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EXISTENCE OF POSITIVE ENTIRE RADIAL SOLUTIONS TO A (k_1, k_2)-HESSIAN SYSTEMS WITH CONVECTION TERMS

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Abstract. In this article, we prove two new results on the existence of positive entire large and bounded radial solutions for nonlinear system with gradient terms

$$\begin{aligned} S_{k_1}(\lambda(D^2u_1)) + b_1(|x|)|\nabla u_1|^{k_1} &= p_1(|x|)f_1(u_1, u_2) \quad \text{for } x \in \mathbb{R}^N, \\ S_{k_2}(\lambda(D^2u_2)) + b_2(|x|)|\nabla u_2|^{k_2} &= p_2(|x|)f_2(u_1, u_2) \quad \text{for } x \in \mathbb{R}^N, \end{aligned}$$

where $S_{k_i}(\lambda(D^2u_i))$ is the k_i-Hessian operator, b_1, p_1, f_1, b_2, p_2 and f_2 are continuous functions satisfying certain properties. Our results expand those by Zhang and Zhou [23]. The main difficulty in dealing with our system is the presence of the convection term.

1. Introduction

The purpose of this article is to present new results concerning the nonlinear Hessian system with convection terms

$$\begin{aligned} S_{k_1}(\lambda(D^2u_1)) + b_1(|x|)|\nabla u_1|^{k_1} &= p_1(|x|)f_1(u_1, u_2), x \in \mathbb{R}^N, \\ S_{k_2}(\lambda(D^2u_2)) + b_2(|x|)|\nabla u_2|^{k_2} &= p_2(|x|)f_2(u_1, u_2), x \in \mathbb{R}^N, \end{aligned} \tag{1.1}$$

where $N \geq 3$, b_1, p_1, f_1, b_2, p_2, f_2 are continuous functions satisfying certain properties, $k_1, k_2 \in \{1, 2, \dots, N\}$ and

$$S_{k_i}(\lambda) = \sum_{1 \leq i_1 < \dots < i_{k_i} \leq N} \lambda_{i_1} \dots \lambda_{i_{k_i}}, \quad \lambda = (\lambda_1, \dots, \lambda_N) \in \mathbb{R}^N,\ i = 1, 2,$$

denotes the k_i-th elementary symmetric function. In the literature $S_{k_i}(\lambda(D^2u_i))$ it is called the k_i-Hessian operator. For instance, the following well known operators are included in this class: $S_1(\lambda(D^2u_i)) = \sum_{i=1}^N \lambda_i = \Delta u_i$ the Laplacian operator and $S_N(\lambda(D^2u_i)) = \prod_{i=1}^N \lambda_i = \det(D^2u_i)$ the Monge-Ampère.

In recent years equations of the type (1.1) have been the subject of rather deep investigations since appears from many branches of mathematics and applied mathematics. For more surveys on these questions we refer the paper to: Alves and Holanda [1], Bao-Ji and Li [3], Bandle and Giarrusso [2], Clément-Manásevich and Mitidieri [4], De Figueiredo and Jianfu [6], Galaktionov and Vázquez [7], Jiang and Lv [9], Salani [20], Ji and Bao [12], Jian [11], Peterson and Wood [16], Pripoae

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[17], Quittner [18], Li and Yang [13], Li-Zhang and Zhang [14], Viaclovsky [21, 22], Zhang and Zhou [23] and not the last Zhang [24].

The motivation for studying (1.1) comes from the work of Jiang and Lv [9] where they study the system

$$\begin{gathered}\Delta u_1 + |\nabla u_1| = p_1(|x|) f_1(u_1, u_2) \quad \text{for } x \in \mathbb{R}^N \ (N \geq 3),\\ \Delta u_2 + |\nabla u_2| = p_2(|x|) f_2(u_1, u_2) \quad \text{for } x \in \mathbb{R}^N \ (N \geq 3),\end{gathered}$$

and from the recently work of Zhang and Zhou [23] where the authors considered the system

$$\begin{gathered}S_k(\lambda(D^2 u_1)) = p_1(|x|) f_1(u_2) \quad \text{for } x \in \mathbb{R}^N \ (N \geq 3),\\ S_k(\lambda(D^2 u_2)) = p_2(|x|) f_2(u_1) \quad \text{for } x \in \mathbb{R}^N \ (N \geq 3).\end{gathered}$$

