ASYMPTOTIC PROFILE OF A RADially SYMMETRIC
SOLUTION WITH TRANSITION LAYERS FOR AN
UNBALANCED BISTABLE EQUATION

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ABSTRACT. In this article, we consider the semilinear elliptic problem
\[-\varepsilon^2 \Delta u = h(|x|)^2(\vert u - a(|x|)\vert)(1 - u^2)\]
in $B_1(0)$ with the Neumann boundary condition. The function $a$ is a $C^1$ function satisfying $|a(x)| < 1$ for $x \in [0,1]$ and $a'(0) = 0$. In particular we consider the case $a(x) = 0$ on some interval $I \subset [0,1]$. The function $h$ is a positive $C^1$ function satisfying $h'(0) = 0$. We investigate an asymptotic profile of the global minimizer corresponding to the energy functional as $\varepsilon \to 0$. We use the variational procedure used in [4] with a few modifications prompted by the presence of the function $h$.

1. Introduction and Statement of Main Results

In this article, we consider the boundary value problem
\[-\varepsilon^2 \Delta u = h(|x|)^2(\vert u - a(|x|)\vert)(1 - u^2) \quad \text{in } B_1(0)\]
\[\frac{\partial u}{\partial n} = 0 \quad \text{on } \partial B_1(0)\]  
(1.1)

where $\varepsilon$ is a small positive parameter, $B_1(0)$ is a unit ball in $\mathbb{R}^N$ centered at the origin, and the function $a$ is a $C^1$ function on $[0, 1]$ satisfying $-1 < a(|x|) < 1$ and $a'(0) = 0$. The function $h$ is a positive $C^1$ function on $[0,1]$ satisfying $h'(0) = 0$. We set $r = |x|$.

Problem (1.1) appears in various models such as population genetics, chemical reactor theory and phase transition phenomena. See [1] and the references therein. If the function $h$ satisfies $h(r) \equiv 1$ and the function $a$ satisfies $a(r) \neq 0$, then this problem (1.1) has been studied in [3], [4] and [7]. In this case, it is shown that there exist radially symmetric solutions with transition layers near the set $\{x \in B_1(0) | a(|x|) = 0\}$. If the set $\{r \in \mathbb{R} | a(r) = 0\}$ contains an interval $I$, then the problem to decide the configuration of transition layer on $I$ is more delicate.

When $N = 1$, if the function $h$ satisfies $h(r) \neq 1$ and the function $a$ satisfies $a(r) \equiv 0$, then problem (1.1) has been studied in [8] and [9]. In this case, it is
shown that there exist stable solutions with transition layers near prescribed local minimum points of $h$.

In this paper, we consider the case where the function $a$ satisfies $a(r) \neq 0$ with $a(r) = 0$ on some interval $I \subset (0, 1)$. We show the minimum point of the function $r^{N-1}h(r)$ on $I$ has very important role to decide the configuration of transition layer on $I$ in this case.

We note that in [4], Dancer and Shusen Yan considered a problem similar to ours. They assume that $N \geq 2$, $h \equiv 1$ and the nonlinear term is $u(u - a|x|)(1 - u)$ satisfying $a(r) = 1/2$ on $I = [l_1, l_2]$ and $a(r) < 1/2$ for $l_1 - r > 0$ small and $a(r) > 1/2$ for $r - l_2 > 0$ small, then a global minimizer of the corresponding functional has a transition layer near the $l_1$, that is, the minimum point of $r^{N-1}$ on $I$ (see [4, Theorem 1.3]). In this sense, we can say that our results are natural extension of the results in [4]. We are going to follow throughout the variational procedure used in [4] with a few modifications prompted by the presence of the function $h$.

Here we state the energy functional, corresponding to (1.1),

$$J_{\varepsilon}(u) = \int_{B_1(0)} \frac{\varepsilon^2}{2} |\nabla u|^2 - F(|x|, u) dx,$$

where $F(|x|, u) = \int_0^{|x|} f(|x|, s) ds$ and $f(|x|, u) = h(|x|)^2(u - a(|x|))(1 - u^2)$. It is easy to see that the following minimization problem has a minimizer

$$\inf\{J_{\varepsilon}(u) | u \in H^1(B_1(0))\}. \tag{1.2}$$

Let $A_- = \{x \in B_1(0)|a(|x|) < 0\}$ and $A_+ = \{x \in B_1(0)|a(|x|) > 0\}$.

In this paper, we will analyze the profile of the minimizer of (1.2), and prove the following results.

**Theorem 1.1.** Let $u_{\varepsilon}$ be a global minimizer of (1.2). Then $u_{\varepsilon}$ is radially symmetric and

$$u_{\varepsilon} \to \begin{cases} 1, & \text{uniformly on each compact subset of } A_-, \\ -1, & \text{uniformly on each compact subset of } A_+, \end{cases}$$

as $\varepsilon \to 0$. In particular $u_{\varepsilon}$ converges uniformly near the boundary of $B_1(0)$, that is, if $a(r) < 0$ on $[r_0, 1]$ for some $r_0 > 0$, $u_{\varepsilon} \to 1$ uniformly on $\overline{B_1(0) \setminus B_{r_0}(0)}$ and if $a(r) > 0$ on $[r_0, 1]$ for some $r_0 > 0$, $u_{\varepsilon} \to -1$ uniformly on $\overline{B_1(0) \setminus B_{r_0}(0)}$.

