Electronic Journal of Differential Equations, Vol. 2016 (2016), No. 332, pp. 1–8.
ISSN: 1072-6691. URL: http://ejde.math.txstate.edu or http://ejde.math.unt.edu

BLOW-UP OF SOLUTIONS FOR VISCOELASTIC EQUATIONS OF KIRCHHOFF TYPE WITH ARBITRARY POSITIVE INITIAL ENERGY

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Abstract. We consider the viscoelastic equation

$$u_{tt}(x,t) - M(\|\nabla u\|_2^2)\Delta u(x,t) + \int_0^t g(t-s)\Delta u(x,s)ds + u_t = |u|^{p-1}u$$

with suitable initial data and boundary conditions. Under certain assumptions on the kernel g and the initial data, we establish a new blow-up result for arbitrary positive initial energy, by using simple analysis techniques.

1. Introduction

The wave equation

$$u_{tt} - \Delta u + h(u_t) = f(u) \tag{1.1}$$

with suitable initial data and boundary conditions has been extensively studied and several results concerning existence and blow-up have been established (see [1, 2, 10, 16]). Here h represents the friction or damping, and f the source. To describe the nonlinear vibrations of an elastic string, the so-called Kirchhoff equation

$$u_{tt} - M(\|\nabla u\|_2^2)\Delta u + h(u_t) = f(u) \tag{1.2}$$

was introduced [8], where $M(s) = m_0 + bs^\gamma$ is a positive C^1-function ($m_0 > 0$, $b \geq 0$, $\gamma > 0$, $s \geq 0$). In this case the existence and blow-up of solutions have been discussed by many authors (see [5, 6, 14, 15, 21] and the references cited therein).

When we take the viscoelastic materials into consideration, the models (1.1) and (1.2) become

$$u_{tt} - \Delta u + \int_0^t g(t-s)\Delta u(s)ds + h(u_t) = f(u) \tag{1.3}$$

and

$$u_{tt} - M(\|\nabla u\|_2^2)\Delta u + \int_0^t g(t-s)\Delta u(s)ds + h(u_t) = f(u) \tag{1.4}$$

respectively, where g represents the kernel of the memory.

For (1.3), many existence and blow-up results have been proved. See in this regard [7, 11, 12, 17, 18, 20]. For example, Messaoudi [11] studied (1.3) with $h(u_t) = |u_t|^{m-2}u_t$ and $f(u) = |u|^{p-2}u$ and proved a blow-up result for solutions with negative initial energy if $p > m \geq 2$ and a global existence result for $2 \leq p \leq m$.

2010 *Mathematics Subject Classification.* 35L05, 35L55, 35L70.

Key words and phrases. Viscoelastic equation; blow-up; arbitrary positive initial energy.

Submitted June 2, 2016. Published December 28, 2016.

This result has been improved by the same author in [12] to the case of positive initial energy. In [17], Song and Zhang consider (1.3) with $h(u_t) = -\Delta u_t$ and $f(u) = |u|^{p-2}u$ and prove a blow-up result for solutions with positive initial energy by using potential well theory introduced by Payne and Sattinger[16]. Later, Song [18] obtained the blow-up result of (1.3) in the case of $h(u_t) = |u_t|^{m-2}u_t$.

The model (1.4) states that the dynamic equilibrium of a body depends not only on the present state of deformation, but also on the previous history of the deformation[13]. This model was first studied by Torrejón and Young [19], who proved the existence of weakly asymptotic stable solution for a large analytical datum. Later, Munoz Rivera [13] showed the global existence for small datum and the total energy decays to zero exponentially under some restrictions. In [21] and [22], Wu and Tsai studied the model (1.4) with strong damping and nonlinear damping respectively and proved the existence and blow-up of solutions. In [22], a blow-up result of the model (1.4) with $m_0 = 1$, $h(u_t) = a|u_t|^{\nu-2}u_t + a|u_t|^{m-2}u_t$ and $f(u) = |u|^{p-2}u$ is obtained under some assumptions on the kernel g, the exponential p and the initial data. But this result holds only in the case $0 \le E(0) < E_1$, where $E(0)$ is the initial energy of the solution and E_1 is some a positive constant. Recently, by using concavity method, Liu and Liang [9] improved the results of [22] to the case of arbitrary positive initial energy. They considered the following initial-boundary value problem

$$
\begin{gathered}
u_{tt} - M(\|\nabla u\|_2^2)\Delta u + \int_0^t g(t-s)\Delta u(s)ds + u_t = f(u), \\
(x,t) \in \Omega \times (0,T), \\
u(x,t) = 0, \quad (x,t) \in \partial\Omega \times (0,T), \\
u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad x \in \Omega,
\end{gathered} \tag{1.5}
$$

