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GLOBAL ATTRACTOR FOR AN EQUATION MODELLING A THERMOSTAT

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Abstract. In this work we show that the system considered by Guidotti and Merino in [2] as a model for a thermostat has a global attractor and, assuming that the parameter is small enough, the origin is globally asymptotically stable.

1. Introduction

The purpose of this note is to answer a question proposed by Guidotti and Merino [2], concerning the global stability for the trivial solution of the nonlinear and nonlocal boundary-value problem

$$\begin{gathered} u_t = u_{xx}, \quad x \in (0,\pi)\ t > 0 \\ u_x(0,t) = \tanh(\beta u(\pi,t)), \quad t > 0\ \beta > 0 \\ u_x(\pi,t) = 0, \quad t > 0 \\ u(x,0) = u_0(x), \quad x \in (0,\pi). \end{gathered} \tag{1.1}$$

This problem was proposed in [2] as a rudimentary model for a thermostat. To achieve this goal, we first show the existence of a global compact attractor $\mathcal{A}_\beta$ for (1.1), for any positive value of the parameter β. We then prove that $\mathcal{A}_\beta = \{0\}$ if $0 < \beta < 1/\pi$, thus showing that the trivial solution is globally asymptotically stable in the phase space, for these values of β.

2. Global semi-flux in a fractional power space X^α

As in [2] we adopt here the following weak formulation for (1.1): u is a solution of (1.1) if

$$\begin{gathered} \int_0^\pi u_t\varphi dx + \int_0^\pi u_x\varphi_x dx = -\tanh(\beta u(\pi))\varphi(0),\ t > 0, \\ u(0) = u_0, \end{gathered} \tag{2.1}$$

for all $\varphi \in H^1(0,\pi) = H^1$.

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Consider the linear operator $A \in \mathcal{L}(H^1, (H^1)')$ induced by the continuous bilinear form $a(\cdot,\cdot) : H^1 \times H^1 \to \mathbb{R}$ given by $a(u,v) = ((u,v))_{H^1}$, that is,

$$\langle Au, v\rangle_{(H^1)'\times H^1} = a(u,v) = ((u,v))_{H^1}, \forall u,v \in H^1.$$

We may interpret A as the unbounded closed nonnegative self-adjoint operator $A : D(A) \subset L^2(0,\pi) \to L^2(0,\pi) = L^2$ defined by

$$Au(x) = -u''(x) + u(x), x \in (0,\pi),$$

for any $u \in D(A) = \{u \in H^2(0,\pi) : u'(0) = u'(\pi) = 0\}$. Let $\{\lambda_n\}$ and $\{e_n\}$ denote the eigenvalues and eigenfuctions of A, respectively. As it is easy to see, A is a sectorial operator in $L^2(0,\pi)$ and, therefore, its fractional powers are well defined (cf. Henry [4]). Let $X^\alpha = D(A^\alpha)$, $\alpha \ge 0$, be the domain of A^α. It is well known that X^α endowed with the inner product

$$(u,v)_\alpha = (A^\alpha u, A^\alpha v)_{L^2} = \sum_{n=0}^{\infty} |\lambda_n|^{2\alpha}(u,e_n)_{L^2}(v,e_n)_{L^2}$$

is a Hilbert space. In particular, we have $X^0 = L^2$, $X^1 = D(A)$ and $X^{1/2} = H^1$.

Following Amann [1] or Teman [5] we have, for any $\theta \in [0,1]$

$$X^{\frac{1-\theta}{2}} = [H^1, L^2]_\theta,$$

where $[\cdot,\cdot]_\theta$ denotes the complex interpolation functor. On the other hand, for any $s \in [0,1]$,

$$H^s(0,\pi) = [H^1, L^2]_{1-s}.$$

Letting $\theta = 1 - s$, we obtain $X^\alpha = H^{2\alpha}$, for any $\alpha \in [0, 1/2]$.