Our purpose is to expand and improve the results in [23] for the more general system (1.1). By analogy with the work of Zhang and Zhou [23] we introduce the following notations

$$\begin{gathered}C_0 = (N-1)!/[k_1!(N-k_1)!], C_{00} = (N-1)!/[k_2!(N-k_2)!],\\ B_1^-(\xi) = \frac{\xi^{k_1-N}}{C_0} \exp\Big(-\int_0^\xi \frac{1}{C_0} t^{k_1-1} b_1(t)dt\Big),\\ B_1^+(\xi) = \xi^{N-1} \exp\Big(\int_0^\xi \frac{1}{C_0} t^{k_1-1} b_1(t)dt\Big) p_1(\xi),\\ B_2^-(\xi) = \frac{\xi^{k_2-N}}{C_{00}} \exp\Big(-\int_0^\xi \frac{1}{C_{00}} t^{k_2-1} b_2(t)dt\Big),\\ B_2^+(\xi) = \xi^{N-1} \exp\Big(\int_0^\xi \frac{1}{C_{00}} t^{k_2-1} b_2(t)dt\Big) p_2(\xi),\\ P_1(r) = \int_0^r \Big(B_1^-(r) \int_0^r B_1^+(t)dt\Big)^{1/k_1} dr,\\ P_2(r) = \int_0^r \Big(B_2^-(r) \int_0^r B_2^+(t)dt\Big)^{1/k_2} dr,\\ F_{1,2}(r) = \int_{a_1+a_2}^r \frac{1}{f_1^{1/k_1}(t,t) + f_2^{1/k_2}(t,t)} dt\\ \text{for } r \geq a_1 + a_2 > 0,\ a_1 \geq 0,\ a_1 \geq 0,\\ P_1(\infty) = \lim_{r\to\infty} P_1(r), \quad P_2(\infty) = \lim_{r\to\infty} P_2(r), \quad F_{1,2}(\infty) = \lim_{s\to\infty} F_{1,2}(s).\end{gathered}$$

We will always assume that the variable weights functions b_1, b_2, p_1, p_2 and the nonlinearities f_1, f_2 satisfy:

- (A1) $b_1, b_2 : [0,\infty) \to [0,\infty)$ and $p_1, p_2 : [0,\infty) \to [0,\infty)$ are spherically symmetric continuous functions (i.e., $p_i(x) = p_i(|x|)$ and $b_i(x) = b_i(|x|)$ for $i = 1, 2$);
- (A2) $f_1,\ f_2 : [0,\infty) \times [0,\infty) \to [0,\infty)$ are continuous and increasing.

Here is a first result.

Theorem 1.1. *We assume that $F_{1,2}(\infty) = \infty$ and (A1), hold. Furthermore, if f_1 and f_2 satisfy (A2), then system* (1.1) *has at least one positive radial solution $(u_1, u_2) \in C^2([0,\infty)) \times C^2([0,\infty))$ with central value in (a_1, a_2). Moreover, the following hold:*

(1) *If* $P_1(\infty)+P_2(\infty)<\infty$, *then* $\lim_{r\to\infty}u_1(r)<\infty$ *and* $\lim_{r\to\infty}u_2(r)<\infty$.
(2) *If* $P_1(\infty)=\infty$ *and* $P_2(\infty)=\infty$, *then*

$$\lim_{r\to\infty}u_1(r)=\infty \quad \textit{and} \quad \lim_{r\to\infty}u_2(r)=\infty.$$

In the same spirit we also have, our next result.

Theorem 1.2. *Assume that the hypotheses (A1) and (A2) are satisfied. If* $P_1(\infty)+P_2(\infty)<F_{1,2}(\infty)<\infty$, *then system* (1.1) *has one positive bounded radial solution* $(u_1,u_2)\in C^2([0,\infty))\times C^2([0,\infty))$, *with central value in* (a_1,a_2), *such that*

$$\begin{aligned}
a_1+f_1^{1/k_1}(a_1,a_2)P_1(r)\le u_1(r)\le F_{1,2}^{-1}(P_1(r)+P_2(r)),\\
a_2+f_2^{1/k_2}(a_1,a_2)P_2(r)\le u_2(r)\le F_{1,2}^{-1}(P_1(r)+P_2(r)).
\end{aligned}$$

2. Proofs of main results

In this section we give the proof of Theorems 1.1 and 1.2. For the readers' convenience, we recall the radial form of the k-Hessian operator.

