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ENTIRE SOLUTIONS FOR NONLINEAR DIFFERENTIAL-DIFFERENCE EQUATIONS

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Abstract. In this article, we study entire solutions of the nonlinear differential-difference equation

$$q(z)f^n(z) + a(z)f^{(k)}(z+1) = p_1(z)e^{q_1(z)} + p_2(z)e^{q_2(z)}$$

where $p_1(z)$, $p_2(z)$ are nonzero polynomials, $q_1(z)$, $q_2(z)$ are nonconstant polynomials, $q(z)$, $a(z)$ are nonzero entire functions of finite order, $n \geq 2$ is an integer. We obtain additional results for case: $n = 3$, $q_1(z) = -q_2(z)$, and $p_1(z)$, $p_2(z)$ nonzero constants.

1. Introduction and main results

In this article, we assume that the reader is familiar with standard symbols and fundamental results of Nevanlinna Theory. We denote by $S(r, f)$ any quantity satisfying $S(r, f) = o(T(r, f))$, as $r \to \infty$, possibly outside of a set E with finite linear measure. We use $\lambda(\frac{1}{f})$ and $\lambda(f)$ to denote the exponents of convergence of poles and zeros of $f(z)$ respectively, $\sigma(f)$ to denote the order of $f(z)$. The hyper-order of $f(z)$ is defined as

$$\sigma_2(f) = \limsup_{r\to\infty} \frac{\log\log T(r, f)}{\log r},$$

the lower hyper-order of $f(z)$ is defined as

$$\mu_2(f) = \liminf_{r\to\infty} \frac{\log\log T(r, f)}{\log r},$$

the hyper exponent of convergence of zeros of $f(z)$ is defined by

$$\lambda_2(f) = \limsup_{r\to\infty} \frac{\log\log N(r, \frac{1}{f})}{\log r},$$

and the deficiency of a with respect to $f(z)$ is defined by

$$\delta(a, f) = 1 - \limsup_{r\to\infty} \frac{N(r, \frac{1}{f-a})}{T(r, f)}.$$

A differential polynomial of $f(z)$ means that it is a polynomial in $f(z)$ and its derivatives with small functions of $f(z)$ as coefficients. A differential-difference

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polynomial of $f(z)$ means that it is a polynomial in $f(z)$, its derivatives and its shifts $f(z+c)$ with small functions of $f(z)$ as coefficients. We shall use $P_d(f)$ to denote a differential polynomial or a differential-difference polynomial of $f(z)$ with degree d.

In previous two decades, the existence and growth of meromorphic solutions of difference equations have been investigated in many papers [1-7, 9-12, 15]. Recently, there has been a renewed interest in studying meromorphic solutions of differential-difference equations, see [13, 14, 17]. For instance, many authors have considered the equation $f^n(z) + P_d(f) = p_1(z)e^{q_1(z)} + p_2(z)e^{q_2(z)}$. when $P_d(f)$ is a differential polynomial, Li and Yang [11, 15] investigated the properties of solutions of the above equation. When $P_d(f)$ is a differential-difference polynomial, Zhang and Liao [17] proved that if the above equation satisfies some conditions, it doesn't have any transcendental entire solution of finite order.

Theorem 1.1 ([17, Theorem 3]). *Let $n \geq 4$ be an integer and $P_d(f)$ denote an algebraic differential-difference polynomial in $f(z)$ of degree $d \leq n-3$. If $p_1(z)$, $p_2(z)$ are nonzero polynomials, α_1, α_2 are nonzero constants with $\frac{\alpha_1}{\alpha_2} \neq (\frac{d}{n})^{\pm 1}, 1$. Then the equation*

$$f^n(z) + P_d(f) = p_1(z)e^{\alpha_1 z} + p_2(z)e^{\alpha_2 z},$$

does not have any transcendental entire solution of finite order.

Peng and Chen [13] considered the special case for difference equations and obtained some results.

Theorem 1.2 ([13, Theorem 2.1]). *Consider the nonlinear difference equation*

$$f^n(z) + a(z)f(z+1) = c \sin bz,$$

where $a(z)$ is a nonconstant polynomial, b, c are nonzero constants and $n \geq 2$ is an integer. Suppose that an entire function $f(z)$ satisfies any one of the following three conditions:

(1) $\lambda(f) < \sigma(f) = \infty$;
(2) $\lambda_2(f) < \sigma_2(f)$;
(3) $\mu_2(f) < 1$.