Denoting $X^{-1/2} = \big(X^{1/2}\big)' = (H^1)'$ and considering the linear operator $A \in \mathcal{L}(H^1, (H^1)')$ as a unbounded operator in $(H^1)' = X^{-1/2}$ given by $D(A) = X^{1/2}$ and

$$\langle Au, \varphi\rangle_{-1/2,1/2} = (u,\varphi)_{1/2} = ((u,\varphi))_{H^1},$$

for any $u, \varphi \in H^1 = X^{1/2}$, we rewrite equation (2.1) as an evolution equation

$$\begin{gathered} u_t = -Au + F(u) \quad \text{in } X^{-1/2}\ t > 0, \\ u(0) = u_0 \end{gathered} \tag{2.2}$$

where $F : X^\alpha \to X^{-1/2}$ is defined by

$$\langle F(u), \varphi\rangle_{-1/2,1/2} = -\tanh(\beta u(\pi))\varphi(0) + \int_0^\pi u\varphi dx,$$

for $u \in X^\alpha$ and $\varphi \in X^{1/2}$, that is, $F(u) = -\gamma_0^* \tanh(\beta\gamma_\pi(u)) + u$ in $X^{-1/2}$, where $\gamma_\pi \in \mathcal{L}(X^\alpha, \mathbb{R})$ is given by $\gamma_\pi(u) = u(\pi)$ and $\gamma_0^* \in \mathcal{L}(\mathbb{R}, X^{-1/2})$ is the adjoint operator of $\gamma_0 \in \mathcal{L}(X^{1/2}, \mathbb{R})$ given by $\gamma_0(u) = u(0)$.

To have a well-posed problem in X^α, we make some restrictions on α. We impose first that $X^\alpha \hookrightarrow \mathcal{C}([0,\pi])$, which is accomplished by requiring that $\alpha > 1/4$. Now, according to [1, 4], $-A$ is the infinitesimal generator of an analytic semigroup $\{e^{-At}; t \ge 0\}$ in $\mathcal{L}(X^{-1/2})$; since F maps X^α into $X^{-1/2}$, we impose also that $0 \le \alpha - (-\frac{1}{2}) < 1$. It turns out that the condition $\frac{1}{4} < \alpha < \frac{1}{2}$ implies that (2.2) has an unique global solution $u : [0,\infty) \to X^\alpha$, for any $u_0 \in X^\alpha$. This follows immediately from Theorem 3.3.3 in [4] and from the fact that F is globally Lipschitz continuous:

$$\begin{aligned}
&|\langle F(u)-F(v),\varphi\rangle_{-1/2,1/2}|\\
&\le |\tanh(\beta\gamma_\pi(u))-\tanh(\beta\gamma_\pi(v))||\varphi(0)|+|(u-v,\varphi)_{L^2}|\\
&\le \beta\|\gamma_\pi\|_{\mathcal{L}(X^\alpha,\mathbb{R})}\|\gamma_0\|_{\mathcal{L}(X^{1/2},\mathbb{R})}\|u-v\|_\alpha\|\varphi\|_{1/2}+\|u-v\|_{L^2}\|\varphi\|_{L^2}\\
&\le \big(\beta\|\gamma_\pi\|_{\mathcal{L}(X^\alpha,\mathbb{R})}\|\gamma_0\|_{\mathcal{L}(X^{1/2},\mathbb{R})}+k\big)\|u-v\|_\alpha\|\varphi\|_{1/2},
\end{aligned}$$

for all φ in $X^{1/2}$ and any u, v in X^α, which implies

$$\|F(u)-F(v)\|_{-1/2}\le K\|u-v\|_\alpha,$$

for all $u,v\in X^\alpha$, where $K=\big(\beta\|\gamma_\pi\|_{\mathcal{L}(X^\alpha,\mathbb{R})}\|\gamma_0\|_{\mathcal{L}(X^{1/2},\mathbb{R})}+k\big)$ and k is the embedding constant of X^α in L^2.

Since F maps bounded sets of X^α into bounded sets of $X^{-1/2}$, it follows by [4, Theorem 3.3.4] that the flow defined by (2.2) is global.