where Ω is a bounded domain in $\mathbb{R}^n$ with a smooth boundary $\partial\Omega$. u_0 and u_1 are given initial data. M and g are two functions which stated as in (1.2) and (1.3). For this model, they obtained a blow-up result under some basic assumptions on f, g, M and the initial data u_0, u_1. (Readers can see [9, Conditions A1–A4, (2.3) and (2.4)].) However, we find that [9, conditions (A4) and (2.4)] are inessential. Moreover, it is difficult to construct a concrete model according to all the assumptions in [9], especially for (A4) and (2.4). So, motivated by [18, 22, 9], we try to consider the blow-up properties of the model (1.5) with $m_0 = 1$ and $f(u) = |u|^{p-2}u$. That is, we study the following problem

$$
\begin{gathered}
u_{tt} - M(\|\nabla u\|_2^2)\Delta u + \int_0^t g(t-s)\Delta u(s)ds + u_t = |u|^{p-2}u, \\
(x,t) \in \Omega \times (0,T), \\
u(x,t) = 0, \quad (x,t) \in \partial\Omega \times (0,T), \\
u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad x \in \Omega,
\end{gathered} \tag{1.6}
$$

where $M(s) = 1 + bs^{\gamma} (b \ge 0, \gamma > 0, s \ge 0)$ is a positive C^1 -function. We hope to get some more concise sufficient conditions.

2. Preliminaries and statement of main result

In this article, C denotes a generic positive constant. It may be different from line to line. And we use the standard Lebesgue space $L^p(\Omega)$ with their usual norms $\|\cdot\|_p$. Moreover, we denote by $(\cdot,\cdot)$ the usual $L^2(\Omega)$ inner product.

We first state the general assumptions on g and p as follows:

(A1) $g \in C^1([0,\infty))$ is a non-negative and non-increasing function satisfying

$$0 < k := \int_0^\infty g(s)ds < 1. \tag{2.1}$$

(A2) If the space dimension $n = 1, 2$, then $2(\gamma+1) < p < \infty$; If $n \geq 3$, then

$$2(\gamma+1) < p \leq \frac{2(n-1)}{n-2}.$$

To simplify the notation, we set

$$(\phi \circ \psi)(t) := \int_0^t \phi(t-s) \int_\Omega |\psi(t) - \psi(s)|^2 dxds,$$

where ψ may be a scalar, or a vector valued function. A direct computation shows that, for any $g \in C^1(\mathbb{R})$ and $u \in H^2(0,T,L^2(\Omega))$, the following identity holds:

$$\begin{aligned}
&\int_0^t g(t-s)\big(\nabla u(s), \nabla u_t(t)\big)ds \\
&= \frac{1}{2}(g' \circ \nabla u)(t) - \frac{1}{2}g(t)\|\nabla u(t)\|_2^2 \\
&\quad - \frac{1}{2}\frac{d}{dt}\Big\{(g \circ \nabla u)(t) - \Big(\int_0^t g(s)ds\Big)\|\nabla u(t)\|_2^2\Big\}.
\end{aligned} \tag{2.2}$$

Now, we state a local existence theorem that can be established by adopting the arguments of [22].

Theorem 2.1 (Local solution)**.** *Assume that* (A1) *and* (A2) *hold. Let* $u_0 \in H_0^2(\Omega)$ *and* $u_1 \in H_0^1(\Omega)$ *be given. Then, there exists a unique weak solution* $u(t)$ *of* (1.5) *such that*

$$u \in C([0,T]; H_0^2(\Omega)) \cap C^1([0,T]; L^2(\Omega)), \quad u_t \in L^2([0,T]; H_0^1(\Omega)). \tag{2.3}$$

for a small enough $T > 0$.