Then $f(z)$ can not be an entire solution of this equation.

In this paper, we consider a general differential-difference equation and obtain the following theorem.

Theorem 1.3. *Consider the nonlinear differential-difference equation*

$$q(z)f^n(z) + a(z)f^{(k)}(z+1) = p_1(z)e^{q_1(z)} + p_2(z)e^{q_2(z)}, \tag{1.1}$$

where $p_1(z)$, $p_2(z)$ are two nonzero polynomials, $q(z)$, $a(z)$ are two nonzero entire functions of finite order, $q_1(z)$, $q_2(z)$ are two nonconstant polynomials, $n \geq 2$ is an integer. Suppose that an entire function $f(z)$ satisfies any one of the following two conditions:

(1) $\lambda(f) < \sigma(f) = \infty$, $\sigma_2(f) < \infty$;
(2) $\lambda_2(f) < \sigma_2(f) < \infty$.

Then $f(z)$ can not be an entire solution of (1.1).

Zhang and Liao [17] also considered the existence of transcendental entire solutions of finite order to

$$f^3(z) + a(z)f(z+1) = p_1 e^{\lambda z} + p_2 e^{-\lambda z}$$

and obtained the following theorem.

Theorem 1.4 ([17, Theorem 4]). *Let p_1, p_2 and λ be nonzero constants, for the difference equation*

$$f^3(z) + a(z)f(z+1) = p_1 e^{\lambda z} + p_2 e^{-\lambda z},$$

where $a(z)$ is a polynomial, we have

(1) *if $a(z)$ is not a constant, then the equation does not have any transcendental entire solution of finite order;*
(2) *if $a(z)$ is a nonzero constant, then the equation admits transcendental entire solutions of finite order if and only if the condition $e^{\lambda/3} = \mp 1$ and $p_1 p_2 = \pm(a/3)^3$ holds. Furthermore if the condition above holds, then the transcendental entire solution of finite order of the equation has the form*

$$f(z) = \sigma_j e^{2k\pi iz} - \frac{a}{3\sigma_j} e^{-2k\pi iz}$$

or

$$f(z) = \sigma_j e^{2k\pi iz + \pi iz} + \frac{a}{3\sigma_j} e^{-(2k\pi iz + \pi iz)}.$$

In this article, we consider the more general case for differential-difference equations and obtain the following theorem.

Theorem 1.5. *Let p_1, p_2 and λ be nonzero constants, $a(z)$ be an entire function with zero order, $q(z)$ be a nonconstant polynomial. Then any transcendental entire solution $f(z)$ of finite order of the equation*

$$f^3(z) + a(z)f^{(k)}(z+1) = p_1 e^{\lambda q(z)} + p_2 e^{-\lambda q(z)}, \tag{1.2}$$

satisfies $\delta(0, f) = 0$.

For the special case of $q(z) \equiv z$, we have the following result.

Theorem 1.6. *Consider the differential-difference equation*

$$f^3(z) + a(z)f^{(k)}(z+1) = p_1 e^{\lambda z} + p_2 e^{-\lambda z}, \tag{1.3}$$

where p_1, p_2 and λ are nonzero constants, $a(z)$ is an entire function with zero order. We have

(1) *if $a(z)$ is not a constant, then the equation does not have any transcendental entire solution of finite order;*
(2) *if $a(z)$ is a nonzero constant, k is an even number, then the equation admits transcendental entire solutions of finite order if and only if the condition $e^{\lambda/3} = \mp 1$ and $p_1 p_2 = \pm(a/3)^3$ holds. Furthermore if the condition above holds, then the transcendental entire solution of finite order of the equation has the form*

$$f(z) = \sigma_j e^{2k\pi iz} - \frac{a}{3\sigma_j} e^{-2k\pi iz}$$

or

$$f(z) = \sigma_j e^{2k\pi iz + \pi iz} + \frac{a}{3\sigma_j} e^{-(2k\pi iz + \pi iz)};$$