Moreover, for any $0 < r_1 \leq r_2 < 1$ with $a(r_i) = 0$, $i = 1, 2$, $a(r) \neq 0$ for $r_1 - r > 0$ small and for $r - r_2 > 0$ small, $a(r) = 0$ if $r \in [r_1, r_2]$, we have:

(i) If $a(r) < 0$ for $r_1 - r > 0$ small and $a(r) > 0$ for $r - r_2 > 0$, then for any small $\eta > 0$ and any small $\theta > 0$, there exists a positive number $\varepsilon_0$ which has the following properties:

(a) For all $\varepsilon \in (0, \varepsilon_0)$, there exist $t_{\varepsilon, 1} < t_{\varepsilon, 2}$ such that

$$u_{\varepsilon}(r) > 1 - \eta \quad \text{for } r \in [r_1 - \theta, t_{\varepsilon, 1}),$$

$$u_{\varepsilon}(t_{\varepsilon, 1}) = 1 - \eta,$$

$$u_{\varepsilon}(t_{\varepsilon, 2}) = -1 + \eta,$$

$$u_{\varepsilon}(r) < -1 + \eta \quad \text{for } r \in (t_{\varepsilon, 2}, r_2 + \theta].$$

(b) The function $u_{\varepsilon}(r)$ is decreasing on the interval $(t_{\varepsilon, 1}, t_{\varepsilon, 2})$

(c) The inequality $0 < R_1 \leq \frac{t_{\varepsilon, 2} - t_{\varepsilon, 1}}{\varepsilon} \leq R_2$ holds, where $R_1$ and $R_2$ are two constants independent of $\varepsilon > 0$. 
(d) If \( t_{\varepsilon,1}, t_{\varepsilon,2} \to \bar{t} \) for some positive sequence \( \{\varepsilon_j\} \) converging to zero as \( j \to \infty \), then \( \bar{t} \) satisfies 
\[
 h(\bar{t})\bar{t}^{N-1} = \min_{s \in [r_1, r_2]} h(s)s^{N-1}.
\]

(ii) If \( a(r) > 0 \) for \( r_1 - r > 0 \) small and \( a(r) < 0 \) for \( r - r_2 > 0 \), then for each small \( \eta > 0 \) and for each small \( \theta > 0 \), there exists a positive number \( \varepsilon_0 \) which has the following properties: For each \( \varepsilon \in (0, \varepsilon_0) \), there exist \( t_{\varepsilon,1} < t_{\varepsilon,2} \) such that 

(a) 
\[
 u_\varepsilon(r) < -1 + \eta \quad \text{for } r \in [r_1 - \theta, t_{\varepsilon,1}),
 u_\varepsilon(t_{\varepsilon,1}) = -1 + \eta,
 u_\varepsilon(t_{\varepsilon,2}) = 1 - \eta,
 u_\varepsilon(r) > 1 - \eta, \quad \text{for } r \in (t_{\varepsilon,2}, r_2 + \theta] .
\]

(b) The function \( u_\varepsilon(r) \) is increasing in \( (t_{\varepsilon,1}, t_{\varepsilon,2}) \).

(c) The inequality \( 0 < R_1 \leq \frac{t_{\varepsilon,2} - t_{\varepsilon,1}}{\varepsilon} \leq R_2 \) holds, where \( R_1 \) and \( R_2 \) are two constants independent of \( \varepsilon > 0 \).

(d) If \( t_{\varepsilon,1}, t_{\varepsilon,2} \to \bar{t} \) for some positive sequence \( \{\varepsilon_j\} \) converging to zero as \( j \to \infty \), then \( \bar{t} \) satisfies 
\[
 h(\bar{t})\bar{t}^{N-1} = \min_{s \in [r_1, r_2]} h(s)s^{N-1}.
\]

Figure 1. Profile of the global minimizer \( u_\varepsilon \)

Remarks.

- Note that results from (a) to (c) both in cases (i) and (ii) are not related to the presence of the function \( h \). The effect of presence of function \( h \) appears in the result (d) in (i) and (ii).
- If \( \min_{s \in [r_1, r_2]} s^{N-1}h(s) \) is attained at a unique point \( \bar{t} \), we can show \( t_{\varepsilon,1}, t_{\varepsilon,2} \to \bar{t} \) as \( \varepsilon \to 0 \) without taking subsequences.
- If the function \( r^{N-1}h(r) \) is constant on \( [r_1, r_2] \), it is a very difficult problem to know the location of the point \( \bar{t} \in [r_1, r_2] \).

This paper is organized as follows: In section 2, we present some preliminary results. In section 3, we prove the main theorem.

2. Preliminary Results

Let \( D \) be a bounded domain in \( \mathbb{R}^N \). Let \( \bar{f}(x, t) \) be a function defined on \( \overline{D} \times \mathbb{R} \) which is bounded on \( \overline{D} \times [-1, 1] \). Suppose \( \bar{f} \) is continuous on \( t \in \mathbb{R} \) for each \( x \in \overline{D} \).
and is measurable in $D$ for each $t \in \mathbb{R}$. We also assume
\[
\mathcal{J}(x, t) > 0 \quad \text{for } x \in \overline{D}, \ t < -1;
\]
\[
\mathcal{J}(x, t) < 0 \quad \text{for } x \in \overline{D}, \ t > 1.
\] (2.1)

Consider the minimization problem
\[
\inf \left\{ \mathcal{J}_\epsilon(u, D) := \int_D \frac{\epsilon^2}{2} |\nabla u|^2 - \mathcal{F}(x, u) \, dx : u - \eta \in H^1_0(D) \right\},
\] (2.2)
where $\eta \in H^1(D)$ with $-1 \leq \eta \leq 1$ on $D$ and
\[
\mathcal{F}(x, t) = \int_{-1}^t \mathcal{J}(x, s) \, ds.
\]
We can prove next two lemmas by methods similar to [4]. For the readers convenience, we prove these lemmas in this section.

**Lemma 2.1.** Suppose that $\mathcal{J}(x, t)$ satisfies (2.1). Let $u_\epsilon$ be a minimizer of (2.2). Then $-1 \leq u_\epsilon \leq 1$ on $D$.

