

DYNAMICS OF LOGISTIC SYSTEMS DRIVEN BY LÉVY NOISE UNDER REGIME SWITCHING

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ABSTRACT. This article concerns the stochastic logistic models under regime switching with Lévy noise. In the model, the color noise and Lévy noise are taken into account at the same time. This model is new and more feasible and more accordance with the actual. Some dynamical behaviors are investigated and sufficient conditions for stochastic permanence, extinction, non-persistence in the mean and weak persistence are established. The critical value among the extinction, non-persistence in the mean and weak persistence is obtained. Our results demonstrate that the asymptotic properties of the model have close relations with the Lévy noise and stationary distribution of the color noise.

1. INTRODUCTION

Due to the importance in both ecology and mathematical ecology, the logistic model has been studied a lot, and many results have been reported, see [9, 10, 12, 15, 16, 17, 18, 19, 20, 21] and the references cited therein. The classical autonomous logistic equation is expressed by

$$\dot{x}(t) = x(t)[b - ax(t)] \quad (1.1)$$

for $t \geq 0$ with initial value $x(0) > 0$. In this model, $x(t)$ is the population size at time t , b denotes the intrinsic growth rate and b/a is the carrying capacity. However, in the real world the population systems are inevitably subject to stochastic environmental noise which is important in ecosystem (see e.g. Gard [7, 8]). If environmental noise is taken into account, the system will change significantly.

In practice, population systems may suffer from sudden environmental shocks, e.g., ocean red tide, soaring, tsunami, earthquakes, hurricanes, epidemics and so on, see [3, 4]. These events are so abrupt that they break the continuity of the solution. So models with only white noise can not explain these phenomena. In this case, introducing Lévy noise into the underlying population models may be a reasonable way to describe these phenomena, see [3, 4, 22]. Incorporating the effect of Lévy noise, model (1.1) changes into

$$dx(t) = x(t^-) \left[(b - ax(t^-))dt + \sigma x(t^-)dB(t) + \int_{\mathbb{Y}} \gamma(u)N(dt, du) \right]. \quad (1.2)$$

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In the model, $x(t^-)$ denotes the left limit of $x(t)$. $B(t)$ is a standard Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathbb{P})$ with a filtration $\{\mathcal{F}_t\}_{t \geq 0}$ satisfying the usual conditions, σ^2 denotes the intensity of the noise. N is a Poisson counting measure with characteristic measure λ on a measurable subset \mathbb{Y} of $(0, \infty)$ with $\lambda(\mathbb{Y}) < \infty$, $\tilde{N}(dt, du) = N(dt, du) - \lambda(du)dt$ is the corresponding martingale measure. The pair (B, N) is called a Lévy noise.

Models with Lévy noise have received considerable attention in recent years. Many scholars have examined the effects of Lévy noise on the population model. The famous result is that Mao, Marion, Renshaw [25] showed the environmental Brownian noise suppresses explosion in population dynamics. Bao et al. [3, 4] studied Lotka-Volterra population dynamics with Lévy noise, and analyzed the impacts of Lévy noise on the population dynamics. Since then, Liu and Wang [22] investigated the Leslie-Gower Holling-type II predator-prey system with Lévy noise. About the knowledge of Lévy noise, Situ [28], Applebaum [2] and Kunita [11] are all good references.

Now let us take a further step by considering another important type of environmental noise, the color noise, also called telegraph noise [24, 30]. The color noise can be regarded as a switching between two or more regimes of environment, which differ by factors such as rain falls or nutrition [5, 29]. Since the switching among the different environments is memoryless and the waiting time for the next switch has an exponential distribution, we can make use of a right-continuous Markov chain $r(t)$ with finite state space $S = \{1, \dots, N\}$ to model the regime switching. So far as our knowledge is concerned, the models which consider Lévy noise and the color noise at the same time have not been reported, not to mention the properties of the solution.

Inspired by the above discussions, we impose the color noise into model (1.2) and obtain the model

$$\begin{aligned} dx(t) = & x(t^-) \left[(b(r(t)) - a(r(t))x(t^-))dt + \sigma(r(t))x(t^-)dB(t) \right. \\ & \left. + \int_{\mathbb{Y}} \gamma(r(t), u)N(dt, du) \right]. \end{aligned} \quad (1.3)$$

As pointed out in [20], the mechanism of the ecosystem described by (1.3) can be explained by follows. If the initial state $r(0) = i \in S$, then (1.3) obeys

$$dx(t) = x(t^-) \left[(b(i) - a(i)x(t^-))dt + \sigma(i)x(t^-)dB(t) + \int_{\mathbb{Y}} \gamma(i, u)N(dt, du) \right]$$

till time τ_1 when the Markov chain switches to $r(1) = j \in S$ from $r(0)$; then the system obeys

$$dx(t) = x(t^-) \left[(b(j) - a(j)x(t^-))dt + \sigma(j)x(t^-)dB(t) + \int_{\mathbb{Y}} \gamma(j, u)N(dt, du) \right]$$

until the next switching. The system will continue to switch as long as the Markov chain switches. The Markov chain has significant impacts on the population dynamics. Takeuchi et al. [30] considered a two-dimensional autonomous Lotka-Volterra predator-prey system with regime switching and showed that the stochastic population system is neither permanent nor dissipative (see [6]) which is an important result because it reveals the significant effect of the environmental noise to the population system: both its subsystems develop periodically but switching between them makes them become neither permanent nor dissipative.

In this article, we attempt to explore the effects of the color noise and Lévy noise on the dynamical properties of system (1.3). As we know that, the extinction and stochastic permanence are two important and interesting properties in the biomathematics, and the threshold value of extinction and survival is valuable in practice. So in this paper we consider the extinction and stochastic permanence of system (1.3), and try to give the threshold value of extinction and survival.

2. GLOBAL POSITIVE SOLUTIONS

Throughout this paper, we assume $\min_{k \in S} a(k) > 0$. For the biological background, (see [34]), we assume $\gamma(k, u) > -1$, for all $k \in S$, $u \in \mathbb{Y}$. Write $\mathbb{R}_+ = [0, \infty)$. Moreover, for a matrix or vector G , $G \gg 0$ means all elements of G are positive.