(3) *if $a(z)$ is a nonzero constant, k is an odd number, then the equation admits transcendental entire solutions of finite order if and only if the condition $e^{\frac{1}{3}\lambda} = \mp i$ and $p_1p_2 = \pm(\frac{ai}{3})^3$ holds.*

Furthermore if the condition above holds, then the transcendental entire solution of finite order of the equation has the form

$$f(z) = \sigma_j e^{2k\pi iz+\frac{\pi}{2}iz} - \frac{ai}{3\sigma_j}e^{-(2k\pi iz+\frac{\pi}{2}iz)}$$

or

$$f(z) = \sigma_j e^{2k\pi iz-\frac{\pi}{2}iz} + \frac{ai}{3\sigma_j}e^{-(2k\pi iz-\frac{\pi}{2}iz)}.$$

2. Lemmas

To prove our results, we need some lemmas.

Lemma 2.1 ([16]). *Suppose that $f_1(z), f_2(z), \dots, f_n(z), (n \geq 2)$ are meromorphic functions and $g_1(z), g_2(z), \dots, g_n(z)$ are entire functions satisfying the following conditions:*

(1) $\sum_{j=1}^{n} f_j(z)e^{g_j(z)} \equiv 0$;
(2) *$g_j(z) - g_k(z)$ are not constants for $1 \leq j < k \leq n$;*
(3) *For $1 \leq j \leq n, 1 \leq h < k \leq n$, $T(r, f_j) = o(T(r, e^{g_h - g_k}))$ $(r \to \infty, r \notin E)$.*

Then $f_j(z) \equiv 0 (j = 1, 2, \dots, n)$.

Lemma 2.2 ([3]). *Let $f(z)$ be a transcendental entire function of infinite order and $\sigma_2(f) = \alpha < \infty$. Then $f(z)$ can be represented as*

$$f(z) = Q(z)e^{g(z)},$$

where Q and g are entire functions such that

$$\lambda(Q) = \sigma(Q) = \lambda(f), \lambda_2(Q) = \sigma_2(Q) = \lambda_2(f),$$
$$\sigma_2(f) = \max\{\sigma_2(Q), \sigma_2(e^g)\}.$$

Lemma 2.3 ([9]). *Let $f(z)$ be a transcendental meromorphic solution of finite order σ of a difference equation of the form*

$$U(z, f)P(z, f) = Q(z, f),$$

where $U(z, f), P(z, f), Q(z, f)$ are difference polynomials such that the total degree of $U(z, f)$ in $f(z)$ and its shifts is n, and that the total degree of $Q(z, f)$ is at most n. If $U(z, f)$ just contains one term of maximal total degree, then for any $\varepsilon > 0$,

$$m(r, P(z, f)) = O(r^{\sigma-1+\varepsilon}) + S(r, f)$$

holds possibly outside of an exceptional set of finite logarithmic measure.

Lemma 2.4 ([15]). *Suppose that c is a nonzero constant and α is a nonconstant meromorphic function. Then the equation*

$$f^2(z) + (cf^{(n)}(z))^2 = \alpha$$

has no transcendental meromorphic solution $f(z)$ satisfying $T(r, \alpha) = S(r, f)$.

3. Proofs main results

Proof of Theorem 1.3. (1) Let f be an entire solution of equation (1.1) and satisfy $\lambda(f) < \sigma(f) = \infty, \sigma_2(f) < \infty$. By Lemma 2.2, $f(z)$ can be rewritten as $f(z) = Q(z)e^{g(z)}$, where Q is an entire function, g is a transcendental entire function such that $\lambda(Q) = \sigma(Q) = \lambda(f)$, $\lambda_2(Q) = \sigma_2(Q) = \lambda_2(f)$, $\sigma_2(f) = \max\{\sigma_2(Q), \sigma_2(e^g)\}$.