**Proof.** We prove $-1 \leq u_\epsilon$ on $D$. Let $M = \{ x : u_\epsilon(x) < -1 \}$. Define $\tilde{u}_\epsilon$ by
\[
\tilde{u}_\epsilon(x) = \begin{cases} 
    u_\epsilon(x) & \text{if } x \in D \setminus M \\
    -1 & \text{if } x \in M.
\end{cases}
\]
Since $u_\epsilon(x) = \eta \geq -1$ on $\partial D$, we see that $M$ is compactly contained in $D$. Thus $\tilde{u} - \eta \in H^1_0(D)$. If the measure $m(M)$ of $M$ is positive, we have $\mathcal{J}_\epsilon(\tilde{u}_\epsilon, D) < \mathcal{J}_\epsilon(u_\epsilon, D)$. Because $u_\epsilon$ is a minimizer, we see $m(M) = 0$, where $m(A)$ denotes the Lebesgue measure of the set $A$. Thus $u_\epsilon \geq -1$. Similarly we can prove that $u_\epsilon \leq 1$. \hfill \Box

**Lemma 2.2.** Suppose that $\mathcal{J}_1(x, t)$ and $\mathcal{J}_2(x, t)$ both satisfy (2.1) and the same regularity assumption on $\mathcal{J}$. Assume that $\eta_i \in H^1(D)$ satisfy $-1 \leq \eta_i \leq 1$ on $D$ for $i = 1, 2$. Let $u_{\epsilon,i}$ be a corresponding minimizer of (2.2), where $\mathcal{J} = \mathcal{J}_i$ and $\eta = \eta_i$, $i = 1, 2$. Suppose that $\mathcal{J}_1(x, t) \geq \mathcal{J}_2(x, t)$ for all $(x, t) \in D \times [-1, 1]$ and $1 \geq \eta_1 \geq \eta_2 \geq -1$. Then $u_{\epsilon,1} \geq u_{\epsilon,2}$.

**Proof.** Let $M = \{ x \in D : u_{\epsilon,2} > u_{\epsilon,1} \}$. Define $\varphi_\epsilon = (u_{\epsilon,2} - u_{\epsilon,1})^+$. Since $\eta_1 \geq \eta_2$, we have $\varphi_\epsilon \in H^1_0(D)$. Set $\tilde{F}_i(x, u) = \int_{-1}^u \mathcal{J}_i(x, s) \, ds$. Since $u_{\epsilon,i}$ is a minimizer of

\[
J_{\epsilon,i}(u) := \int_D \frac{\epsilon^2}{2} |\nabla u|^2 - \mathcal{F}_i(x, u) \, dx
\]
and $\varphi_\epsilon = 0$ for $x \in D \setminus M$, we have
\[
0 \leq J_{\epsilon,1}(u_{\epsilon,1} + \varphi_\epsilon) - J_{\epsilon,1}(u_{\epsilon,1})
= \int_M \frac{\epsilon^2}{2} (|\nabla (u_{\epsilon,1} + \varphi_\epsilon)|^2 - |\nabla u_{\epsilon,1}|^2) \, dx - \int_M \int_{u_{\epsilon,1}}^{u_{\epsilon,1} + \varphi_\epsilon} \mathcal{J}_1(x, s) \, ds
\leq \int_M \frac{\epsilon^2}{2} (|\nabla (u_{\epsilon,1} + \varphi_\epsilon)|^2 - |\nabla u_{\epsilon,1}|^2) \, dx - \int_M \int_{u_{\epsilon,1}}^{u_{\epsilon,1} + \varphi_\epsilon} \mathcal{J}_2(x, s) \, ds
= J_{\epsilon,2}(u_{\epsilon,2}) - J_{\epsilon,2}(u_{\epsilon,2} - \varphi_\epsilon) \leq 0.
\]
This implies that \( u_{\varepsilon,1} + \varphi_{\varepsilon} \) is also a minimizer of \( J_{\varepsilon,1}(u) \). Let \( L > 0 \) be large enough such that \( \tilde{f}_1(x,t) + Lt \) is strictly increasing for \( x \in D, \ t \in [-1,1] \). From

\[-\varepsilon^2 \Delta(u_{\varepsilon,1} + \varphi_{\varepsilon}) = \tilde{f}_1(u_{\varepsilon,1} + \varphi_{\varepsilon}),\]

we obtain

\[-\varepsilon^2 \Delta \varphi_{\varepsilon} = \tilde{f}_1(u_{\varepsilon,1} + \varphi_{\varepsilon}) - \tilde{f}_1(u_{\varepsilon,1}).\]

Thus

\[-\varepsilon^2 \Delta \varphi_{\varepsilon} + L \varphi_{\varepsilon} = \tilde{f}_1(u_{\varepsilon,1} + \varphi_{\varepsilon}) + L(u_{\varepsilon,1} + \varphi_{\varepsilon}) - (\tilde{f}_1(u_{\varepsilon,1}) + Lu_{\varepsilon,1}) > 0\]

in \( D \). Fix \( z_0 \in M \). Let \( x_0 \in \partial M \) such that \( |x_0 - z_0| = \text{dist}(z_0, \partial M) \). Using the Strong maximum principle and Hopf’s lemma in \( B_{\text{dist}(z_0, \partial M)}(z_0) \), we obtain that \( \frac{\partial \varphi_{\varepsilon}}{\partial \nu}(x_0) < 0 \), where \( \nu = (x_0 - z_0)/|x_0 - z_0| \). But \( \varphi_{\varepsilon}(x) = 0 \) for \( x \notin M \). Thus, \( \frac{\partial \varphi_{\varepsilon}}{\partial \nu}(x_0) = 0 \). This is a contradiction. Thus we obtain \( M = \emptyset \).

3. Proof of Main Theorem

To prove Theorem 1.1, the following proposition is used as the first step.

**Proposition 3.1.** Let \( u_{\varepsilon} \) be a global minimizer of the problem \([1.2]\). Then \( u_{\varepsilon} \) satisfies

\[ u_{\varepsilon} \rightarrow \begin{cases} 1 & \text{uniformly on each compact subset of } A_- \\ -1 & \text{uniformly on each compact subset of } A_+ \end{cases} \]
as \( \varepsilon \to 0 \).