Let $r(t)$ be a right-continuous Markov chain taking values in a finite state space $S = \{1, 2, \dots, N\}$ with the generator $Q = (q_{ij})_{N \times N}$ given by

$$\mathbb{P} = \{r(t + \Delta t) = j | r(t) = i\} = \begin{cases} q_{ij}\Delta t + o(\Delta t), & \text{if } j \neq i; \\ 1 + q_{ii}\Delta t + o(\Delta t), & \text{if } j = i, \end{cases}$$

where $\Delta t > 0$, $q_{ij} \geq 0$ is transition rate from i to j if $i \neq j$ while $\sum_{j=1}^N q_{ij} = 0$. Further assume that Markov chain $r(t)$ is irreducible which means that the system can switch from any regime to any other regime. It is known that (see [1]) the irreducibility implies that the Markov chain has a unique stationary distribution $\pi = (\pi_1, \pi_2, \dots, \pi_N) \in R^{1 \times N}$ satisfying

$$\pi Q = 0 \tag{2.1}$$

and

$$\sum_{i=1}^N \pi_i = 1 \quad \text{and} \quad \pi_i > 0, \quad \forall i \in S.$$

In the sequel, for convenience and simplicity, we adopt the following symbols:

$$\begin{aligned} \hat{f} &= \min_{k \in S} f(k), & \check{f} &= \max_{k \in S} f(k), & \overline{f(t)} &= t^{-1} \int_0^t f(s) ds, \\ f^* &= \limsup_{t \rightarrow +\infty} f(t), & f_* &= \liminf_{t \rightarrow +\infty} f(t). \end{aligned}$$

Due to biology, for model (1.3), we are only interested in positive solutions.

For the jump-diffusion coefficient, we assume

(A1) There exists a positive constant c such that

$$\int_{\mathbb{Y}} [\ln(1 + \gamma(i, u))]^2 \lambda(du) < c, \quad \text{for all } i \in S.$$

About the rationality and biological significance of this assumption, the readers can refer to [34].

Before we consider the properties of the solutions, first we should guarantee the existence of positive solutions. We have the following result.

Theorem 2.1. *Under Assumption (A1), for any initial value $r(0) \in S$ and $x(0) > 0$, Equation (1.3) admits a unique positive solution $x(t)$ on $t \geq 0$.*

Proof. Our proof is motivated by Bao and Yuan [4]. Since the coefficients of the equation are local Lipschitz continuous, then for any initial data $x(0) > 0$, Equation (1.3) has a unique local solution $x(t)$ on $[0, \tau_e)$, where τ_e is the explosion time [2].

To show this solution is global, we only need to show that $\tau_e = \infty$. Let $k_0 > 0$ be so large that $x(0) \in [1/k_0, k_0]$. For each integer $k > k_0$, define a sequence of stopping time expressed by

$$\tau_k = \inf\{t \in [0, \tau_e) : x(t) \notin (1/k, k)\}.$$

So τ_k is increasing as $k \rightarrow \infty$. Let $\tau_\infty = \lim_{k \rightarrow \infty} \tau_k$, then $\tau_\infty \leq \tau_e$ a.s. If we can show $\tau_\infty = \infty$, then $\tau_e = \infty$. Namely, if we have $\tau_\infty = \infty$, then we complete the proof. For any $p \in (0, 1)$, define a C^2 -function $V : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ by

$$V(x) = x^p. \quad (2.2)$$

Let $T > 0$ be arbitrary, for any $0 \leq t \leq \tau_k \wedge T$, applying generalized Itô formula with jumps results in

$$\begin{aligned} dV(x(t)) &= px^{p-1}[x(b(r(t)) - a(r(t))x)dt + \sigma(r(t))x^2dB(t)] \\ &\quad + \frac{1}{2}p(p-1)x^{p-2}\sigma^2(r(t))x^4dt + \int_{\mathbb{Y}} [(x + x\gamma(r(t), u))^p - x^p]N(dt, du) \\ &= x^p \left[\frac{1}{2}p(p-1)\sigma^2(r(t))x^2 - pa(r(t))x + pb(r(t)) \right. \\ &\quad \left. + \int_{\mathbb{Y}} [(1 + \gamma(r(t), u))^p - 1]\lambda(du) \right] dt \\ &\quad + x^p \int_{\mathbb{Y}} [(1 + \gamma(r(t), u))^p - 1]\tilde{N}(dt, du) + p\sigma(r(t))x^{p+1}dB(t) \\ &= LV(x(t))dt + x^p \int_{\mathbb{Y}} [(1 + \gamma(r(t), u))^p - 1]\tilde{N}(dt, du) + p\sigma(r(t))x^{p+1}dB(t), \end{aligned} \quad (2.3)$$

where

$$\begin{aligned} LV(x) &= x^p \left[\frac{1}{2}p(p-1)\sigma^2(r(t))x^2 - pa(r(t))x + pb(r(t)) \right. \\ &\quad \left. + \int_{\mathbb{Y}} [(1 + \gamma(r(t), u))^p - 1]\lambda(du) \right] \\ &\leq x^p \left[\frac{1}{2}p(p-1)(\hat{\sigma})^2x^2 - p\hat{a}x + p\hat{b} + \int_{\mathbb{Y}} [(1 + \check{\gamma}(u))^p - 1]\lambda(du) \right]. \end{aligned} \quad (2.4)$$

Here, for simplicity, we omit t^- in $x(t^-)$. By the value $p \in (0, 1)$, there exists a constant M such that

$$LV(x) \leq M. \quad (2.5)$$

For each $u > 0$, define

$$\mu(u) = \inf\{V(x), |x| \geq u\}.$$

It is easy to see that

$$\lim_{u \rightarrow \infty} \mu(u) = \infty. \quad (2.6)$$

Using (2.5) it follows that

$$\mu(k)\mathbb{P}(\tau_k \leq T) \leq \mathbb{E}\left(V(x(\tau_k))I_{\tau_k \leq T}\right) \leq \mathbb{E}V\left(x(\tau_k \wedge T)\right) \leq MT.$$

Letting $k \rightarrow \infty$ and using (2.6), it results that $\mathbb{P}(\tau_\infty \leq T) = 0$. By the arbitrariness of T , we must have $\mathbb{P}(\tau_\infty = \infty) = 1$. This completes the proof. \square

Now, it follows that system (1.3) admits a unique global positive solution. From the biological point of view, the nonexplosion property and positivity in a population dynamical system are often not good enough. Further, in the next we will investigate asymptotic properties of the solutions.

3. CRITICAL VALUE BETWEEN EXTINCTION AND PERSISTENCE

In the next we present a lemma which plays important roles in our paper.

Lemma 3.1 ([14]). *Suppose that $M(t), t \geq 0$, is a local martingale with $M(0) = 0$. Then*

$$\lim_{t \rightarrow +\infty} \rho_M(t) < \infty \Rightarrow \lim_{t \rightarrow +\infty} \frac{M(t)}{t} = 0 \quad a.s.,$$

where

$$\rho_M(t) = \int_0^t \frac{d\langle M \rangle(s)}{(1+s)^2}, \quad t \geq 0$$

and $\langle M \rangle(t)$ is Meyer's angle bracket process (see e.g. [11])

In the sequel, we will consider long time behaviors of the positive solutions which are important in applications, because they can predict the future properties of the solutions. First, we give several definitions, then we will try to illustrate sufficient conditions for them.