The energy functional of the solution u of (1.5) is defined as

$$\begin{aligned}
E(t) := &\frac{1}{2}\|u_t\|_2^2 + \frac{1}{2}\Big(1 - \int_0^t g(s)ds\Big)\|\nabla u\|_2^2 + \frac{b}{2(\gamma+1)}\|\nabla u\|_2^{2(\gamma+1)} \\
&+ \frac{1}{2}(g \circ \nabla u)(t) - \frac{1}{p}\|u\|_p^p.
\end{aligned} \tag{2.4}$$

By (2.2) and assumption (A1), direct computations yield

$$E'(t) = \frac{1}{2}(g' \circ \nabla u)(t) - \frac{1}{2}g(t)\|\nabla u\|_2^2 - \|u_t\|_2^2 \leq -\|u_t\|_2^2 \leq 0. \tag{2.5}$$

According to [22], we can obtain the following blow-up with negative initial energy:

Theorem 2.2. *Assume that* (A1), (A2) *and* $k < \frac{2(p-2)}{2p-3}$ *hold. if* $E(0) < 0$, *then for all the initial data* $u_0 \in H_0^2(\Omega)$ *and* $u_1 \in H_0^1(\Omega)$, *the corresponding solution* $u(x,t)$ *of the problem* (1.5) *blows up in finite time.*

Our main result is a blow-up with positive initial energy that reads as follows.

Theorem 2.3. *Assume that* (A1), (A2) *and* $k < \frac{p(p-2)}{(p-1)^2}$ *hold. Moreover,* $E(0) > 0$ *(maybe large enough) is a given initial energy state. If we choose initial data* $u_0 \in H_0^2(\Omega)$ *and* $u_1 \in H_0^1(\Omega)$ *satisfying*

$$\int_\Omega u_0 u_1 dx > \beta E(0), \tag{2.6}$$

where $\beta = \frac{1}{2\varepsilon_0}, \varepsilon_0 \in (0,1)$ *is a positive constant, then the corresponding solution* $u(x,t)$ *of the problem* (1.5) *blows up in finite time.*

In [9], the kernel g must be the so-called positive type function. But, we do not need that assumption. Moreover, our kernel function space is bigger than the one in [22] since $\frac{p(p-2)}{(p-1)^2} > \frac{2(p-2)}{2p-3}$.

3. Proof of main result

Assume u is a global solution of problem (1.6). Let

$$Q(t) = \int_\Omega u u_t dx.$$

Multiplying the first equation of (1.6) by u and integrating over Ω, we get

$$\int_\Omega u u_{tt} dx + M(\|\nabla u\|_2^2)\|\nabla u\|_2^2 - \int_\Omega \Big(\int_0^t g(t-s)\Delta u(s) ds\Big) u dx + \int_\Omega u u_t dx = \|u\|_p^p.$$

Then, we easily obtain

$$\begin{aligned} Q'(t) &= \|u_t\|_2^2 - M(\|\nabla u\|_2^2)\|\nabla u\|_2^2 + \|u\|_p^p \\ &\quad - \int_\Omega \left(\int_0^t g(t-s)\Delta u(s) ds\right) u dx - \int_\Omega u u_t dx. \end{aligned} \tag{3.1}$$

For the last term on the right side of (3.1), using Cauchy inequality, we deduce that

$$\begin{aligned} &- \int_\Omega \Big(\int_0^t g(t-s)\Delta u(s) ds\Big) u dx \\ &= \int_0^t g(t-s) \int_\Omega \nabla u(s) \nabla u(t) dx ds \\ &= \int_0^t g(t-s) \int_\Omega \nabla u(t)(\nabla u(s) - \nabla u(t)) dx ds + \int_0^t g(s) ds \|\nabla u\|_2^2 \\ &\geq -\frac{p(1-\varepsilon)}{2}(g \circ \nabla u)(t) + \big(1 - \frac{1}{2p(1-\varepsilon)}\big) \int_0^t g(s) ds \|\nabla u\|_2^2 \end{aligned} \tag{3.2}$$

for all $\varepsilon \in (0,1)$. By (3.2) and (2.4), we have

$$\begin{aligned} Q'(t) &\geq \|u_t\|_2^2 - \big(1 - \int_0^t g(s)ds\big)\|\nabla u\|_2^2 - b\|\nabla u\|_2^{2(\gamma+1)} + \|u\|_p^p - \int_\Omega u u_t dx \\ &\quad - \frac{p(1-\varepsilon)}{2}(g\circ\nabla u)(t) - \frac{1}{2p(1-\varepsilon)}\int_0^t g(s)ds\|\nabla u\|_2^2 \\ &= \big(\frac{p(1-\varepsilon)}{2}+1\big)\|u_t\|_2^2 + \big(\frac{p(1-\varepsilon)}{2}-1\big)\Big(1-\int_0^t g(s)ds\Big)\|\nabla u\|_2^2 \\ &\quad - \frac{1}{2p(1-\varepsilon)}\int_0^t g(s)ds\|\nabla u\|_2^2 - p(1-\varepsilon)E(t) + \varepsilon\|u\|_p^p - \int_\Omega u u_t dx \\ &\quad + \Big(\frac{bp(1-\varepsilon)}{2(\gamma+1)} - b\Big)\|\nabla u\|_2^{2(\gamma+1)}. \end{aligned} \tag{3.3}$$