**Proof.** Let \( x_0 \in A_- \). Choose \( \delta > 0 \) small so that \( B_\delta(x_0) \subset A \). Take \( b \in (\max_{x \in B_\delta(x_0)} a(z), 1/2) \). Define \( f_{x_0,\delta,b}(t) = (\min_{z \in B_\delta(x_0)} h(|z|)^2)(t - b)(1 - t^2) \).

Then for \( x \in B_\delta(x_0), t \in [-1,1] \), we have \( f(|x|,t) \geq f_{x_0,\delta,b}(t) \). Let \( u_{\varepsilon,x_0,\delta,b} \) be the minimizer of

\[ \inf \left\{ \int_{B_\delta(x_0)} \frac{\varepsilon^2}{2} |\nabla u|^2 - F_{x_0,\delta,b}(u) dx : u \in H^1_0(B_\delta(x_0)) \right\}, \]

where \( F_{x_0,\delta,b}(t) = \int_{-1}^{t} f_{x_0,\delta,b}(s) ds \). It follows from Lemmas 2.1 and 2.2 that

\[ u_{\varepsilon,x_0,\delta,b}(x) \leq u_{\varepsilon}(x) \leq 1, \quad \text{for } x \in B_\delta(x_0). \]

Since \( \int_{-1}^{1} f_{x_0,\delta,b}(s) ds > 0 \), it follows from \([2,3]\) that \( u_{\varepsilon,x_0,\delta,b}(x) \to 1 \) as \( \varepsilon \to 0 \) uniformly in \( B_{\delta/2}(x_0) \), thus \( u_{\varepsilon}(x) \to 1 \) as \( \varepsilon \to 0 \) uniformly in \( B_{\delta/2}(x_0) \).

To prove the rest of Theorem 1.1 we need the following proposition and lemma.

**Proposition 3.2.** Let \( u \) be a local minimizer of the problem

\[ \inf \left\{ \int_{B_1(0)} \frac{1}{2} |\nabla u|^2 - G(|x|,u) dx : u \in H^1(B_1(0)) \right\}. \]

Here \( G(r,t) = \int_{-1}^{t} g(r,s) ds \), \( g(r,t) \) is \( C^1 \) in \( t \in \mathbb{R} \) for each \( r \geq 0 \), \( g(r,t) \) and \( q_t(r,t) \) are measurable on \([0,\infty)\) for each \( t \in \mathbb{R} \), and \( g_t(r,t) < 0 \) if \( t < -1 \) or \( t > 1 \) and \( |g(r,t)| + |q_t(r,t)| \) is bounded on \([0,k] \times [-2,2] \) for any \( k > 0 \). Then \( u \) is radial, i.e., \( u(x) = u(|x|) \).

The proof of the above proposition can be found in \([4]\) Proposition 2.6].
Lemma 3.3. Let $0 < \eta < 1$ be any fixed constant and $w$ satisfies
\[ -w_{zz} = w(1 - w^2) \quad \text{on } \mathbb{R}, \]
\[ w(0) = -1 + \eta \quad (\text{resp. } w(0) = 1 - \eta), \]
\[ w(z) \leq -1 + \eta \quad (\text{resp. } w(z) \geq 1 - \eta) \quad \text{for } z \leq 0, \]
\[ w \text{ is bounded on } \mathbb{R}. \]

Then $w$ is a unique solution of
\[ -w_{zz} = w(1 - w^2) \quad \text{on } \mathbb{R}, \]
\[ w(0) = -1 + \eta \quad (\text{resp. } w(0) = 1 - \eta), \]
\[ w'(z) > 0 \quad (\text{resp. } w'(z) < 0) \quad z \in \mathbb{R}, \]
\[ w(z) \to \pm 1 \quad (\text{resp. } w(z) \to \mp 1) \quad \text{as } z \to \pm \infty. \]

The proof of the above lemma can be found in [6]. Now we prove the rest of Theorem 1.1.

Proof of Theorem 1.1. For the sake of simplicity, we prove for the case where $a(r) < 0$ on $[0, r_1)$, $a(r) = 0$ on $[r_1, r_2]$ and $a(r) > 0$ on $(r_2, 1]$ for some $0 < r_1 < r_2 < 1$ (see Figure 1 in Section 1).

Part 1. First we show that $u_\varepsilon$ converges uniformly near the boundary of $B_1(0)$, that is, $u_\varepsilon \to -1$ uniformly on $B_1(0) \setminus B_{r_2 + \varepsilon}(0)$ for any small $\tau > 0$. We note that we have $u_\varepsilon \to -1$ uniformly on $B_{1 - \tau}(0) \setminus B_{r_2 + \varepsilon}(0)$ as $\varepsilon \to 0$. Now we claim that $u_\varepsilon(r) \leq u_\varepsilon(1 - \tau) =: T_\varepsilon$ for $r \in [1 - \tau, 1]$. We define the function $\bar{u}_\varepsilon$ by

\[
\bar{u}_\varepsilon(r) = \begin{cases} u_\varepsilon(r) & \text{if } r \in [0, 1 - \tau] \\
 u_\varepsilon(r) & \text{if } u_\varepsilon(r) < T_\varepsilon \text{ and } r \in [1 - \tau, 1], \\
 T_\varepsilon & \text{if } u_\varepsilon(r) \geq T_\varepsilon \text{ and } r \in [1 - \tau, 1]. \end{cases}
\]

We note that $\bar{u}_\varepsilon \in H^1(B_1(0))$ and $-F(r, T_\varepsilon) \leq -F(r, t)$ for $\varepsilon > 0$ and $|r - 1|$ small and $t \geq T_\varepsilon$. Hence we obtain $J_\varepsilon(\bar{u}_\varepsilon) < J_\varepsilon(u_\varepsilon)$ and we have a contradiction if we assume that the measure of the set $\{r \in [0, 1]|u_\varepsilon(r) > T_\varepsilon \text{ and } r \in [1 - \tau, 1]\}$ is positive. Hence $-1 < u_\varepsilon(r) \leq T_\varepsilon$ and $u_\varepsilon \to -1$ uniformly on $B_1(0) \setminus B_{r_2 + \varepsilon}(0)$.