Definition 3.2 ([17]). Let $x(t)$ be the solution of (1.3),

- (a) if $\lim_{t \rightarrow +\infty} x(t) = 0$, we call the species modeled by (1.3) is extinction.
- (b) if $\lim_{t \rightarrow +\infty} \overline{x(t)} = \lim_{t \rightarrow +\infty} t^{-1} \int_0^t x(s) ds = 0$, species modeled by (1.3) is called non-persistence in the mean.
- (c) if $x^* = \limsup_{t \rightarrow +\infty} x(t) > 0$, we call species modeled by (1.3) is weakly persistence.

Definition 3.3 ([12]). The solutions $x(t)$ of (1.3) are called stochastically ultimate bounded, if for any initial value $x(0) > 0$, and for all $\epsilon \in (0, 1)$, there exists $H = H_\epsilon > 0$, such that the solutions $x(t)$ of (1.3) satisfy

$$\limsup_{t \rightarrow +\infty} \mathbb{P}[|x(t)| > H] < \epsilon.$$

Definition 3.4 ([17]). The solution $x(t)$ of (1.3) is said to be stochastically permanent, if for any $\epsilon \in (0, 1)$, there is a pair of positive constants $H_1 = H_1(\epsilon)$ and $H_2 = H_2(\epsilon)$ such that

$$\liminf_{t \rightarrow +\infty} \mathbb{P}[|x(t)| \leq H_1] \geq 1 - \epsilon, \quad \liminf_{t \rightarrow +\infty} \mathbb{P}[|x(t)| \geq H_2] \geq 1 - \epsilon.$$

where $x(t)$ is an arbitrary solution of the equation with initial value $x(0) > 0$, $r(0) \in S$.

From the above definitions we can see that extinction implies non-persistence in the mean, stochastically ultimate boundedness means the solution will be ultimately bounded with the large probability, and the stochastic permanence is the strongest property, we will consider them one by one.

Theorem 3.5. *Let Assumption (A1) hold, then for the initial value $x(0) > 0$ and $r(0) \in S$, the solution $x(t)$ of (1.3) satisfies*

$$\limsup_{t \rightarrow \infty} \frac{\ln x(t)}{t} \leq \sum_{i=1}^N h(i)\pi_i.$$

Particularly, if $\sum_{i=1}^N h(i)\pi_i < 0$, then species $x(t)$ will go to extinction a.s., where $h(i) = b(i) + \int_{\mathbb{Y}} (\ln(1 + \gamma(i, u)))\lambda(du)$.

Proof. For (1.3), applying generalized Itô's formula with jumps to $\ln x$ yields

$$\begin{aligned} d \ln x(t) &= \frac{1}{x} \left[x(b(r(t)) - a(r(t))x)dt + \sigma(r(t))x^2 dB(t) \right] + \frac{1}{2} \cdot -\frac{1}{x^2} \sigma^2(r(t))x^4 dt \\ &\quad + \int_{\mathbb{Y}} \left[\ln(x + x\gamma(r(t), u)) - \ln x \right] N(dt, du) \\ &= \left[b(r(t)) - a(r(t))x - \frac{1}{2} \sigma^2(r(t))x^2 + \int_{\mathbb{Y}} (\ln(1 + \gamma(r(t), u))\lambda(du)) \right] dt \\ &\quad + \sigma(r(t))x dB(t) + \int_{\mathbb{Y}} \ln(1 + \gamma(r(t), u)) \tilde{N}(dt, du). \end{aligned}$$

In other words,

$$\begin{aligned} &\ln x(t) - \ln x(0) \\ &= \int_0^t h(r(s))ds - \int_0^t a(r(s))x(s)ds - \frac{1}{2} \int_0^t \sigma^2(r(s))x^2(s)ds \\ &\quad + \int_0^t \sigma(r(s))x(s)dB(s) + \int_0^t \int_{\mathbb{Y}} \ln(1 + \gamma(r(s), u)) \tilde{N}(ds, du) \\ &= \int_0^t h(r(s))ds - \int_0^t a(r(s))x(s)ds - \frac{1}{2} \int_0^t \sigma^2(r(s))x^2(s)ds + M(t) + Q(t). \end{aligned} \tag{3.1}$$

Where $M(t) = \int_0^t \sigma(r(s))x(s)dB(s)$, $Q(t) = \int_0^t \int_{\mathbb{Y}} \ln(1 + \gamma(r(s), u)) \tilde{N}(ds, du)$. The quadratic variation of $M(t)$ is

$$\langle M(t), M(t) \rangle = \int_0^t \sigma^2(r(s))x^2(s)ds.$$

By the exponential martingale inequality [27], for any positive numbers T, α and β , we have

$$\mathbb{P} \left(\sup_{0 \leq t \leq T} [M(t) - \frac{\alpha}{2} \langle M(t), M(t) \rangle] > \beta \right) \leq e^{-\alpha\beta}.$$

Choose $T = n, \alpha = 1, \beta = 2 \ln n$, we have

$$\mathbb{P} \left(\sup_{0 \leq t \leq n} [M(t) - \frac{1}{2} \langle M(t), M(t) \rangle] > 2 \ln n \right) \leq \frac{1}{n^2}.$$

Since $\sum_{n=1}^{\infty} 1/n^2 < \infty$, making using of Borel-Cantelli lemma [27] follows that for almost all $\omega \in \Omega$, there is a random integer $n_0 = n_0(\omega)$ such that for $n \geq n_0$

$$\sup_{0 \leq t \leq n} [M(t) - \frac{1}{2} \langle M(t), M(t) \rangle] \leq 2 \ln n.$$

This is equivalent to

$$M(t) \leq 2 \ln n + \frac{1}{2} \langle M(t), M(t) \rangle = 2 \ln n + \frac{1}{2} \int_0^t \sigma^2(r(s))x^2(s)ds, \tag{3.2}$$

for all $0 \leq t \leq n, n \geq n_0$. Substituting (3.2) into (3.1) results in

$$\ln x(t) - \ln x(0) \leq \int_0^t h(r(s))ds - \int_0^t a(r(s))x(s)ds + 2 \ln n + Q(t). \tag{3.3}$$

On the other hand, by Assumption (A1),

$$\langle Q(t), Q(t) \rangle = \int_0^t \int_{\mathbb{Y}} [\ln(1 + \gamma((r(s)), u))]^2 \lambda(du) ds \leq ct.$$

In view of Lemma 2, we obtain

$$\lim_{t \rightarrow +\infty} \frac{Q(t)}{t} = 0 \quad \text{a.s.} \quad (3.4)$$

Dividing (3.3) by t , for $n - 1 \leq t \leq n$, $n \geq n_0$, we obtain

$$\begin{aligned} t^{-1} [\ln x(t) - \ln x(0)] &\leq \frac{1}{t} \int_0^t h(r(s)) ds - \frac{1}{t} \int_0^t a(r(s)) x(s) ds + \frac{2 \ln n}{n-1} + \frac{Q(t)}{t} \\ &\leq \frac{1}{t} \int_0^t h(r(s)) ds + \frac{2 \ln n}{n-1} + \frac{Q(t)}{t}. \end{aligned}$$

Taking the superior limit and using (3.4) and the ergodic property of the Markov chain, we follow our desired assertion. This completes the proof. \square

Remark 3.6. It is evident that $x(t) \equiv 0$ is the trivial solution of (1.3), by Theorem 3.5, we conclude that if $\sum_{i=1}^N h(i)\pi_i < 0$, the trivial solution of system (1.3) is almost surely exponentially stable.