From condition $\sigma_2(f) < \infty$, so $\sigma(g) = \sigma_2(e^g) < \infty$. Substituting $f(z) = Q(z)e^{g(z)}$ into (1.1) we obtain that

$$q(z)Q^n(z)e^{ng(z)} + a(z)H(z)e^{g(z+1)} = p_1(z)e^{q_1(z)} + p_2(z)e^{q_2(z)}, \tag{3.1}$$

where $H(z)$ is a differential polynomial in $Q(z+1)$ and $g(z+1)$, $\sigma(H) < \infty$. Set $G(z) = g(z+1) - ng(z)$, then (3.1) becomes

$$q(z)Q^n(z) + a(z)H(z)e^{G(z)} = e^{-ng(z)}\Big(p_1(z)e^{q_1(z)} + p_2(z)e^{q_2(z)}\Big). \tag{3.2}$$

If $G(z)$ is a polynomial, then

$$\sigma\Big(q(z)Q^n(z) + a(z)H(z)e^{G(z)}\Big) < \infty,$$

but

$$\sigma\Big(e^{-ng(z)}\Big(p_1(z)e^{q_1(z)} + p_2(z)e^{q_2(z)}\Big)\Big) = \infty.$$

Then by (3.2), we obtain a contradiction.

If $G(z)$ is a transcendental entire function, then (3.1) can be rewritten as

$$q(z)Q^n(z)e^{ng(z)} + a(z)H(z)e^{g(z+1)} - e^{h(z)}\Big(p_1(z)e^{q_1(z)} + p_2(z)e^{q_2(z)}\Big) = 0, \tag{3.3}$$

where $h(z) \equiv 0$. By Lemma 2.1, we deduce

$$q(z)Q^n(z) \equiv 0, a(z)H(z) \equiv 0, -p_1(z)e^{q_1(z)} - p_2(z)e^{q_2(z)} \equiv 0,$$

for $Q^n(z) \equiv 0$, so $f(z) \equiv 0$, but $\sigma(f) = \infty$, this is a contradiction.

(2) Suppose that f is an entire solution of equation (1.1) and satisfies $\lambda_2(f) < \sigma_2(f) < \infty$. By Lemma 2.2, $f(z)$ can be rewritten as $f(z) = Q(z)e^{g(z)}$, where Q is an entire function, g is a transcendental entire function such that

$$\lambda(Q) = \sigma(Q) = \lambda(f), \lambda_2(Q) = \sigma_2(Q) = \lambda_2(f), \sigma_2(f) = \max\{\sigma_2(Q), \sigma_2(e^g)\}.$$

From condition, we obtain $\sigma_2(f) = \sigma_2(e^g) < \infty$, so $\sigma_2(Q) < \sigma_2(e^g) = \sigma(g) < \infty$. Substituting $f(z) = Q(z)e^{g(z)}$ into (1.1), we obtain (3.2). Since $\sigma(q(z)) < \infty$, so $\sigma_2(q(z)) = 0$.

If $\sigma(G) < \sigma(g)$, then

$$\begin{aligned}\sigma_2\Big(q(z)Q^n(z) + a(z)H(z)e^{G(z)}\Big) &\le \max\{\sigma_2(Q), \sigma(G)\} < \sigma(g) \\ &= \sigma_2\Big(e^{-ng(z)}\Big(p_1(z)e^{q_1(z)} + p_2(z)e^{q_2(z)}\Big)\Big),\end{aligned}$$

which is a contradiction.

If $\sigma(G) = \sigma(g)$, then we can get (3.3). Using the same method as in the proof of (1), by Lemma 2.1, we also get a contradiction. □

Proof of Theorem 1.5. Let $f(z)$ be a transcendental entire solution of finite order of (1.2) with $\delta(0, f) > 0$. By differentiating both sides of (1.2), we obtain

$$3f^2(z)f'(z) + a'(z)f^{(k)}(z+1) + a(z)f^{(k+1)}(z+1) = \lambda q'(z)\Big(p_1e^{\lambda q(z)} - p_2e^{-\lambda q(z)}\Big). \tag{3.4}$$

By taking both squares of (1.2) and (3.4), and eliminating $e^{\pm\lambda q(z)}$, we deduce

$$
\begin{aligned}
&(\lambda q'(z))^2\Big(f^3(z)+a(z)f^{(k)}(z+1)\Big)^2\\
&-\Big(3f^2(z)f'(z)+a'(z)f^{(k)}(z+1)+a(z)f^{(k+1)}(z+1)\Big)^2\\
&=4p_1p_2\lambda^2(q'(z))^2,
\end{aligned}
\tag{3.5}
$$

Set $\alpha(z)=\lambda^2(q'(z))^2f^2(z)-9(f'(z))^2$, thus $\alpha(z)$ is an entire function. Then we rewrite (3.5) in the form $f^4\alpha=Q(f)$, where $Q(f)$ is a differential-difference polynomial in $f(z)$ with total degree 4. By Lemma 2.3, we obtain

$$T(r,\alpha)=m(r,\alpha)=O(r^{\sigma-1+\varepsilon})+S(r,f).$$

Thus α is a small function of $f(z)$. Next, we consider two cases.