Now, by assumption (A2), we select ε small enough to ensure that

$$\frac{bp(1-\varepsilon)}{2(\gamma+1)} - b > 0.$$

Moreover, using Hölder inequality and Young inequality, we can get

$$\big|\int_\Omega u u_t dx\big| \leq \|u\|_2\|u_t\|_2 \leq \frac{\varepsilon}{2}\|u\|_2^2 + \frac{1}{2\varepsilon}\|u_t\|_2^2.$$

Then, by assumption (A1), (2.5) and Poincaré's inequality, we have

$$\begin{aligned} \Big(Q(t) - \frac{E(t)}{2\varepsilon}\Big)' &\geq Q'(t) + \frac{1}{2\varepsilon}\|u_t\|_2^2 \\ &\geq \Big(\frac{p(1-\varepsilon)}{2}+1\Big)\|u_t\|_2^2 - p(1-\varepsilon)E(t) - \frac{\varepsilon}{2}\|u\|_2^2 \\ &\quad + \Big(\Big(\frac{p(1-\varepsilon)}{2}-1\Big)(1-k) - \frac{k}{2p(1-\varepsilon)}\Big)\|\nabla u\|_2^2 \\ &\geq \Big(\frac{p(1-\varepsilon)}{2}+1\Big)\|u_t\|_2^2 - p(1-\varepsilon)E(t) \\ &\quad + \big(f(\varepsilon)\lambda_1 - \frac{\varepsilon}{2}\big)\|u\|_2^2. \end{aligned} \tag{3.4}$$

where λ_1 is the first eigenvalue of $-\Delta$ and

$$f(\varepsilon) = \Big(\frac{p(1-\varepsilon)}{2} - 1\Big)(1-k) - \frac{k}{2p(1-\varepsilon)}. \tag{3.5}$$

Since $k < \frac{p(p-2)}{(p-1)^2}$ and $p > 2$, we deduce that $1 - k > \frac{1}{(p-1)^2}$ and

$$\theta := (p-2)(1-k) - \frac{k}{p} > 0.$$

Moreover, we note that $f(\varepsilon) \to \frac{\theta}{2}$ as $\varepsilon \to 0^+$. So, we can select ε small enough such that $f(\varepsilon)\lambda_1 - \frac{\varepsilon}{2} > 0$. Then, using Cauchy inequality to (3.4), we have

$$\begin{aligned} \Big(Q(t) - \frac{E(t)}{2\varepsilon}\Big)' &\geq h(\varepsilon)Q(t) - p(1-\varepsilon)E(t) \\ &= h(\varepsilon)\Big(Q(t) - \frac{p(1-\varepsilon)}{h(\varepsilon)}E(t)\Big), \end{aligned} \tag{3.6}$$

where

$$h(\varepsilon) = 2\sqrt{\big(\frac{p(1-\varepsilon)}{2}+1\big)\big(f(\varepsilon)\lambda_1-\frac{\varepsilon}{2}\big)}.$$

Denote

$$\varphi(\varepsilon) = \Big(\frac{p(1-\varepsilon)}{2}+1\Big)\Big(f(\varepsilon)\lambda_1-\frac{\varepsilon}{2}\Big).$$

It is easy to see that

$$\begin{gathered} f(\varepsilon)\lambda_1-\frac{\varepsilon}{2}\to\frac{\theta\lambda_1}{2},\quad \varphi(\varepsilon)\to\theta\lambda_1(p+2),\quad \text{as } \varepsilon\to 0^+,\\ f(\varepsilon)\to-\infty,\quad f(\varepsilon)\lambda_1-\frac{\varepsilon}{2}\to-\infty,\quad \varphi(\varepsilon)\to-\infty\quad \text{as } \varepsilon\to 1^-. \end{gathered}$$

Hence, by the continuity of $\varphi(\varepsilon)$, there exists $\tilde{\varepsilon}\in(0,1)$ such that $\varphi(\tilde{\varepsilon})=0$ and $\varphi(\varepsilon)>0$ for all $\varepsilon\in(0,\tilde{\varepsilon})$. So, we have $h(\tilde{\varepsilon})=2\sqrt{\varphi(\tilde{\varepsilon})}=0$ and $h(\varepsilon)=2\sqrt{\varphi(\varepsilon)}>0$ for all $\varepsilon\in(0,\tilde{\varepsilon})$. And then, we easily deduce that

$$\begin{gathered} \frac{p(1-\varepsilon)}{h(\varepsilon)}\to\frac{p}{\sqrt{\theta\lambda_1(p+2)}},\quad \frac{1}{2\varepsilon}\to+\infty,\quad \text{as } \varepsilon\to 0^+,\\ \frac{p(1-\varepsilon)}{h(\varepsilon)}\to+\infty,\quad \frac{1}{2\varepsilon}\to\frac{1}{2\tilde{\varepsilon}},\quad \text{as } \varepsilon\to\tilde{\varepsilon}^-. \end{gathered}$$