Theorem 3.7. *If $\sum_{i=1}^N h(i)\pi_i = 0$, then species modeled by (1.3) will be non-persistence in the mean a.s.*

Proof. By the fact that $\lim_{t \rightarrow +\infty} t^{-1} \int_0^t h(r(s)) ds = \sum_{i=1}^N h(i)\pi_i$ and (3.4), for all $\varepsilon > 0$, there exists a positive constant T_1 , for $t > T_1$ we have

$$t^{-1} \int_0^t h(r(s)) ds \leq \sum_{i=1}^N h(i)\pi_i + \varepsilon/4 = \varepsilon/4, \quad Q(t)/t \leq \varepsilon/4.$$

Then, for $T_1 < t \leq n$, $n \geq n_0$, (3.3) changes into

$$\ln x(t) - \ln x(0) \leq \varepsilon t/2 - \hat{a} \int_0^t x(s) ds + 2 \ln n.$$

Note that for sufficiently large t with $T_1 < T < n - 1 \leq t \leq n$, $n \geq n_0$, we have $(\ln n)/t \leq \varepsilon/4$. So we follow that

$$\ln x(t) - \ln x(0) \leq \varepsilon t - \hat{a} \int_0^t x(s) ds, \quad t > T.$$

Using Lemma 2 [23], we have $\bar{x}^* \leq \varepsilon/\hat{a}$, by the arbitrariness of ε , we get our required assertion. This completes the proof. \square

Lemma 3.8. *For any initial value $x(0) > 0$ and $\alpha(0) \in S$, the solution $x(t)$ of (1.3) has the property*

$$\limsup_{t \rightarrow +\infty} \frac{\ln x(t)}{t} \leq 0 \quad \text{a.s.} \quad (3.5)$$

The proof of the above lemma is similar to that of [33, Theorem 3.3]; we omit it here.

Theorem 3.9. *If $\sum_{i=1}^N h(i)\pi_i > 0$, then species modeled by (1.3) will be weak persistence a.s.*

Proof. Suppose that the result is not true, then $\mathbb{P}(E) > 0$, where $E = \{x^* = 0\}$. By (3.1), we find

$$t^{-1}[\ln x(t) - \ln x(0)] = \overline{h(r(t)) - a(r(t))x(t)} - \frac{1}{2}\overline{\sigma^2(r(t))x(t)} + M(t)/t + Q(t)/t. \quad (3.6)$$

Note that $\lim_{t \rightarrow +\infty} x(t, \omega) = 0$ for all $\omega \in E$. Since σ is bounded, by Lemma 3.1, we have $\lim_{t \rightarrow +\infty} M(t)/t = 0$. Substituting (3.4) in (3.6), we obtain $[t^{-1} \ln x(t, \omega)]^* = \overline{h(r(t))}^* = \sum_{i=1}^N h(i)\pi_i > 0$, then $\mathbb{P}\{[t^{-1} \ln x(t)]^* > 0\} > 0$ which contradicts with (3.5). This completes the proof. \square

Remark 3.10. Theorems 3.5–3.9 have an obvious and interesting biological interpretation. It is evident that the extinction and persistence of species $x(t)$ modeled by (1.3) depend only on the value $\sum_{i=1}^N h(i)\pi_i$. By $h(i) = b(i) + \int_{\mathbb{Y}} (\ln(1 + \gamma(i, u)))\lambda(du)$, we can see that the white noise $\sigma(t)$ imposed on the intraspecific competition coefficient has no impact on the extinction and persistence of the species, which coincides with the special case (see [17]) when $\gamma(i, u) \equiv 0$.

Remark 3.11. Let us consider the effect of jump-diffusion coefficient $\gamma(i, u)$ on the extinction and persistence of species. If $\gamma(i, u) < 0$, which means that the jumping noise is always disadvantage for a ecosystem, e.g. tsunami, earthquakes, then $h(i) < b(i)$, so the jump noise can make the species extinctive; if $\gamma(i, u) > 0$, which implies that the jumping noise is always advantage for a ecosystem, e.g. ocean red tide, soaring, then $h(i) > b(i) > 0$, so the jump noise guarantees the population of (1.3) will be weak persistence.

Remark 3.12. Let us consider the subsystem

$$dx(t) = x(t^-) \left[(b(i) - a(i)x(t^-))dt + \sigma(i)x(t^-)dB(t) + \int_{\mathbb{Y}} \gamma(i, u)N(dt, du) \right]. \quad (3.7)$$

Similarly, we can prove that if $h(i) < 0$, then species $x(t)$ of (3.7) will go to extinction, $h(i) = 0$, then species $x(t)$ of (3.7) will non-persistence in the mean, if $h(i) > 0$, then species $x(t)$ of (3.7) will weak persistence.

Remark 3.13. Let us turn to see the impact on the model of the Markov switching. If for some $i \in S$, $h(i) < 0$, then the corresponding subsystem (3.7) is extinctive. Theorem 3.5 tells us that if every individual of (1.3) is extinctive, then as a result of Markovian switching, the overall behavior of (1.3) remains extinctive. However, Theorem 3.5-3.9 imply an interesting result that if some individual subsystem is extinction, again as a result of Markovian switching, the value $\sum_{i=1}^N h(i)\pi_i$ may be equal to zero or large than zero, then the overall behavior of (1.3) may be non-persistence in the mean or weak persistence.

4. STOCHASTIC PERMANENCE

Stochastic permanence is an important asymptotic behavior, it implies that the population will survive forever, so it is interesting in the biomathematics. In the following, we strengthen the condition to get the stochastic permanence. We use the assumptions

(A2) For some $u \in S$, $q_{iu} > 0$, for all $i \neq u$.