3. Main Results

We denote by $\{T(t);\ t\ge 0\}\subset\mathcal{L}(X^{-1/2})$ the semigroup generated by (2.2). Since the spectrum of $A: X^{1/2}\subset X^{-1/2}\to X^{-1/2}$ is given by $\sigma(A)=\{n^2+1; n=0,1,...\}$, for any $0<\delta<1$, we have, by [4, Theorem 1.4.3],

$$\|e^{-At}\|_{\mathcal{L}(X^{-1/2})}\le Ce^{-\delta t},\quad \|A^\alpha e^{-At}\|_{\mathcal{L}(X^{-1/2})}\le C_\alpha t^{-\alpha}e^{-\delta t}, \tag{3.1}$$

for $t>0$. Since

$$\begin{aligned}
|\langle F(u),\varphi\rangle_{-1/2,1/2}| &\le |\tanh(\beta u(\pi))||\varphi(0)| + |(u,\varphi)_{L^2}|\\
&\le |\varphi(0)|+\|u\|_{L^2}\|\varphi\|_{L^2}\\
&\le \sqrt{2\pi}\|\varphi\|_{1/2}+\|u\|_{L^2}\|\varphi\|_{1/2},
\end{aligned}$$

for all $\varphi\in X^{1/2}$, we have that for all $u\in X^\alpha$,

$$\|F(u)\|_{-1/2}\le\sqrt{2\pi}+\|u\|_{L^2}\,. \tag{3.2}$$

Lemma 3.1. *Let $\beta\in(0,\infty)$, $\alpha\in(1/4,1/2)$. Denote by B_ε the ball with center 0 and radius $\pi(\sqrt{\pi}+\varepsilon)$ in L^2. Then we have*

1. *For any $u_0\in X^\alpha$ there exists $t^*=t^*(u_0)$, depending only on the L^2-norm of u_0, such that the positive semiorbit $T(t)u_0$ is in B_ε for $t\ge t^*(u_0)$;*
2. *While $T(t)u_0$ is outside B_ε its L^2-norm is decreasing.*

Proof. Let $u_0\in X^\alpha$, $\epsilon>0$ and, for simplicity, denote by $u(\cdot,t)=T(t)u_0$ the solution of (1.1) through u_0. Then, we have

$$\begin{aligned}
\frac{d}{dt}\frac{1}{2}\int_0^\pi u(x,t)^2dx &= \int_0^\pi u(x,t)u_t(x,t)dx\\
&= -\tanh(\beta u(\pi,t))u(0,t)-\int_0^\pi u_x(x,t)^2dx,\ t>0.
\end{aligned} \tag{3.3}$$

To obtain estimates for this derivative we consider the subsets

$$\begin{gathered}
S_1(u_0)=\{t\in(0,\infty): u(0,t)u(\pi,t)\ge 0\},\\
S_2(u_0)=\{t\in(0,\infty): u(0,t)u(\pi,t)<0\}=(0,\infty)\setminus S_1(u_0).
\end{gathered}$$

If $t \in S_2(u_0)$, there exists $y(t) \in (0,\pi)$ such that $u(y(t),t) = 0$ and then

$$|u(x,t)| \leq |u(y(t),t)| + \int_0^\pi |u_x(x,t)|dx \leq \sqrt{\pi}\|u_x(\cdot,t)\|_{L^2},$$

for all $x \in [0,\pi]$. Therefore,

$$\|u(\cdot,t)\|_{L^2}^2 \leq \pi^2\|u_x(\cdot,t)\|_{L^2}^2,$$

for any $t \in S_2(u_0)$. Hence, for all $t \in S_2(u_0)$,

$$\begin{aligned} \frac{d}{dt}\frac{1}{2}\|u(\cdot,t)\|_{L^2}^2 &= |\tanh(\beta u(\pi,t))||u(0,t)| - \|u_x(\cdot,t)\|_{L^2}^2 \\ &\leq \sqrt{\pi}\|u_x(\cdot,t)\|_{L^2} - \|u_x(\cdot,t)\|_{L^2}^2 \,. \end{aligned} \tag{3.4}$$

If $\|u(\cdot,t)\|_{L^2} > \pi(\sqrt{\pi}+\epsilon)$, then

$$\frac{d}{dt}\frac{1}{2}\|u(\cdot,t)\|_{L^2}^2 \leq -\epsilon(\sqrt{\pi}+\epsilon). \tag{3.5}$$