Remark 2.1 (see [12, 20])**.** Assume $\varphi\in C^2[0,R)$ is radially symmetric with $\varphi'(0)=0$. Then, for $k\in\{1,2,\dots,N\}$ and $u(x)=\varphi(r)$ where $r=|x|<R$, we have that $u\in C^2(B_R)$, and

$$\begin{gathered}
\lambda(D^2u(r))=\begin{cases}(\varphi''(r),\frac{\varphi'(r)}{r},\dots,\frac{\varphi'(r)}{r}) & \text{for } r\in(0,R),\\ (\varphi''(0),\varphi''(0),\dots,\varphi''(0)) & \text{for } r=0\,;\end{cases}\\
S_k(\lambda(D^2u(r)))=\begin{cases}C_{N-1}^{k-1}\varphi''(r)\big(\frac{\varphi'(r)}{r}\big)^{k-1}+C_{N-1}^{k-1}\frac{N-k}{k}\big(\frac{\varphi'(r)}{r}\big)^{k}, & r\in(0,R),\\ C_N^k(\varphi''(0))^k & \text{for } r=0,\end{cases}
\end{gathered}$$

where the prime denotes differentiation with respect to $r=|x|$ and $C_{N-1}^{k-1}=(N-1)!/[(k-1)!(N-k)!]$.

Proof of the Theorems 1.1 and 1.2. We start by showing that system (1.1) has positive radial solutions. For this purpose, we show that the system of ordinary differential equations

$$\begin{gathered}
\frac{C_{N-1}^{k_1-1}}{r^{N-1}}[\frac{r^{N-k_1}}{k_1}e^{\int_0^r\frac{1}{C_0}t^{k_1-1}b_1(t)dt}(u_1')^{k_1}]'\\
=e^{\int_0^r\frac{1}{C_0}t^{k_1-1}b_1(t)dt}p_1(r)f_1(u_1,u_2),\quad r>0,\\
\frac{C_{N-1}^{k_2-1}}{r^{N-1}}[\frac{r^{N-k_2}}{k_2}e^{\int_0^r\frac{1}{C_{00}}t^{k_2-1}b_2(t)dt}(u_2')^{k_2}]'\\
=e^{\int_0^r\frac{1}{C_{00}}t^{k_2-1}b_2(t)dt}p_2(r)f_2(u_1,u_2), r>0,\\
u_1'(r)\ge 0 \quad\text{and}\quad u_2'(r)\ge 0 \quad \text{for } r\in[0,\infty),\\
u_1(0)=a_1 \quad\text{and}\quad u_2(0)=a_2,
\end{gathered}\tag{2.1}$$

has a solution. Therefore, at least one solution of (2.1) can be obtained using successive approximation by defining the sequences $\{u_1^m\}_{m\ge0}$ and $\{u_2^m\}_{m\ge0}$ on

$[0, \infty)$ in the following way

$$\begin{aligned}
u_1^0 &= a_1, \quad u_2^0 = a_2 \quad \text{for } r \geq 0 \\
u_1^m(s) &= a_1 + \int_0^r \Big[B_1^-(t) \int_0^t B_1^+(s) f_1(u_1^{m-1}(s), u_2^{m-1}(s))ds\Big]^{1/k_1} dt, \\
u_2^m(s) &= a_2 + \int_0^r \Big[B_2^-(t) \int_0^t B_2^+(s) f_2(u_1^{m-1}(s), u_2^{m-1}(s))ds\Big]^{1/k_2} dt.
\end{aligned} \tag{2.2}$$

It is easy to see that $\{u_1^m\}_{m\geq 0}$ and $\{u_2^m\}_{m\geq 0}$ are non-decreasing on $[0, \infty)$. Indeed, we consider

$$\begin{aligned}
u_1^1(r) &= a_1 + \int_0^r \Big[B_1^-(t) \int_0^t B_1^+(s) f_1(u_1^0(s), u_2^0(s))ds\Big]^{1/k_1} dt \\
&= a_1 + \int_0^r \Big[B_1^-(t) \int_0^t B_1^+(s) f_1(a_1, a_2)ds\Big]^{1/k_1} dt \\
&\leq a_1 + \int_0^r \Big[B_1^-(t) \int_0^t B_1^+(s) f_1(u_1^1(s), u_2^1(s))ds\Big]^{1/k_1} dt = u_1^2(r).
\end{aligned}$$