Lemma 4.1. *Let Assumption (A2) hold. If $\bar{h} = \sum_{i=1}^N \pi_i \bar{h}(i) > 0$, then there exists a constant $\theta > 0$ such that the matrix*

$$A(\theta) := \text{diag}(\xi_1(\theta), \xi_2(\theta), \dots, \xi_N(\theta)) - Q \tag{4.1}$$

is a nonsingular M-matrix, where $\bar{h}(i) = 2b(i) - \int_{\mathbb{Y}} (\frac{1}{(1+\gamma(i,u))^2} - 1) \lambda(du)$, $\xi_i(\theta) = \theta \bar{h}(i)$.

Proof. This proof is motivated by [13]. It is known that a determinant will not change its value if we switch the i th row with the j th row and then switch the i th column with the j th column. It is also known that given a nonsingular M-matrix, if we switch the i th row with the j th row and then switch the i th column with the j th column, then the new matrix is still a nonsingular M-matrix. Without loss of generality, we assume $u = N$ in Assumption (A2), namely

$$q_{iN} > 0, \quad 1 \leq i \leq N - 1.$$

Using $\sum_{i=1}^N q_{ij} = 0$, $i = 1, 2, \dots, N$ it follows that

$$\det A(\theta) = \begin{vmatrix} \xi_1(\theta) & -q_{12} & \dots & -q_{1N} \\ \xi_2(\theta) & \xi_2(\theta) - q_{22} & \dots & -q_{2N} \\ \vdots & \vdots & \dots & -q_{N-1,N} \\ \xi_N(\theta) & -q_{N2} & \dots & \xi_N(\theta) - q_{NN} \end{vmatrix} = \sum_{k=1}^N \xi_k(\theta) M_k(\theta),$$

where $M_k(\theta)$ is the corresponding minor of $\xi_k(\theta)$ in the first column; i.e.,

$$M_1(\theta) = (-1)^{1+1} \begin{vmatrix} \xi_2(\theta) - q_{22} & \dots & -q_{2N} \\ \vdots & \dots & \vdots \\ -q_{N-1,2} & \dots & -q_{N-1,N} \\ -q_{N,2} & \dots & \xi_N(\theta) - q_{NN} \end{vmatrix},$$

$$\dots$$

$$M_N(\theta) = (-1)^{N+1} \begin{vmatrix} -q_{12} & \dots & -q_{1N} \\ \xi_2(\theta) - q_{22} & \dots & -q_{2N} \\ \vdots & \dots & \vdots \\ -q_{N-1,2} & \dots & -q_{N-1,N} \end{vmatrix}.$$

Note that

$$\xi_k(0) = 0, \quad \frac{d}{d\theta} \xi_k(0) = \bar{h}(k);$$

so we have

$$\frac{d}{d\theta} \det A(0) = \sum_{k=1}^N \bar{h}(k) M_k(0).$$

This means that

$$\frac{d}{d\theta} \det A(0) = \begin{vmatrix} \bar{h}(1) & -q_{12} & \dots & -q_{1N} \\ \bar{h}(2) & -q_{22} & \dots & -q_{2N} \\ \vdots & \vdots & \dots & \vdots \\ \bar{h}(N) & -q_{N2} & \dots & -q_{NN} \end{vmatrix}. \tag{4.2}$$

According to [26, Appendix A], the condition $\sum_{k=1}^N \pi_k \bar{b}(k) > 0$ is equivalent to

$$\begin{vmatrix} \bar{h}(1) & -q_{12} & \cdots & -q_{1N} \\ \bar{h}(2) & -q_{22} & \cdots & -q_{2N} \\ \vdots & \vdots & \cdots & \vdots \\ \bar{h}(N) & -q_{N2} & \cdots & -q_{NN} \end{vmatrix} > 0.$$

Together with (4.2), we see that

$$\frac{d}{d\theta} \det A(0) > 0.$$

By $\det A(0) = 0$, we can find a sufficiently small $\theta > 0$ such that $\det A(\theta) > 0$ and

$$\xi_k(\theta) = \theta \left[2b(k) - \int_{\mathbb{Y}} \left(\frac{1}{(1 + \gamma(k, u))^2} - 1 \right) \lambda(du) \right] > -q_{kN}, \quad 1 \leq k \leq N-1. \quad (4.3)$$

For every $1 \leq k \leq N-1$, we consider the leading principle sub-matrix

$$A_k(\theta) := \begin{vmatrix} \xi_1(\theta) - q_{11} & -q_{12} & \cdots & -q_{1k} \\ -q_{21} & \xi_2(\theta) - q_{22} & \cdots & -q_{2k} \\ \vdots & \cdots & \ddots & \vdots \\ -q_{k1} & -q_{k2} & \cdots & \xi_k(\theta) - q_{kk} \end{vmatrix}$$

of $A(\theta)$. Clearly, $A_k(\theta) \in \mathcal{Z}^{N \times N} := \{A = (a_{ij})_{N \times N} : a_{ij} \leq 0, i \neq j\}$. By (4.3) we follow that each row of this sun-matrix has the sum

$$\xi_k(\theta) - \sum_{j=1}^k q_{kj} \geq \xi_k(\theta) + q_{kN} > 0.$$

By [27, Lemma 5.3], we have $\det A_k(\theta) > 0$. In other words, we reach that all the leading principle minors of $A(\theta)$ are positive. According to Theorem 2.10 [27], we obtain the desired assertion. \square

Theorem 4.2. *For any $p \in (0, 1)$, there exists a constant $K(p)$ such that the solution of (1.3) has the property*

$$\limsup_{t \rightarrow +\infty} \mathbb{E}|x(t)|^p \leq K(p).$$

Proof. For any $p \in (0, 1)$, let V be defined by (2.2). For any $|x(0)| < k$, define a stopping time

$$\sigma_k = \inf\{t \geq 0, |x(t)| > k\}.$$

Then $\sigma_k \uparrow \infty$ a.s. as $k \rightarrow \infty$. Applying Itô's formula yields

$$\mathbb{E} \left[e^{t \wedge \sigma_k} V(x(t \wedge \sigma_k)) \right] = V(x(0)) + \mathbb{E} \int_0^{t \wedge \sigma_k} e^s [V(x(s)) + LV(x(s))] ds,$$

where $LV(x)$ is defined as (2.4). Since the leading term of $V(x) + LV(x)$ is less than zero, then there exists a constant $K(p) > 0$ such that $V(x) + LV(x) \leq K(p)$. Hence $\mathbb{E} [e^t V(x(t))] \leq V(x(0)) + K(p)e^t$. Taking the superior limit for both sides, we have $\limsup_{t \rightarrow +\infty} \mathbb{E}|x(t)|^p \leq K(p)$ which is our desired assertion. This completes the proof. \square

As an application of Theorem 4.2 together with Chebyshev's inequality, we get the following result.