To compute the derivative when $t \in S_1(u_0)$, we need to estimate $\|u_x(\cdot,t)\|_{L^2}$. Let $m : (0,\infty) \to \mathbb{R}^+$ be the continuous function $m(t) = \min\{|u(0,t)|, |u(\pi,t)|\}$ and

$$J(u_0) = \big\{t \in S_1(u_0),\ m(t) \leq \frac{1}{2\pi}\|u(\cdot,t)\|_{L^2}\big\}.$$

From

$$|u(x,t)| \leq \min\{|u(0,t)|, |u(\pi,t)|\} + \int_0^\pi |u_x(x,t)|dx$$

for $x \in [0,\pi]$ and $t > 0$, we have

$$\|u(\cdot,t)\|_{L^2}^2 \leq \pi\left(m(t) + \sqrt{\pi}\|u_x(\cdot,t)\|_{L^2}\right)^2 \leq 2\pi^2\left(m(t)^2 + \|u_x(\cdot,t)\|_{L^2}^2\right) \,.$$

Therefore,

$$\|u_x(\cdot,t)\|_{L^2}^2 \geq \frac{1}{2\pi^2}\|u(\cdot,t)\|_{L^2}^2 - m(t)^2.$$

Thus, if $t \in J(u_0)$, then

$$\|u_x(\cdot,t)\|_{L^2}^2 \geq \frac{1}{2\pi^2}\|u(\cdot,t)\|_{L^2}^2 - \frac{1}{4\pi^2}\|u(\cdot,t)\|_{L^2}^2 = \frac{1}{4\pi^2}\|u(\cdot,t)\|_{L^2}^2.$$

Therefore, for all $t \in J(u_0)$,

$$\begin{aligned} \frac{d}{dt}\frac{1}{2}\|u(\cdot,t)\|_{L^2}^2 &= -\tanh(\beta u(\pi,t))u(0,t) - \|u_x(\cdot,t)\|_{L^2}^2 \\ &\leq -\|u_x(\cdot,t)\|_{L^2}^2 \\ &\leq -\frac{1}{4\pi^2}\|u(\cdot,t)\|_{L^2}^2. \end{aligned} \tag{3.6}$$

If $\|u(\cdot,t)\|_{L^2} > \pi(\sqrt{\pi}+\epsilon)$, we obtain

$$\frac{d}{dt}\frac{1}{2}\|u(\cdot,t)\|_{L^2}^2 \leq -\frac{1}{4}(\sqrt{\pi}+\varepsilon)^2 \tag{3.7}$$

On the other hand, if $t \in S_1(u_0) \setminus J(u_0)$, then

$$\begin{aligned} \frac{d}{dt}\frac{1}{2}\|u(\cdot,t)\|_{L^2}^2 &= -\tanh(\beta u(\pi,t))u(0,t) - \|u_x(\cdot,t)\|_{L^2}^2 \\ &\le -\tanh(\beta u(\pi,t))u(0,t) \\ &= -\tanh(\beta|u(\pi,t)|)|u(0,t)| \\ &\le -\tanh\big(\frac{\beta\|u(\cdot,t)\|_{L^2}}{2\pi}\big)\frac{\|u(\cdot,t)\|_{L^2}}{2\pi} \end{aligned} \tag{3.8}$$

If $\|u(\cdot,t)\|_{L^2} > \pi(\sqrt{\pi}+\epsilon)$, we obtain

$$\frac{d}{dt}\frac{1}{2}\|u(\cdot,t)\|_{L^2}^2 \le -\tanh\big(\frac{\beta(\sqrt{\pi}+\epsilon)}{2}\big)\frac{(\sqrt{\pi}+\epsilon)}{2} \tag{3.9}$$

Letting $\varepsilon_1 = \min\big\{\varepsilon(\sqrt{\pi}+\varepsilon), \frac{1}{4}(\sqrt{\pi}+\varepsilon)^2, \tanh\big(\frac{\beta(\sqrt{\pi}+\epsilon)}{2}\big)\frac{(\sqrt{\pi}+\epsilon)}{2}\big\}$, we conclude using (3.5), (3.7) and (3.9), that

$$\frac{d}{dt}\|u(\cdot,t)\|_{L^2}^2 \le -2\varepsilon_1 \tag{3.10}$$

This proves our second assertion.

