$ in $\mathbb{R}^N \times [0,T)$ with $u(x,0) = \psi(x)$ which satisfies the relation*

$$\hat{v}(x,t) \leq u(x,t) \leq \hat{\omega}(x,t) \ ((x,t) \in \mathbb{R}^N \times [0,T)).$$

At the end of this section, we construct an upper solution to (1.2)-(1.3) and show that the solution is global, that is it is defined for all $t \geq 0$. To this end, let ω be the solution of the equation

$$\begin{gathered} \omega_t = \omega(1 + \alpha\omega - \beta\omega^2), \\ \omega(0) = \max|u_0|. \end{gathered}$$

Since ω is a bounded upper solution to (1.2)-(1.3), and it is trivial to see that 0 is lower solution. From Theorems 2.3, 2.7, 2.2 and 2.6, we have proved the following theorem.

Theorem 2.8. *Assume that* (A1)–(A3) *hold. Then there exists a unique global solution to* (1.2)-(1.3).

Finally, we use the spatially non-uniformly perturbations to discuss the effects of aggregation on the stability of uniformly steady solution of (1.2) when $\beta = 0$. In order to consider stability to perturbations of wave number k, we substitute $u(x,t) = 1 + \varepsilon e^{ikx} e^{\lambda t}$ into (1.2) and neglect high term of ε, then we obtain that

$$\lambda(k) = d[\hat{J}(k) - 1] + \alpha - (1 + \alpha - \beta)\hat{G}(k),$$

where $\hat{J}(k)$ denotes the Fourier transform of J; that is,

$$\hat{J}(k) = \int_{\mathbb{R}^N} J(x) e^{-ikx} dx.$$

In view of that $\hat{J}(0) = \hat{G}(0) = 1$, we have $\lambda(0) = -1 < 0$. However, by the basic properties of Fourier transform, we have

$$\lim_{k \to \infty} \hat{J}(k) = \lim_{k \to \infty} \hat{G}(k) = 0.$$

Thus, if k is large, we can take α large enough such that $\lambda(k) > 0$. In this case, we know that the uniform steady state 1 of (1.2) may become unstable. Under the assumption that α is large, there is a constant $k_1 > 0$ such that if $k > k_1$, the steady state of (1.2) is unstable to perturbations including the wave number k, which is quite different from (1.4), see for example [2].

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