Theorem 4.3. *Equation (1.3) is stochastically ultimate bounded.*

We are now in position to present our main result of this section.

Theorem 4.4. *Under Assumption (A2), if $\bar{h} = \sum_{i=1}^N \pi_i \bar{h}(i) > 0$, then species $x(t)$ modeled by (1.3) will be stochastic permanence.*

Proof. As applications of Chebyshev's inequality and Theorem 4.2, we can get

$$\liminf_{t \rightarrow +\infty} \mathbb{P}[x(t) \leq H_1] \geq 1 - \varepsilon.$$

In the following, we will prove the another inequality $\liminf_{t \rightarrow +\infty} \mathbb{P}[x(t) \geq H_2] \geq 1 - \varepsilon$. Define $V_1(x) = \frac{1}{x^2}$, using generalized Itô formula results in

$$\begin{aligned} dV_1(x) &= 2V_1 \left[a(k)x - b(k) \right] dt + 3\sigma^2(k)dt - 2\sigma(k)x^{-1}dB(t) \\ &\quad + V_1 \int_{\mathbb{Y}} \left[\frac{1}{(1 + \gamma(k, u))^2} - 1 \right] N(dt, du), \end{aligned}$$

where we drop t from $x(t)$ and $r(k(t))$ etc. again. For θ given in Lemma 4.1, by Theorem 2.10 [27], there exists a vector $\vec{p} = (p_1, p_2, \dots, p_N)^T \gg 0$ such that $A(\theta)\vec{p} \gg 0$ which is equivalent to

$$p_k \theta \left(2b(k) - \int_{\mathbb{Y}} \left(\frac{1}{(1 + \gamma(k, u))^2} - 1 \right) \lambda(du) \right) - \sum_{j=1}^N q_{kj} p_j > 0, \quad \text{for } 1 \leq k \leq N. \quad (4.4)$$

Define function $V_2 : \mathbb{R}_+^n \times \mathbb{S} \rightarrow \mathbb{R}_+$ by

$$V_2(x, k) = p_k (1 + V_1)^\theta.$$

Making use of the generalized Itô formula follows that

$$\mathbb{E}V_2(x(t), r(t)) = V_2(x(0), \alpha(0)) + \mathbb{E} \int_0^t LV_2(x(s), r(s)) ds,$$

where

$$\begin{aligned} LV_2(x, k) &= \theta p_k (1 + V_1)^{\theta-2} \left\{ 2V_1 (1 + V_1) (a(k)x - b(k)) + 3\sigma^2(k) (1 + V_1) \right. \\ &\quad \left. + 2(\theta - 1)\sigma^2(k)V_1 \right\} + \sum_{j=1}^N q_{kj} p_j (1 + V_1)^\theta \\ &\quad + \int_{\mathbb{Z}} p_k \left[(1 + V_1 + V_1 \left(\frac{1}{(1 + \gamma(k, u))^2} - 1 \right))^\theta - (1 + V_1)^\theta \right] \lambda(du). \end{aligned}$$

Note that

$$(1 + V_1 + V_1 \left(\frac{1}{(1 + \gamma(k, u))^2} - 1 \right))^\theta - (1 + V_1)^\theta \leq (1 + V_1)^{\theta-1} \theta V_1 \left(\frac{1}{(1 + \gamma(k, u))^2} - 1 \right).$$

Here, we use the fundamental inequality $x^r \leq 1 + r(x - 1)$, $x \geq 0$, $1 \geq r \geq 0$. Further, we have

$$\begin{aligned}
 & LV_2(x, k) \\
 & \leq (1 + V_1)^{\theta-2} \left\{ -V_1^2(2\theta p_k b(k) - \theta p_k \int_{\mathbb{Y}} (\frac{1}{(1 + \gamma(k, u))^2} - 1) \lambda(du)) \right. \\
 & \quad - \sum_{j=1}^N q_{kj} p_j + 2\theta p_k a(k) V_1^{1.5} + V_1(-2\theta p_k b(k) + (2\theta + 1)\theta p_k \sigma^2(k) + 2 \sum_{j=1}^N q_{kj} p_j \\
 & \quad \left. + \int_{\mathbb{Y}} \theta p_k (\frac{1}{(1 + \gamma(k, u))^2} - 1) \lambda(du)) + 2\theta p_k a(k) V_1^{0.5} + 3\theta p_k \sigma^2(k) + \sum_{j=1}^N q_{kj} p_j \right\}.
 \end{aligned} \tag{4.5}$$

Now, by (4.4) we can choose a sufficiently small η to satisfy

$$p_k \theta \left(2b(k) - \int_{\mathbb{Y}} (\frac{1}{(1 + \gamma(k, u))^2} - 1) \lambda(du) \right) - \sum_{j=1}^N q_{kj} p_j - \eta p_k > 0, \tag{4.6}$$

for $1 \leq k \leq N$. Using generalized Itô formula again, we obtain

$$\mathbb{E}[e^{\eta t} V_2(x(t), r(t))] = V_2(x(0), r(0)) + \mathbb{E} \int_0^t e^{\eta s} [LV_2(x(s), r(s)) + \eta V_2(x(s))] ds. \tag{4.7}$$

By (4.5) it follows that

$$\begin{aligned}
 & LV_2(x, k) + \eta V_2 \\
 & \leq (1 + V_1)^{\theta-2} \left\{ -V_1^2(2\theta p_k b(k) - \theta p_k \int_{\mathbb{Y}} (\frac{1}{(1 + \gamma(k, u))^2} - 1) \lambda(du)) \right. \\
 & \quad - \sum_{j=1}^N q_{kj} p_j - \eta p_k + 2\theta p_k a(k) V_1^{1.5} + V_1(-2\theta p_k b(k) + (2\theta + 1)\theta p_k \sigma^2(k) \\
 & \quad + 2 \sum_{j=1}^N q_{kj} p_j + 2\eta p_k + \int_{\mathbb{Y}} \theta p_k (\frac{1}{(1 + \gamma(k, u))^2} - 1) \lambda(du)) \\
 & \quad \left. + 2\theta p_k a(k) V_1^{0.5} + 3\theta p_k \sigma^2(k) + \eta p_k + \sum_{j=1}^N q_{kj} p_j \right\}.
 \end{aligned}$$

According to (4.6), $LV_2 + \eta V_2$ is bounded, namely, there exists a constant M such that $LV_2 + \eta V_2 \leq M$. Therefore, (4.7) changes into

$$\mathbb{E}[V_2(x, k)] \leq e^{-\eta t} V_2(x(0), r(0)) + M/\eta.$$

Further we have

$$\limsup_{t \rightarrow +\infty} \mathbb{E}[V_1^\theta(x(t))] \leq \limsup_{t \rightarrow +\infty} \mathbb{E}[(1 + V_1(x(t)))^\theta] \leq M/(\eta \hat{p}).$$