Suppose $u(t,u_0)$ is outside B_ε for $0 \le t \le \bar{t}$. Then $\|u(\cdot,\bar{t})\|_{L^2}^2 \le \|u_0\|_{L^2}^2 - 2\varepsilon_1\bar{t}$. Therefore, there must exist a $t^* = t^*(u_0) \le \frac{1}{2\varepsilon_1}\big(\|u_0\|_{L^2}^2 - \pi^2(\sqrt{\pi}+\varepsilon)^2\big)$ such that $u(\cdot,t^*)$ belongs to B_ε. We claim that $\|u(\cdot,t)\|_{L^2} \le \pi(\sqrt{\pi}+\varepsilon)$ for all $t \ge t^*$. Otherwise, there would exist $t_1 \ge t^*$ and $\delta > 0$ such that $\|u(\cdot,t_1)\|_{L^2} = \pi(\sqrt{\pi}+\varepsilon)$ and $\|u(\cdot,t)\|_{L^2} > \pi(\sqrt{\pi}+\varepsilon)$ for $t \in (t_1, t_1+\delta)$, which is a contradiction with the fact that $t \mapsto \|u(\cdot,t)\|_{L^2}$ is non increasing. This proves our first assertion. □

Theorem 3.2. *If $\beta \in (0,\infty)$ and $\alpha \in (1/4, 1/2)$, then $\{T(t); t \ge 0\}$ has a global attractor $\mathcal{A}_\beta$.*

Proof. . Let $u_0 \in X^\alpha$ and $u(\cdot,t) = T(t)u_0$. By the variation of constant formula and estimates (3.1), (3.2), we have

$$\begin{aligned} \|u(\cdot,t)\|_\alpha &\le Ce^{-\delta t}\|u_0\|_\alpha + C_\alpha\int_0^t e^{-\delta(t-s)}(t-s)^{-\alpha}\|F(u(\cdot,s))\|_{-1/2}ds, \\ &\le Ce^{-\delta t}\|u_0\|_\alpha + C_\alpha\int_0^t e^{-\delta(t-s)}(t-s)^{-\alpha}(\sqrt{2\pi} + \|u(\cdot,s)\|_{L^2})ds. \end{aligned} \tag{3.11}$$

If $t^*(u_0)$ is as given by Lemma 3.1, for $t > t^*$ we have

$$\begin{aligned}
\|u(\cdot,t)\|_\alpha &\le Ce^{-\delta t}\|u_0\|_\alpha + C_\alpha\int_0^{t^*} e^{-\delta(t-s)}(t-s)^{-\alpha}(\sqrt{2\pi}+\|u(\cdot,s)\|_{L^2})ds \\
&\quad + C_\alpha\int_{t^*}^{t} e^{-\delta(t-s)}(t-s)^{-\alpha}(\sqrt{2\pi}+\|u(\cdot,s)\|_{L^2})ds \\
&\le Ce^{-\delta t}\|u_0\|_\alpha + C_\alpha\int_0^{t^*} e^{-\delta(t-s)}(t-s)^{-\alpha}(\sqrt{2\pi}+\|u(\cdot,s)\|_{L^2})ds \\
&\quad + C_\alpha(\sqrt{2\pi}+\pi(\sqrt{\pi}+\varepsilon))\int_0^{\infty} e^{-\delta(t-s)}(t-s)^{-\alpha}ds \\
&\le Ce^{-\delta t}\|u_0\|_\alpha + C_\alpha e^{-\delta t}(\sqrt{2\pi}+\|u_0\|_{L^2})\int_0^{t^*} e^{\delta s}(t-s)^{-\alpha}ds + M_1 \\
&\le e^{-\delta t}\Big(C\|u_0\|_\alpha + C_\alpha(\sqrt{2\pi}+\|u_0\|_{L^2})e^{\delta t^*}(t^*)^{1-\alpha}(1-\alpha)^{-1}\Big) + M_1,
\end{aligned} \tag{3.12}$$

where $M_1 = C_\alpha(\sqrt{2\pi}+\pi(\sqrt{\pi}+\varepsilon))\int_0^\infty e^{-\delta(t-s)}(t-s)^{-\alpha}ds$. From this formula, and the continuous inclusion of X_α in L^2, it is easy to see that one can choose $t_1 > 0$, depending only on the norm of u_0 in X_α, so that

$$\|u(\cdot,t)\|_\alpha \le 2M_1,$$

for all $t \ge t_1$ and, therefore, the semigroup $\{T(t); t \ge 0\}$ is bounded dissipative.