Part 2. We remark that, by Proposition 3.1, $u_\varepsilon$ is radially symmetric and we note that for any $t_2 > t_1$, $u_\varepsilon$ is a minimizer of the following problem

\[
\inf \{ J_\varepsilon(u, B_2(0) \setminus B_1(0)) : u - u_\varepsilon \in H^1_0(B_2(0) \setminus B_1(0)) \},
\]

where

\[
J_\varepsilon(u, M) = \int_M \frac{\varepsilon^2}{2} |\nabla u|^2 - F(|x|, u)\,dx
\]

for any open set $M$. Let $m_{\varepsilon, t_1, t_2}$ be the minimum value of this minimization problem.

In this part we show that $u_\varepsilon$ has exactly one layer near the interval $[r_1, r_2]$.

Step 2.1. First we estimate the energy of transition layer. Let $\eta > 0$ and $\theta > 0$ be small numbers. Since $u_\varepsilon \to 1$ uniformly on $[0, r_1 - \theta]$ and $u_\varepsilon \to -1$ uniformly on $[r_2 + \theta, 1 - \theta]$, we can find $r_\varepsilon \in (r_1 - \theta, r_2 + \theta)$ such that $u_\varepsilon(r_\varepsilon) \geq 1 - \eta$ if $r \in [0, r_\varepsilon]$, $u_\varepsilon(r_\varepsilon) < 1 - \eta$ for $r - r_\varepsilon > 0$ small. Let $\tilde{r}_\varepsilon > r_\varepsilon$ be such that $u_\varepsilon(r_\varepsilon) \leq \eta$ if $r \in [\tilde{r}_\varepsilon, 1 - \theta]$, $u_\varepsilon(r_\varepsilon) > \eta$ for $\tilde{r}_\varepsilon - r > 0$ small. We may assume that $r_\varepsilon \to r \in [r_1, r_2]$ and $\tilde{r}_\varepsilon \to \tilde{r} \in [r_1, r_2]$. 

We employ the so-called blow-up argument. Let \( v_\varepsilon(t) = u_\varepsilon(\varepsilon t + \tau_\varepsilon) \). Then

\[
-\varepsilon^N v''_\varepsilon - \varepsilon^{N-1} \frac{N-1}{\varepsilon t + \tau_\varepsilon} v'_\varepsilon = f(\varepsilon t + \tau_\varepsilon, v_\varepsilon),
\]

\(-1 \leq v_\varepsilon \leq 1\) and \( v_\varepsilon(0) = 1 - \eta \). Since \( \tau_\varepsilon \rightarrow \tau \in [r_1, r_2] \), it is easy to see that \( v_\varepsilon \rightarrow v \) in \( C^1_{\text{loc}}(\mathbb{R}) \) and

\[
-v'' = h(\tau)^2 (v - v^3), \quad t \in \mathbb{R},
\]

and \( v(t) \geq 1 - \eta \) for \( t \leq 0 \). If we set \( v(t) = V(h(\tau)t) \), the function \( V(t) \) satisfies

\[
-V'' = V - V^3 \quad \text{on } \mathbb{R},
\]

\[
V(0) = 1 - \eta, \quad V'(t) \geq 1 - \eta \quad t \leq 0.
\]

Hence by Lemma 3.3, the function \( V \) is a unique solution for

\[
-V'' = V - V^3 \quad \text{on } \mathbb{R},
\]

\[
V(0) = 1 - \eta, \quad V'(t) < 0 \quad t \leq 0.
\]

\[
V(t) \rightarrow \pm 1 \quad \text{as } t \rightarrow \mp \infty.
\]

Thus, we can find an \( R > 0 \) large, such that \( v(R) = \eta \). Since \( v_\varepsilon \rightarrow v \) in \( C^1_{\text{loc}}(\mathbb{R}) \), we can find an \( R_\varepsilon \in (R-1, R+1) \), such that \( v'_\varepsilon(r) < 0 \) if \( r \in [0, R_\varepsilon] \) and \( v_\varepsilon(R_\varepsilon) = -1+\eta \). Hence \( u'_\varepsilon(r) < 0 \) if \( r \in [\tau_\varepsilon, \tau_\varepsilon + \varepsilon R_\varepsilon] \) and \( u_\varepsilon(\tau_\varepsilon + \varepsilon R_\varepsilon) = -1+\eta \). Then we have

\[
J_\varepsilon(u_\varepsilon, B_{\tau_\varepsilon + \varepsilon R_\varepsilon}(0)\backslash \overline{B_{\tau_\varepsilon}(0)})
\]

\[
= \omega_{N-1}(\tau_\varepsilon N^{-1} + o_\varepsilon(1)) \int_{\tau_\varepsilon}^{\tau_\varepsilon + \varepsilon R_\varepsilon} \left( \frac{\varepsilon^2}{2} |u'_\varepsilon|^2 - F(t, u_\varepsilon) \right) dt
\]

\[
= \omega_{N-1}(\tau_\varepsilon N^{-1} + o_\varepsilon(1))\varepsilon \int_0^{R_\varepsilon} \left( \frac{1}{2} |u'_\varepsilon|^2 - F(\varepsilon t + \tau_\varepsilon, v_\varepsilon) \right) dt
\]

\[
= \omega_{N-1}(\tau_\varepsilon N^{-1} + o_\varepsilon(1))(\beta_{h(\tau)} + O(\eta) + o_\varepsilon(1))\varepsilon,
\]

where \( \omega_{N-1} \) is the area of the unit sphere in \( \mathbb{R}^N \), \( o_\varepsilon(1) \rightarrow 0 \) as \( \varepsilon \rightarrow 0 \), \( \beta_{h(s)} \) is the positive value defined by

\[
\beta_{h(s)} = \int_{-\infty}^{+\infty} \left( \frac{1}{2} |w'h(s)(t)|^2 + h(s)^2 \frac{(w^2 h(s) - 1)^2}{4} \right) dt
\]

\[
= h(s) \int_{-\infty}^{+\infty} \frac{1}{2} |V'(t)|^2 + \frac{(V(t)^2 - 1)^2}{4} dt
\]

\[
= h(s) \beta_1,
\]

and \( w_{h(s)}(t) = V(h(s)t) \) for \( s \in [0, 1] \). We note that although the function \( V \) depends on \( \eta \), the value

\[
\beta_1 = \int_{-\infty}^{+\infty} \frac{1}{2} |V'(t)|^2 + \frac{(V(t)^2 - 1)^2}{4} dt
\]

is independent of \( \eta \).