Namely,

$$\limsup_{t \rightarrow +\infty} \mathbb{E}[|x(t)|^{-2\theta}] \leq M/(\eta \hat{p}) := K.$$

For any given $\varepsilon > 0$, let $H_2 = (\varepsilon/K)^{\frac{1}{2\theta}}$, by Chebyshev inequality, we see that

$$\mathbb{P}\{|x(t)| \leq H_2\} = \mathbb{P}\{|x(t)|^{-2\theta} \geq H_2^{-2\theta}\} \leq \frac{E(|x(t)|^{-2\theta})}{H_2^{-2\theta}}.$$

So, $\limsup_{t \rightarrow +\infty} \mathbb{P}\{|x(t)| \leq H_2\} \leq \varepsilon$. Therefore, $\liminf_{t \rightarrow +\infty} \mathbb{P}\{|x(t)| \geq H_2\} \geq 1 - \varepsilon$ is obtained. \square

Remark 4.5. If the jump-diffusion coefficient $\gamma(k, u) \equiv 0$, then our result coincides with Theorem 5 in [21] without jumps, this demonstrates that our result is a strictly generalization of [21].

Remark 4.6. For the subsystem (3.7), similarly, we have if $\bar{h}(i) > 0$, then species $x(t)$ of (3.7) will be stochastic permanence. That is to say, if every individual equation in (1.3) is stochastically permanent, then as the result of Markovian switching, the overall behavior of (1.3) remains stochastically permanent. However, Theorem 4.4 reveals a more interesting result. If some individual equations in (1.3) are extinctive, some are stochastically permanent, again as the result of Markovian switching, the overall behavior of (1.3) may be stochastically persistent, depending on the value of $\bar{h} = \sum_{i=1}^N \pi_i \bar{h}(i) > 0$.

5. NUMERICAL SIMULATIONS

In this section, we give two numerical simulations to support the results obtained. In our examples, we assume the Markov chain $r(t)$ takes values in the state space $\mathbb{S} = \{1, 2\}$. Let the generator Q be expressed by $Q = \begin{pmatrix} -7 & 7 \\ 5 & -5 \end{pmatrix}$, then the unique stationary distribution π of $r(t)$ is expressed by $\pi = (\pi_1, \pi_2) = (5/12, 7/12)$.

Example 5.1. The parameters of system (1.3) are chosen as follows: $b(1) = 0.3$, $a(1) = 0.5$, $\sigma(1) = 0.5$, $\gamma(1, u) \equiv -0.3$; $b(2) = 0.2$, $a(2) = 0.4$, $\sigma(2) = 0.1$, $\gamma(2, u) \equiv -0.2$. The initial values are $x(0) = 0.6$, $r(0) = 2$ and $\lambda(\mathbb{Y}) = 1$.

By computation, we have $h(1) = -0.06$, $h(2) = -0.02$, so $\pi_1 h(1) + \pi_2 h(2) < 0$. By Theorem 3.5, the species will go to extinction. Figure 1 shows this.

Example 5.2. let $\lambda(\mathbb{Y}) = 1$, the initial data $x(0) = 0.6$, $r(0) = 2$ and the coefficients be $b(1) = 0.8$, $a(1) = 0.5$, $\sigma(1) = 0.5$, $\gamma(1, u) \equiv -0.3$; $b(2) = 0.5$, $a(2) = 0.4$, $\sigma(2) = 0.1$, $\gamma(2, u) \equiv -0.2$. By simple calculation, we get $\bar{h}(1) = 0.56$, $\bar{h}(2) = 0.44$, so $\pi_1 \bar{h}(1) + \pi_2 \bar{h}(2) > 0$. By Theorem 4.4, the species will be stochastic permanence. Figure 2 shows this.

Concluding remarks. This article concerns the stochastic logistic models under Markovian switching driven by Lévy noise. We establish sufficient conditions for stochastic permanence, extinction, non-persistence in the mean and weak persistence. Our key contributions are as follows.

(A) The model is new. By now, as our knowledge is concerned, the extinction and permanence of the model with three noise at the same time has not been reported.

(B) The critical value among the extinction, non-persistence in the mean and weak persistence is obtained.

(C) Our results show that the asymptotic properties of the model have close relations with the Lévy noise and stationary distribution of the color noise.

(D) From our results we can see that the Markovian switching plays important roles in the model, it can switch the overall property of the system.

Some interesting topics deserve further consideration. One may investigate some realistic but complex systems, for example, some n -species models or the general regime whose generator depend on $x(t)$, see [32, 31].

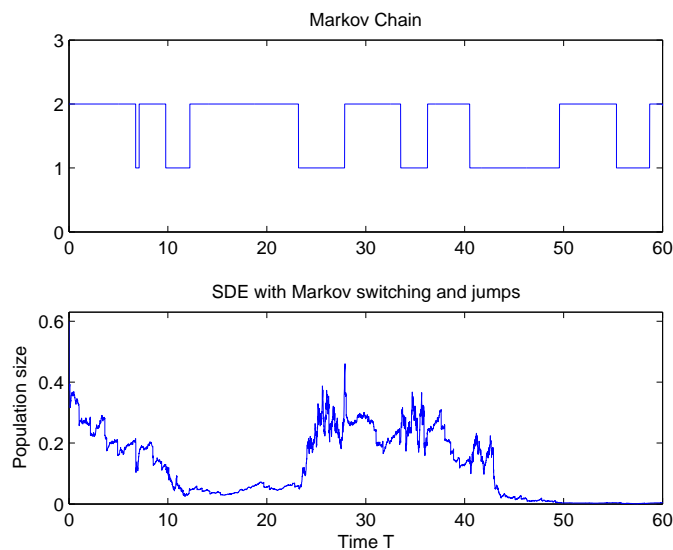


FIGURE 1. For Example 5.1, the first figure shows the numerical simulation of the Markov chain, while the second figure shows the numerical simulation of system (1.3). We can see that the species of (1.3) will go to extinction.