Thus, using the continuity in ε of $\frac{p(1-\varepsilon)}{h(\varepsilon)}$ and $\frac{1}{2\varepsilon}$, there exists $\varepsilon_0\in(0,\tilde{\varepsilon})\subset(0,1)$ such that

$$\frac{1}{2\varepsilon_0}=\frac{p(1-\varepsilon_0)}{h(\varepsilon_0)}.$$

Now, let

$$\beta=\frac{1}{2\varepsilon_0}\quad\text{and}\quad H(t)=Q(t)-\beta E(t). \tag{3.7}$$

By using (2.6), (2.5) and (3.6), we deduce that

$$\begin{gathered} H(0)=Q(0)-\beta E(0)>0,\\ H'(t)\ge Q'(t)\ge h(\varepsilon_0)H(t). \end{gathered}$$

Then, we have

$$H(t)\ge e^{h(\varepsilon_0)t}H(0).$$

Since u is global, by (2.5) and Theorem 2.2, the energy $E(t)$ remains nonnegative, i.e., $0\le E(t)\le E(0)$ for all $t\in[0,+\infty)$. So, we deduce that $Q(t)\ge e^{h(\varepsilon_0)t}H(0)$ and

$$\begin{aligned} \|u(t)\|_2^2 &= \|u(0)\|_2^2+2\int_0^t Q(s)ds\\ &\ge \|u(0)\|_2^2+2\int_0^t e^{h(\varepsilon_0)s}H(0)ds\\ &= \|u(0)\|_2^2+\frac{2H(0)}{h(\varepsilon_0)}\big(e^{h(\varepsilon_0)t}-1\big). \end{aligned}\tag{3.8}$$

By (2.5), Theorem 2.2, and Hölder inequality, we obtain

$$\begin{aligned} \|u(t)\|_2 &\le \|u(0)\|_2 + \int_0^t \|u_s(s)\|_2 ds \\ &\le \|u(0)\|_2 + t^{1/2}\Big(\int_0^t \|u_s(s)\|_2^2 ds\Big)^{1/2} \\ &\le \|u(0)\|_2 + t^{1/2}\left(E(0)-E(t)\right)^{1/2} \\ &\le \|u(0)\|_2 + t^{1/2}(E(0))^{1/2} \end{aligned} \tag{3.9}$$

which contradicts (3.8). □

As a simple example, we consider a one-dimension model with $M(s) = 1+s, \Omega = [0, 2\pi]$ and $p = 5$. Let

$$u_0 = \xi \sin(\eta x), \quad u_1 = \xi\eta^2 \sin(\eta x),$$

where $\xi > 0$ and η is a positive integer. Then, we have $Q(0) = (u_0, u_1) = \xi^2\eta^2\pi$ and

$$\begin{aligned} E(0) &= \frac{1}{2}\|u_1\|_2^2 + \frac{1}{2}\|\nabla u_0\|_2^2 + \frac{1}{4}\|\nabla u_0\|_2^4 - \frac{1}{5}\|u_0\|_5^5 \\ &= \int_0^{2\pi} |\xi\eta^2 \sin(\eta x)|^2 dx - \frac{1}{5}\int_0^{2\pi} |\xi \sin(\eta x)|^5 dx \\ &= \xi^2\eta^4\pi - \frac{32}{75}\xi^5. \end{aligned}$$

Now, we choose $\eta > \sqrt{1/(2\beta)}$ and $\xi = \sqrt[3]{\frac{75}{32}\eta^2\pi(\eta^2 - \frac{1}{2\beta})}$. Then, we can deduce that

$$Q(0) = 2\beta E(0) > \beta E(0).$$

According Theorem 2.3, the corresponding solution blows up in finite time.

Acknowledgments. The author would like to thank the anonymous referees for their invaluable comments and suggestions. This research was supported by the Natural Science Foundation of China (11671128), the Science and Technology Plan Project of Hunan Province (2016TP1020), the Key Construction Disciplines of Hunan Province and the Starting Project of Hengyang Normal University(16D01).

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