Case 1. $\alpha\equiv 0$. Then $f(z)=ce^{\pm\frac{1}{3}\lambda q(z)}$. By substituting this into (1.2), we obtain

$$(c^3-p_1)e^{\lambda q(z)}+\frac{1}{3}\lambda a(z)q'(z+1)e^{\frac{1}{3}\lambda q(z+1)}-p_2e^{-\lambda q(z)}=0,$$

or

$$(c^3-p_2)e^{-\lambda q(z)}-\frac{1}{3}\lambda a(z)q'(z+1)e^{-\frac{1}{3}\lambda q(z+1)}-p_1e^{\lambda q(z)}=0.$$

Since $q(z)$ is a nonconstant polynomial, by Lemma 2.1, we obtain $p_1=0$ or $p_2=0$. This is a contradiction.

Case 2. $\alpha\not\equiv 0$. We rewrite α as

$$\alpha=f^2A(z),$$

where $A(z)=\lambda^2q'-9(\frac{f'}{f})^2$, by the Lemma of Logarithmic Derivative of meromorphic function, then $m(r,A)=S(r,f)$. Since $\alpha\not\equiv 0$, then $A\not\equiv 0$. For any Small $\varepsilon>0$, we have

$$
\begin{aligned}
O(1)+2T(r,f)=T(r,f^2)=m(r,f^2)&=m(r,\frac{\alpha}{A})\\
&\le m(r,\alpha)+m(r,\frac{1}{A})\\
&\le S(r,f)+T(r,A)\\
&\le S(r,f)+N(r,A)\\
&\le S(r,f)+2N(r,\frac{1}{f})\\
&\le 2(1-\delta(0,f)+\varepsilon)T(r,f).
\end{aligned}
$$

This is impossible for $0<\varepsilon<\delta(0,f)$. The proof of Theorem 1.5 is complete. □

Proof of Theorem 1.6. Suppose that $f(z)$ is a transcendental entire solution of (1.3) with finite order. By differentiating both sides of (1.3), we obtain

$$3f^2(z)f'(z)+a'(z)f^{(k)}(z+1)+a(z)f^{(k+1)}(z+1)=\lambda p_1e^{\lambda z}-\lambda p_2e^{-\lambda z}. \tag{3.6}$$

By taking both squares of (1.3) and (3.6), and eliminating $e^{\pm\lambda z}$, we deduce

$$
\begin{aligned}
4\lambda^2p_1p_2=\lambda^2&\Big(f^3(z)+a(z)f^{(k)}(z+1)\Big)^2\\
&-\Big(3f^2(z)f'(z)+a'(z)f^{(k)}(z+1)+a(z)f^{(k+1)}(z+1)\Big)^2,
\end{aligned}
$$

set $\alpha(z)=\lambda^2 f^2(z)-9(f'(z))^2$, thus $\alpha(z)$ is an entire function. Then we rewrite(3.6) in the form $f^4\alpha=Q(f)$, where $Q(f)$ is a differential-difference polynomial in $f(z)$ with total degree 4. By Lemma 2.3, we obtain

$$T(r,\alpha)=m(r,\alpha)=O(r^{\sigma-1+\varepsilon})+S(r,f).$$

Thus α is a small function of $f(z)$. Next, we consider two cases.