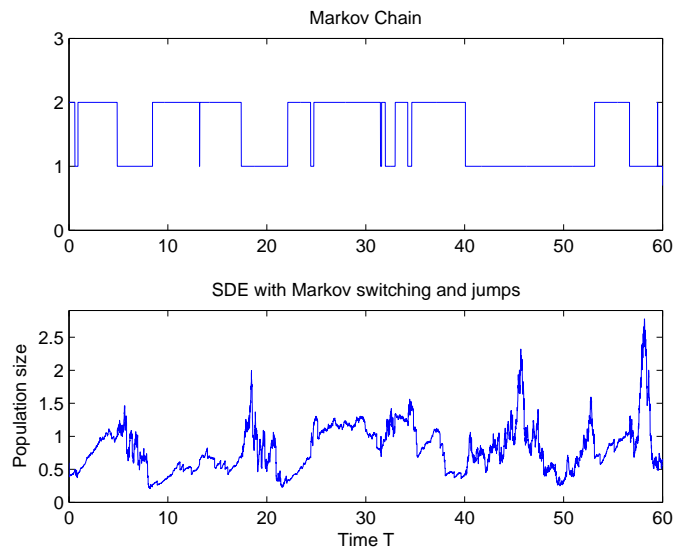


FIGURE 2. For Example 5.2, the first figure shows the numerical simulation of the Markov chain, while the second figure shows the solution of system (1.3). We can see that the species of (1.3) will be stochastic permanence.

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REFERENCES

- [1] W. Anderson; *Continuous-Time Markov Chains*, Springer, 1991.
- [2] D. Applebaum; *Lévy Process and Stochastic Calculus*, 2nd ed., Cambridge University Press, 2009.
- [3] J. Bao, X. Mao, G. Yin, C. Yuan; *Competitive Lotka-Volterra population dynamics with jumps*, *Nonlinear Anal.* 74 (2011) 6601-6616.
- [4] J. Bao, C. Yuan; *Stochastic population dynamics driven by Lévy noise*, *J. Math. Anal. Appl.* 391 (2012) 363-375.
- [5] N. Du, R. Kon, K. Sato, Y. Takeuchi; *Dynamical behaviour of Lotka-Volterra competition systems: Nonautonomous bistable case and the effect of telegraph noise*, *Journal of Computational and Applied Mathematics* 170 (2004) 399-422.
- [6] H. Freedman, S. Ruan; *Uniform persistence in functional differential equations*, *Journal of Differential Equations* 115 (1995) 173-192.
- [7] T. Gard; *Persistence in stochastic food web models*, *Bull. Math. Biol.* 46 (1984) 357-370.
- [8] T. Gard; *Stability for multispecies population models in random environments*, *Nonlinear Anal.* 10 (1986) 1411-1419.
- [9] D. Jiang, N. Shi; *A note on nonautonomous logistic equation with random perturbation*, *J. Math. Anal. Appl.* 303 (2005) 164-172.
- [10] D. Jiang, N. Shi, X. Li; *Global stability and stochastic permanence of a non-autonomous logistic equation with random perturbation*, *J. Math. Anal. Appl.* 340 (2008) 588-597.
- [11] H. Kunita; *Itô stochastic calculus: Its surprising power for applications*, *Stochastic Process. Appl.* 120 (2010) 622-652.
- [12] X. Li, A. Gray, D. Jiang, X. Mao; *Sufficient and necessary conditions of stochastic permanence and extinction for stochastic logistic populations under regime switching*, *Journal of Mathematical Analysis and Applications* 376 (2011) 11-28.
- [13] X. Li, D. Jiang, X. Mao; *Population dynamical behavior of Lotka-Volterra system under regime switching*, *Journal of Computational and Applied Mathematics* 232 (2009) 427-448.
- [14] R. Lipster; *A strong law of large numbers for local martingales*, *Stochastics* 3 (1980) 217-228.
- [15] B. Lisená; *Global attractivity in nonautonomous logistic equations with delay*, *Nonlinear Anal. Real World Appl.* 9 (2008) 53-63.
- [16] M. Liu, K. Wang; *Persistence and extinction of a non-autonomous logistic equation with random perturbation*, *Electron. J. Differential Equations* 9 (2013) 1-13.
- [17] M. Liu, K. Wang; *Persistence and extinction in stochastic non-autonomous logistic systems*, *J. Math. Anal. Appl.* 375 (2011) 443-457.
- [18] M. Liu, K. Wang; *On a stochastic logistic equation with impulse perturbations*, *Comput. Math. Appl.* 63 (2012) 871-886.
- [19] M. Liu, K. Wang; *Dynamics and simulations of a logistic model with impulsive perturbations in a random environment*, *Math. Comput. Simulation* 92 (2013) 53-75.
- [20] M. Liu, K. Wang; *Asymptotic properties and simulations of a stochastic logistic model under regime switching*, *Math. Comput. Modelling* 54 (2011) 2139-2154.
- [21] M. Liu, K. Wang; *Asymptotic properties and simulations of a stochastic logistic model under regime switching II*, *Math. Comput. Modelling* 55 (2012) 405-418.
- [22] M. Liu, K. Wang; *Dynamics of a Leslie-Gower Holling-type II, predator-prey system with Lévy jumps*, *Nonlinear Anal.* 85 (2013) 204-213.
- [23] M. Liu, K. Wang; *Stochastic Lotka-Volterra systems with Lévy noise*, *J. Math. Anal. Appl.* 410 (2014) 750-763.
- [24] Q. Luo, X. Mao; *Stochastic population dynamics under regime Switching*, *Journal of Mathematical Analysis and Applications* 334 (2007) 69-84.

- [25] X. Mao, G. Marion, E. Renshaw; *Environmental Brownian noise suppresses explosions in population dynamics*, Stoch. Process. Their Appl. 97 (2002) 95-110.
- [26] X. Mao, G. Yin, C. Yuan; *Stabilization and destabilization of hybrid systems of stochastic differential equations*, Automatica 43 (2007) 264-273.
- [27] X. Mao, C. Yuan; *Stochastic Differential Equations with Markovian Switching*, Imperial College Press, London 2006.
- [28] R. Situ; *Theory of Stochastic Differential Equation with Jumps and Applications*, Springer-Verlag, New York, 2012.
- [29] M. Slatkin; *The dynamics of a population in a Markovian environment*, Ecology 59 (1978) 249-256.
- [30] Y. Takeuchi, N. Du, N. Hieu, K. Sato; *Evolution of predator-prey systems described by a Lotka-Volterra equation under random environment*, Journal of Mathematical Analysis and applications 323 (2006) 938-957.
- [31] Z. Yang, G. Yin; *Stability of nonlinear regime-switching jump diffusions*, Nonlinear Anal. 75 (2012) 3854-3873.
- [32] G. Yin, F. Xi; *Stability of regime-switching jump diffusions*, SIAM J. Control optim. 48 (2010) 4525-4549.
- [33] C. Zhu, G. Yin; *On competitive Lotka-Volterra model in random environments*, J. Math. Anal. Appl. 357 (2009) 154-170.
- [34] X. Zou, K. Wang; *Numerical simulations and modeling for stochastic biological systems with jumps*, Commun. Nonlinear Sci. Numer. Simul. 19 (2014) 1557-1568.

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