**Step 2.2.** We claim \( u_\varepsilon \) has exactly one layer near the interval \([r_1, r_2]\). To show \( u_\varepsilon \) has exactly one layer near the interval \([r_1, r_2]\), it sufficient to prove the following claim

**Claim.** \( \tilde{r}_\varepsilon = \tau_\varepsilon + \varepsilon R_\varepsilon \).
Suppose that the claim is not true. Then we can find a \( t_\varepsilon > \tau_\varepsilon + R_\varepsilon \eta \) such that \( u_\varepsilon (r) < -1 + \eta \) if \( r \in (\tau_\varepsilon + R_\varepsilon \eta, t_\varepsilon) \), \( u_\varepsilon (t_\varepsilon) = -1 + \eta \). Thus we can use the blow-up argument again at \( t_\varepsilon \) to deduce that there is a \( \hat{t}_\varepsilon = t_\varepsilon + \varepsilon \tilde{R} \tilde{e} \) with \( u'_\varepsilon (r) > 0 \) if \( r \in (t_\varepsilon, \hat{t}_\varepsilon) \), \( u_\varepsilon (\hat{t}_\varepsilon) = 1 - \eta \). We may assume that \( t_\varepsilon, \hat{t}_\varepsilon \to \tilde{t} \) as \( \varepsilon \to 0 \) for some \( \tilde{t} \in [r_2, r_3] \). Moreover
\[
J_\varepsilon (u_\varepsilon, B_{t_\varepsilon} (0) \setminus \overline{B}_{\hat{t}_\varepsilon} (0)) = \omega_{N-1} (t_\varepsilon^{N-1} + o_\varepsilon (1)) (\beta_h (\tau) + O (\eta)) \varepsilon + o_\varepsilon (1) \tag{3.4}
\]
Now we claim \( \hat{t}_\varepsilon \geq r_1 \). Suppose \( \hat{t}_\varepsilon < r_1 \). Let \( F_a (t) = \int_{t-1}^{t} (v - a) (1 - v^2) dv \). Then for any \( t > 0 \) small and \( s \in [-1 + t, 1 - t] \),
\[
F_a (1 - t) - F_a (s) = F_0 (1 - t) - F_0 (s) + F_a (1 - t) - F_0 (1 - t) - F_a (s) + F_0 (s) 
= \left( \frac{v^2 - 1}{4} \right)_{s}^{1-t} - a \int_{s}^{1-t} (1 - v^2) dv 
\tag{3.5}
\]
Thus it follows from \( \text{[3.5]} \) that if \( a < 0 \), then
\[
F_a (1 - t) - F_a (s) > 0 \tag{3.6}
\]
for \( s \in [-1 + t, 1 - t] \). Define
\[
\overline{\nu}_\varepsilon (r) \equiv \begin{cases} \dfrac{1 - \eta}{r - \tau_\varepsilon} & r \in [\tau_\varepsilon, \tau_\varepsilon + R_\varepsilon \varepsilon] \cup [t_\varepsilon, \hat{t}_\varepsilon], \\ -u_\varepsilon (r) & r \in [\tau_\varepsilon + R_\varepsilon, \tau_\varepsilon]. \end{cases}
\]
By the assumption that \( \hat{t}_\varepsilon < r_1 \) and using \( \text{[3.6]} \), we see \( F (r, u_\varepsilon) < F (r, \overline{\nu}_\varepsilon) \) if \( r \in [\tau_\varepsilon, \hat{t}_\varepsilon] \). Hence, we obtain
\[
J_\varepsilon (\overline{\nu}_\varepsilon, B_{\hat{t}_\varepsilon} (0) \setminus \overline{B}_{\tau_\varepsilon} (0)) < J_\varepsilon (u_\varepsilon, B_{t_\varepsilon} (0) \setminus \overline{B}_{\tau_\varepsilon} (0)).
\]
Thus we obtain a contradiction. Therefore we have that \( \hat{t}_\varepsilon \geq r_1 \).