If $t < t^*$ the same estimate (without the last term and with t in the place of t^*) shows that

$$\|u(\cdot,t)\|_\alpha \le e^{-\delta t}C\|u_0\|_\alpha + C_\alpha(\sqrt{2\pi}+\|u_0\|_{L^2})t^{1-\alpha}(1-\alpha)^{-1} \tag{3.13}$$

From 3.12 and 3.13 it follows that orbits of bounded sets are bounded. Since A has compact resolvent and F maps bounded sets in X^α into bounded sets in $X^{-1/2}$, it follows from [3, Theorem 4.2.2] that $T(t)$ is compact for all $t > 0$. The result follows then from [3, Theorem 3.4.6]. □

Remark 3.3. We observe that 3.12 above also gives an estimate for the size of the attractor.

Theorem 3.4. *If $\beta \in (0, 1/\pi)$ and $\alpha \in (1/4, 1/2)$, then $\mathcal{A}_\beta = \{0\}$.*

Proof. Let $\varepsilon > 0$ be given. We will use the estimates obtained in Lemma 3.2 for the decay of the L^2-norm of a solution $u(\cdot,t)$ when $t \in S_1(u_0)$. If $t \in S_2(u_0)$, we have

$$\begin{aligned}
\frac{d}{dt}\frac{1}{2}\|u(\cdot,t)\|_{L^2}^2 &= |\tanh(\beta u(\pi,t))||u(0,t)| - \|u_x(\cdot,t)\|_{L^2}^2 \\
&\le \beta|u(\pi,t))||u(0,t)| - \|u_x(\cdot,t)\|_{L^2}^2 \\
&\le \beta\left(\sqrt{\pi}\|u_x(\cdot,t)\|_{L^2}\right)^2 - \|u_x(\cdot,t)\|_{L^2}^2 \\
&\le (\beta\pi - 1)\|u_x(\cdot,t)\|_{L^2}^2 \\
&\le -\frac{1-\beta\pi}{\pi^2}\|u(t)\|_{L^2}^2
\end{aligned} \tag{3.14}$$

If $\|u(\cdot,t)\|_{L^2} \ge \varepsilon$ and $\varepsilon_2 = \min\left\{\frac{1-\beta\pi}{\pi^2}\varepsilon^2, \frac{\varepsilon^2}{4\pi^2}, \tanh\left(\frac{\beta\varepsilon}{2\pi}\right)\left(\frac{\varepsilon}{2\pi}\right)\right\}$, we obtain using (3.14), (3.6) and (3.8), that

$$\frac{d}{dt}\|u(\cdot,t)\|_{L^2}^2 \le -2\varepsilon_2. \tag{3.15}$$

Suppose $\|u(\cdot,t)\|_{L^2} \geq \varepsilon$ for $0 \leq t \leq \bar{t}$. Then $\|u(\cdot,\bar{t})\|_{L^2}^2 \leq \|u_0\|_{L^2}^2 - 2\varepsilon_2\bar{t}$. Therefore, there must exist a $t^* = t^*(u_0) \leq \frac{1}{2\varepsilon_2}\left(\|u_0\|_{L^2}^2 - \varepsilon^2\right)$ such that $\|u(\cdot,t)\|_{L^2} \leq \varepsilon$ for $t \geq t^*$.

Since the attractor $\mathcal{A}_\beta$ is a bounded subset of L^2, there exists $t^*(\varepsilon)$ such that $\mathcal{A}_\beta = T(t^*)\mathcal{A}_\beta \subset V_\varepsilon$, where V_ε is the ball of radius ε in L^2. Since ε is arbitrary, we conclude that $A_\beta = \{0\}$ as claimed. □

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