This implies that $u_1^1(r) \leq u_1^2(r)$ which further produces $u_1^2(r) \leq u_1^3(r)$. Continuing, an induction argument applied to (2.2) show that for any $r \geq 0$ we have

$$u_1^m(r) \leq u_1^{m+1}(r) \quad \text{and} \quad u_2^m(r) \leq u_2^{m+1}(r) \quad \text{for any } m \in \mathbb{N}$$

i.e., $\{u_1^m\}_{m\geq 0}$ and $\{u_2^m\}_{m\geq 0}$ are non-decreasing on $[0, \infty)$. By their monotonicity, we have the inequalities

$$C_{N-1}^{k_1-1}\{\frac{r^{N-k_1}}{k_1} e^{\int_0^r \frac{1}{C_0} t^{k_1-1} b_1(t)dt} [(u_1^m)']^{k_1}\}' \leq B_1^+(r) f_1(u_1^m, u_2^m), \tag{2.3}$$

$$C_{N-1}^{k_2-1}\{\frac{r^{N-k_2}}{k_2} e^{\int_0^r \frac{1}{C_{00}} t^{k_2-1} b_2(t)dt} [(u_2^m)']^{k_2}\}' \leq B_2^+(r) f_2(u_1^m, u_2^m). \tag{2.4}$$

After integration from 0 to r, an easy calculation yields

$$\begin{aligned}
&(u_1^m(r))' \\
&\leq \Big(B_1^-(r) \int_0^r B_1^+(t) f_1(u_1^m(t), u_2^m(t))dt\Big)^{1/k_1} \\
&\leq \Big(B_1^-(r) \int_0^r B_1^+(t) f_1(u_1^m(t) + u_2^m(t), u_1^m(t) + u_2^m(t))dt\Big)^{1/k_1} \\
&\leq (f_1^{1/k_1} + f_2^{1/k_2})(u_1^m(r) + u_2^m(r), u_1^m(r) + u_2^m(r))(B_1^-(r) \int_0^r B_1^+(t)dt)^{1/k_1}.
\end{aligned} \tag{2.5}$$

As before, exactly the same type of conclusion holds for $(u_2^m(r))'$:

$$\begin{aligned}
(u_2^m(r))' &\leq \Big(B_2^-(r) \int_0^r B_2^+(z) f_2(u_1^m(z), u_2^m(z))dz\Big)^{1/k_2} \\
&\leq (f_1^{1/k_1} + f_2^{1/k_2})(u_1^m + u_2^m, u_1^m + u_2^m)\Big(B_2^-(r) \int_0^r B_2^+(t)dt\Big)^{1/k_2}.
\end{aligned} \tag{2.6}$$

Summing the inequalities (2.5) and (2.6), we obtain

$$\frac{(u_1^m(r) + u_2^m(r))'}{(f_1^{1/k_1} + f_2^{1/k_2})(u_1^m(r) + u_2^m(r), u_1^m(r) + u_2^m(r))} \leq P_1'(r) + P_2'(r). \tag{2.7}$$

Integrating from 0 to r, we obtain

$$\int_{a_1+a_2}^{u_1^m(r)+u_2^m(r)} \frac{1}{f_1^{1/k_1}(t,t)+f_2^{1/k_2}(t,t)}dt \le P_1(r)+P_2(r).$$

We now have

$$F_{1,2}(u_1^m(r)+u_2^m(r)) \le P_1(r)+P_2(r), \tag{2.8}$$

which will play a basic role in the proof of our main results. The inequalities (2.8) can be reformulated as

$$u_1^m(r)+u_2^m(r) \le F_{1,2}^{-1}(P_1(r)+P_2(r)). \tag{2.9}$$

This can be easily seen from the fact that $F_{1,2}$ is a bijection with the inverse function $F_{1,2}^{-1}$ strictly increasing on $[0,\infty)$. So, we have found upper bounds for $\{u_1^m\}_{m\ge 0}$ and $\{u_2^m\}_{m\ge 0}$ which are dependent of r. We are now ready to give a complete proof of the Theorems 1.1 and 1.2.