Since \( a(r) \geq 0 \) for \( r \in [r_1, 1] \), we see \( F (r, t) \leq F (r, -1) = 0 \) if \( r \in [r_1, 1] \). Since \( u_\varepsilon (r) \in (-1, -1 + \eta) \) for \( r \in [\tau_\varepsilon + R_\varepsilon, t_\varepsilon] \), we have
\[
m_{\tau_\varepsilon, \overline{\nu}_\varepsilon, \tilde{t}_\varepsilon} = J_\varepsilon (\overline{\nu}_\varepsilon, B_{\tau_\varepsilon + R_\varepsilon} (0) \setminus \overline{B}_{\hat{t}_\varepsilon} (0)) + J_\varepsilon (\overline{\nu}_\varepsilon, B_{t_\varepsilon} (0) \setminus \overline{B}_{\hat{t}_\varepsilon} (0)) 
+ J_\varepsilon (\overline{\nu}_\varepsilon, B_{t_\varepsilon} (0) \setminus \overline{B}_{\tau_\varepsilon} (0)) + J_\varepsilon (\overline{\nu}_\varepsilon, B_{\hat{t}_\varepsilon} (0) \setminus \overline{B}_{\tau_\varepsilon} (0)) 
\geq \omega_{N-1} (t_\varepsilon^{N-1} - \beta_h (\tau) \varepsilon + t_\varepsilon^{N-1} \beta_h (\tau) \varepsilon) + O (\eta \varepsilon) + o (\varepsilon)
+ \inf \left\{ - \int_{B_{\tau_\varepsilon} (0) \setminus \overline{B}_{\hat{t}_\varepsilon} (0)} F (r, w) : -1 \leq w \leq -1 + \eta \right\} \tag{3.7}
+ \inf \left\{ - \int_{B_{\hat{t}_\varepsilon} (0) \setminus \overline{B}_{\tau_\varepsilon} (0)} F (r, w) : -1 \leq w \leq 1 \right\}
\geq \omega_{N-1} (t_\varepsilon^{N-1} - \beta_h (\tau) \varepsilon + t_\varepsilon^{N-1} \beta_h (\tau) \varepsilon) + O (\eta \varepsilon) + o (\varepsilon)
\]
Now we give an upper bound for \( m_{\tau_\varepsilon, \overline{\nu}_\varepsilon, \hat{t}_\varepsilon} \). Let \( R > 0 \) be such that \( V (h (\tau) R) = \eta \), where \( V \) is a unique solution to \( \text{[3.2]} \). Define \( \overline{\nu}_\varepsilon \) by
\[
\overline{\nu}_\varepsilon (r) \equiv \begin{cases} \dfrac{V (h (\tau)) - \tau_\varepsilon}{r - \tau_\varepsilon} & r \in [\tau_\varepsilon, \tau_\varepsilon + R_\varepsilon] \\ -1 + \eta - \dfrac{2}{\varepsilon} (r - \tau_\varepsilon - \varepsilon R) & r \in [\tau_\varepsilon + \varepsilon R, \tau_\varepsilon + \varepsilon R + \varepsilon + \tilde{R}_\varepsilon - \varepsilon] \\ -1 & r \in [\tau_\varepsilon + \varepsilon R + \varepsilon, \tilde{R}_\varepsilon] \\ -1 + \dfrac{2}{\varepsilon} (r - \tilde{R}_\varepsilon + \varepsilon) & r \in [\tilde{R}_\varepsilon - \varepsilon, \tilde{R}_\varepsilon] \end{cases} \tag{3.8}
\]
Now we note that $|F(r,t)| = O(\eta)$ for $r \in [r_\varepsilon, \bar{r}_\varepsilon]$ and $-1 \leq t \leq -1 + \eta$. Then we have

$$m_{r_\varepsilon, r_\varepsilon, \bar{r}_\varepsilon} \leq J_\varepsilon(\pi_\varepsilon, B_r(0) \setminus \partial B_{r'}(0))$$

$$\leq J_\varepsilon(\pi_\varepsilon, B_{r_\varepsilon+R}(0) \setminus \partial B_{r'}(0)) + J_\varepsilon(\pi_\varepsilon, B_{r_\varepsilon}(0) \setminus \partial B_{r'-\varepsilon}(0))$$

$$+ J_\varepsilon(\pi_\varepsilon, B_{r_\varepsilon-\varepsilon}(0) \setminus \partial B_{r'}(0))$$

$$\leq \omega_{N-1} \pi_\varepsilon^{N-1} \beta_{h(r')} + O(\eta)\varepsilon + o(\varepsilon) + O(\varepsilon\eta) + o(\varepsilon)$$

(3.9)

By (3.7) and (3.9), we have

$$\omega_{N-1} \pi_\varepsilon^{N-1} \beta_{h(r')} + t_\varepsilon^{N-1} \beta_{h(r)}\varepsilon \leq \omega_{N-1} \pi_\varepsilon^{N-1} \beta_{h(r)}\varepsilon + O(\varepsilon\eta) + o(\varepsilon)$$

This is a contradiction. So we can conclude $\bar{r}_\varepsilon = r_\varepsilon + \varepsilon R_\varepsilon$.

**Part 3.** It remains to prove that if $r_j \to r$ for some positive sequence $\{\varepsilon_j\}$ converging to zero as $j \to \infty$ then $r$ satisfies

$$\pi^{N-1} h(r) = \min_{s \in [r_1, r_2]} s^{N-1} h(s).$$

**Step 3.1.** First we note that from Part 1, the function $u_\varepsilon$ satisfies $-1 \leq u_\varepsilon \leq -1 + \eta$ for $r \in [r_\varepsilon + \eta R_\varepsilon, 1]$ in this case.

**Step 3.2.** Set $H(s) = s^{N-1} h(s)$. Assume that the result is not true. Then there exists a subsequence of $\{r_\varepsilon\}$ (denoted by $r_\varepsilon$) such that $r_\varepsilon \to r' \in [r_1, r_2]$ and $H(r') > \min_{u \in [r_1, r_2]} H(u)$. Then we can find a point $\bar{r} \in (r_1, r_2)$ such that $H(\bar{r}) = H(r') > H(\bar{r})$.

Now we give a lower estimate for $J_\varepsilon(u_\varepsilon)$. We have

$$J_\varepsilon(u_\varepsilon) = J_\varepsilon(u_\varepsilon, B_{r_\varepsilon}(0)) + J_\varepsilon(u_\varepsilon, B_{r_\varepsilon+\varepsilon R_\varepsilon}(0) \setminus \partial B_{r'}(0)) + J_\varepsilon(u_\varepsilon, B_1(0) \setminus \partial B_{r_\varepsilon+\varepsilon R_\varepsilon}(0)).$$

(3.10)