Case 1. $\alpha\equiv 0$. Then $f(z)=ce^{\pm\frac{1}{3}\lambda z}$. By substituting this into (1.3), we obtain

$$(c^3-p_1)e^{\lambda z}+(\frac{1}{3}\lambda)^k a(z)e^{\frac{1}{3}\lambda(z+1)}-p_2e^{-\lambda z}=0,$$

or

$$(c^3-p_2)e^{-\lambda z}+(-\frac{1}{3}\lambda)^k a(z)e^{-\frac{1}{3}\lambda(z+1)}-p_1e^{\lambda z}=0.$$

By Lemma 2.1, we obtain $p_1=0$ or $p_2=0$. This is a contradiction.

Case 2. $\alpha\not\equiv 0$. By Lemma 2.4, we obtain α is a nonzero constant. Thus

$$\alpha'=2f'(\lambda^2 f-9f^{''})=0.$$

Since $f(z)$ is transcendental, then

$$\lambda^2 f-9f^{''}=0.$$

By a simple calculation,

$$f(z)=c_1e^{\frac{1}{3}\lambda z}+c_2e^{-\frac{1}{3}\lambda z},$$

where c_1,c_2 are nonzero constants. By substituting this into (1.3) and simple calculation, get

$$\begin{aligned}&(c_1^3-p_1)e^{\lambda z}+(c_2^3-p_2)e^{-\lambda z}+\Big(3c_1^2c_2+c_1a(z)(\frac{1}{3}\lambda)^k e^{\frac{1}{3}\lambda}\Big)e^{\frac{1}{3}\lambda z}\\&+\Big(3c_1c_2^2+c_2a(z)(-\frac{1}{3}\lambda)^k e^{-\frac{1}{3}\lambda}\Big)e^{-\frac{1}{3}\lambda z}=0,\end{aligned}$$

by Lemma 2.1, we deduce

$$c_1^3=p_1,c_2^3=p_2,3c_1c_2+a(z)(\frac{1}{3}\lambda)^k e^{\frac{1}{3}\lambda}\equiv 0,3c_1c_2+a(z)(-\frac{1}{3}\lambda)^k e^{-\frac{1}{3}\lambda}\equiv 0.$$

If $a(z)$ is not a nonzero constant, we can get a contradiction. Then equation (1.3) does not admit any transcendental entire solution of finite order.

If $a(z)$ is a nonzero constant, k is an even number, then

$$a(\frac{1}{3})^k\lambda^k\Big(e^{\frac{1}{3}\lambda}-e^{-\frac{1}{3}\lambda}\Big)=0,$$

so

$$e^{\frac{1}{3}\lambda}=\mp 1,p_1p_2=\pm(\frac{a}{3})^3,c_1c_2=\pm\frac{a}{3}.$$

Thus c_1 can assume $\sigma_j(j=1,2,3)$, where σ_j satisfies $\sigma_j^3=p_1(j=1,2,3)$ and $c_2=\pm\frac{a}{3c_1}$. Hence $f(z)$ is of the following forms $f(z)=\sigma_je^{2k\pi iz}-\frac{a}{3\sigma_j}e^{-2k\pi iz}$ or $f(z)=\sigma_je^{2k\pi iz+\pi iz}+\frac{a}{3\sigma_j}e^{-(2k\pi iz+\pi iz)}$.

If $a(z)$ is a nonzero constant, k is an odd number, then

$$a(\frac{1}{3})^k\lambda^k\Big(e^{\frac{1}{3}\lambda}+e^{-\frac{1}{3}\lambda}\Big)=0,$$

so

$$e^{\frac{1}{3}\lambda}=\mp i,p_1p_2=\pm(\frac{ai}{3})^3,c_1c_2=\pm\frac{ai}{3}.$$

Thus c_1 can assume σ_j $(j = 1, 2, 3)$, where σ_j satisfies $\sigma_j^3 = p_1(j = 1, 2, 3)$ and $c_2 = \pm\frac{ai}{3c_1}$. Hence $f(z)$ is of the following forms $f(z) = \sigma_j e^{2k\pi iz+\frac{\pi}{2}iz} - \frac{ai}{3\sigma_j}e^{-(2k\pi iz+\frac{\pi}{2}iz)}$ or $f(z) = \sigma_j e^{2k\pi iz-\frac{\pi}{2}iz} + \frac{ai}{3\sigma_j}e^{-(2k\pi iz-\frac{\pi}{2}iz)}$. Therefore, the proof of Theorem 1.6 is complete. $\square$

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