Proof of Theorem 1.1 completed. When $F_{1,2}(\infty)=\infty$ it follows that the sequences $\{u_1^m\}_{m\ge 0}$ and $\{u_2^m\}_{m\ge 0}$ are bounded and equicontinuous on $[0,c_0]$ for arbitrary $c_0>0$. By the Arzela-Ascoli theorem, $\{(u_1^m,u_2^m)\}_{m\ge 0}$ has a subsequence converging uniformly to (u_1,u_2) on $[0,c_0]\times[0,c_0]$. Since $\{u_1^m\}_{m\ge 0}$ and $\{u_2^m\}_{m\ge 0}$ are non-decreasing on $[0,\infty)$ we see that $\{(u_1^m,u_2^m)\}_{m\ge 0}$ itself converges uniformly to (u_1,u_2) on $[0,c_0]\times[0,c_0]$. At the end of this process, we conclude by the arbitrariness of $c_0>0$, that (u_1,u_2) is positive entire solution of system (2.1). The solution constructed in this way will be radially symmetric. Since the radial solutions of the ordinary differential equations system (2.1) are solutions (1.1) it follows that the radial solutions of (1.1) with $u_1(0)=a_1$, $u_2(0)=a_2$ satisfy

$$u_1(r)=a_1+\int_0^r \Big(B_1^-(y)\int_0^y B_1^+(t)f_1(u_1(t),u_2(t))dt\Big)^{1/k_1}dy, \tag{2.10}$$

$$u_2(r)=a_2+\int_0^r \Big(B_2^-(y)\int_0^y B_2^+(t)f_2(u_1(t),u_2(t))dt\Big)^{1/k_2}dy, \tag{2.11}$$

for all $r\ge 0$. Next, it is easy to verify that the Cases 1. and 2. occur.

Case 1. When $P_1(\infty)+P_2(\infty)<\infty$, it is not difficult to deduce from (2.10) and (2.11) that

$$u_1(r)+u_2(r) \le F_{1,2}^{-1}(P_1(\infty)+P_2(\infty))<\infty \quad \text{for all } r\ge 0,$$

and so (u_1,u_2) is bounded. We next consider:

Case 2. When $P_1(\infty)=P_2(\infty)=\infty$, we observe that

$$\begin{aligned} u_1(r)&=a_1+\int_0^r \Big(B_1^-(t)\int_0^t B_1^+(s)f_1(u_1(s),u_2(s))ds\Big)^{1/k_1}dt\\ &\ge a_1+f_1^{1/k_1}(a_1,a_2)\int_0^r \Big(B_1^-(t)\int_0^t B_1^+(s)ds\Big)^{1/k_1}dt\\ &=a_1+f_1^{1/k_1}(a_1,a_2)P_1(r). \end{aligned} \tag{2.12}$$

The same computations as in (2.12) yields

$$u_2(r)\ge a_2+f_2^{1/k_2}(a_1,a_2)P_2(r).$$

and passing to the limit as $r \to \infty$ in (2.12) and in the above inequality we conclude that

$$\lim_{r\to\infty} u_1(r) = \lim_{r\to\infty} u_2(r) = \infty,$$

which yields the result. $\square$

Proof of Theorem 1.2 completed. In view of the above analysis, the proof can be easily deduced from

$$F_{1,2}(u_1^m(r) + u_2^m(r)) \le P_1(\infty) + P_2(\infty) < F_{1,2}(\infty) < \infty,$$

Indeed, since $F_{1,2}^{-1}$ is strictly increasing on $[0,\infty)$, we find that

$$u_1^m(r) + u_2^m(r) \le F_{1,2}^{-1}(P_1(\infty) + P_2(\infty)) < \infty,$$

and then the non-decreasing sequences $\{u_1^m\}_{m\ge 0}$ and $\{u_2^m\}_{m\ge 0}$ are bounded above for all $r \ge 0$ and all m. The final step, is to conclude that

$$(u_1^m(r), u_2^m(r)) \to (u_1(r), u_2(r)) \quad \text{as } m \to \infty$$

and the limit functions u_1 and u_2 are positive entire bounded radial solutions of system (1.1). $\square$

Remark 2.2. Make the same assumptions as in Theorem 1.1 or Theorem 1.2 on b_1, p_1, f_1, b_2, p_2, f_2. If, in addition,

$$p_1(|x|) \ge \Big(C_{N-1}^{k_1-1}\frac{N-k_1}{k_1|x|^N} - \frac{b_1(|x|)}{|x|^{N-k_1}}\Big)\int_0^{|x|}\frac{s^{N-1}}{C_0}p_1(s)ds, \quad x \in \mathbb{R}^N, \tag{2.13}$$

$$p_2(|x|) \ge \Big(C_{N-1}^{k_2-1}\frac{N-k_2}{k_2|x|^N} - \frac{b_2(|x|)}{|x|^{N-k_2}}\Big)\int_0^{|x|}\frac{s^{N-1}}{C_{00}}p_2(s)ds, \quad x \in \mathbb{R}^N, \tag{2.14}$$

then the solution (u_1, u_2) is convex.

Proof. It is clear that

$$\frac{C_{N-1}^{k_1-1}}{r^{N-1}}\Big[\frac{r^{N-k_1}}{k_1}e^{\int_0^r \frac{1}{C_0}t^{k_1-1}b_1(t)dt}(u_1')^{k_1}\Big]' = e^{\int_0^r \frac{1}{C_0}t^{k_1-1}b_1(t)dt}p_1(r)f_1(u_1,u_2), \tag{2.15}$$

and integrating from 0 to r yields

$$\begin{aligned}
&r^{N-k_1}e^{\int_0^r \frac{1}{C_0}t^{k_1-1}b_1(t)dt}(u_1'(r))^{k_1}\\
&= \int_0^r \frac{s^{N-1}}{C_0}e^{\int_0^s \frac{1}{C_0}t^{k_1-1}b_1(t)dt}p_1(s)f_1(u_1(s),u_2(s))ds\\
&\le f_1(u_1(r),u_2(r))\int_0^r \frac{s^{N-1}}{C_0}e^{\int_0^s \frac{1}{C_0}t^{k_1-1}b_1(t)dt}p_1(s)ds,
\end{aligned}$$

which yields

$$\begin{aligned}
(\frac{u_1'(r)}{r})^{k_1} &\le \frac{f_1(u_1(r),u_2(r))}{r^N e^{\int_0^r \frac{1}{C_0}t^{k_1-1}b_1(t)dt}}\int_0^r \frac{s^{N-1}}{C_0}e^{\int_0^s \frac{1}{C_0}t^{k_1-1}b_1(t)dt}p_1(s)ds\\
&\le \frac{f_1(u_1(r),u_2(r))}{r^N}\int_0^r \frac{s^{N-1}}{C_0}p_1(s)ds.
\end{aligned} \tag{2.16}$$

On the other hand inequality (2.15) can be written in the form

$$\begin{aligned}
&C_{N-1}^{k_1-1}u_1''(r)(\frac{u_1'}{r})^{k_1-1} + C_{N-1}^{k_1-1}\frac{N-k_1}{k_1}(\frac{u_1'}{r})^{k_1} + b_1(r)(u_1')^{k_1}\\
&= p_1(r)f_1(u_1,u_2).
\end{aligned} \tag{2.17}$$

Using inequality (2.16) in (2.17) we obtain

$$p_1(r)f_1(u_1,u_2) \le C_{N-1}^{k_1-1}u_1''(\frac{u_1'}{r})^{k_1-1} + C_{N-1}^{k_1-1}\frac{N-k_1}{r^N k_1}f_1(u_1,u_2)\int_0^r \frac{s^{N-1}}{C_0}p_1(s)ds + \frac{b_1(r)f_1(u_1,u_2)}{r^{N-k_1}}\int_0^r \frac{s^{N-1}}{C_0}p_1(s)ds,$$

from which we have

$$\begin{aligned} &f_1(u_1,u_2)[p_1(r) - (C_{N-1}^{k_1-1}\frac{N-k_1}{k_1 r^N} - \frac{b_1(r)}{r^{N-k_1}})\int_0^r \frac{s^{N-1}}{C_0}p_1(s)ds] \\ &\le C_{N-1}^{k_1-1}u_1''(\frac{u_1'}{r})^{k_1-1}, \end{aligned}$$

which completes the proof of $u_1''(r) \ge 0$. A similar argument produces $u_2''(r) \ge 0$. We also remark that, in the simple case $b_1 = b_2 = 0$, $s^{N-1}p_1(s)$ and $s^{N-1}p_2(s)$ are increasing then (2.13) and (2.14) hold. $\square$

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