First we note that $1 - \eta \leq u_\varepsilon(r) \leq 1$ for $r \leq r_\varepsilon$ and for sufficiently small $\eta > 0$, $-F(r, u) \geq -F(r, 1) (u \in [1 - \eta, 1])$. We also remark that since $a(r) < 0$ for $r < r_1$ and $a(r) = 0$ for $r_1 \leq r \leq r_2$, we have $-F(r, 1) < 0$ for $r < r_1$ and $-F(r, 1) = 0$ for $r_1 \leq r \leq r_2$ and $-F(r, 1) > 0$ for $r > r_2$. Hence we have $-\int_{r_1}^{r_\varepsilon} r^{N-1} F(r, 1) dr \geq 0$ and we obtain the estimate

$$J_\varepsilon(u_\varepsilon, B_{r_\varepsilon}(0)) \geq - \int_0^{r_\varepsilon} r^{N-1} F(r, u_\varepsilon) dr$$

$$\geq - \int_0^{r_\varepsilon} r^{N-1} F(r, 1) dr$$

$$= - \int_0^{r_1} r^{N-1} F(r, 1) dr - \int_{r_1}^{r_\varepsilon} r^{N-1} F(r, 1) dr$$

$$\geq - \int_0^{r_1} r^{N-1} F(r, 1) dr =: A.$$  

(3.11)

Using methods similar to those in the proof of (3.3), we obtain

$$J_\varepsilon(u_\varepsilon, B_{r_\varepsilon+\varepsilon R_\varepsilon}(0) \setminus \partial B_{r'}(0)) \geq \omega_{N-1} H(r') \varepsilon + O(\varepsilon\eta) + o(\varepsilon).$$

(3.12)
Since \(-1 \leq u_\varepsilon(r) \leq -1 + \eta\) for \(r \geq \tau_\varepsilon + \varepsilon R_\varepsilon\) and for sufficiently small \(\eta > 0\), 
\(-F(r, u) \geq -F(r, -1) = 0\) \((u \in [-1, -1 + \eta])\), we obtain the estimate

\[
J_\varepsilon(u_\varepsilon, B_1(0) \setminus B_{\tau_\varepsilon + R_\varepsilon}(0)) \geq - \int_{\tau_\varepsilon + \varepsilon R_\varepsilon}^{1} r^{N-1} F(r, u_\varepsilon) dr \\
\geq - \int_{\tau_\varepsilon + \varepsilon R_\varepsilon}^{1} r^{N-1} F(r, -1) dr = 0.
\] (3.13)

Thus we obtain

\[
J(u_\varepsilon) \geq A + \omega_{N-1} H(t') \beta_1 \varepsilon + O(\eta \varepsilon) + o(\varepsilon) .
\] (3.14)

Next we give an upper bound for \(J_\varepsilon(u_\varepsilon)\). Consider the function

\[
\bar{w}_\varepsilon(r) := \begin{cases} 
1 & r \in [0, \bar{t} - \varepsilon] \\
1 - \frac{2}{\varepsilon}(r - \bar{t} + \varepsilon) & r \in [\bar{t} - \varepsilon, \bar{t}]
V(h(\bar{t}) \frac{r - \bar{t} + \varepsilon}{\varepsilon}) & r \in [\bar{t}, \bar{t} + \varepsilon R'] \\
-1 - \frac{2}{\varepsilon}(r - \bar{t} - \varepsilon R' - \varepsilon) & r \in [\bar{t} + \varepsilon R', \bar{t} + \varepsilon R' + \varepsilon] \\
-1 & r \in [\bar{t} + \varepsilon R' + \varepsilon, 1],
\end{cases}
\]

where \(R' > 0\) is the number satisfying \(V(h(\bar{t}) R') = -1 + \eta\). Then

\[
J_\varepsilon(u_\varepsilon) \leq J_\varepsilon(\bar{w}_\varepsilon) \leq A + \omega_{N-1} H(\bar{t}) \beta_1 \varepsilon + O(\eta \varepsilon) + o(\varepsilon). 
\] (3.15)

By (3.14) and (3.15) we have a contradiction. The proof of Theorem 1.1 is complete.

The more complicate case, can be shown by a similar method (see Remark below).

\[\square\]

Remark. We briefly show the more complicate case, that is, when \(a\) is the function as in Figure 2. More precisely we set \(I_1 := [r_1, r_2]\) and \(I_2 := [r_3, r_4]\) and we assume \(a > 0\) on \([0, r_1) \cup (r_4, 1]\) and \(a < 0\) on \((r_3, r_4)\).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Special case of coefficient \(a(t)\)}
\end{figure}

Let \(\eta > 0\) and \(\theta > 0\) be small numbers. As in Part 1, we can find pairs of numbers \((\tau_{1,\varepsilon}, \tau_{2,\varepsilon})\) and \((R_{1,\varepsilon}, R_{2,\varepsilon})\) satisfying \(\tau_{1,\varepsilon} \in (r_1 - \theta, r_2 + \theta), \tau_{2,\varepsilon} \in (r_3 - \theta, r_4 + \theta)\),
sup \varepsilon |R_{1,\varepsilon}| < \infty, \sup \varepsilon |R_{2,\varepsilon}| < \infty and
\begin{align*}
  u_\varepsilon(r) &< -1 + \eta \quad \text{for } 0 < r < \tau_{1,\varepsilon} \\
  u_\varepsilon(\tau_{1,\varepsilon}) &= -1 + \eta \\
  u_\varepsilon(\tau_{1,\varepsilon} + \varepsilon R_{1,\varepsilon}) &= 1 - \eta \\
  u_\varepsilon(r) &> 1 - \eta \quad \text{for } \tau_{1,\varepsilon} + \varepsilon R_{1,\varepsilon} < r < \tau_{2,\varepsilon} \\
  u_\varepsilon(\tau_{2,\varepsilon}) &= 1 - \eta \\
  u_\varepsilon(\tau_{2,\varepsilon} + \varepsilon R_{2,\varepsilon}) &= -1 + \eta \\
  u_\varepsilon(r) &< -1 + \eta \quad \text{for } \tau_{2,\varepsilon} + \varepsilon R_{2,\varepsilon} < r < 1
\end{align*}

We assume that \( \tau_{1,\varepsilon} \to \tau_1 \in I_1 \) and that \( \tau_{2,\varepsilon} \to \tau_2 \in I_2 \) for some sequence \( \{ \varepsilon_j \} \) which converges to 0 as \( j \to \infty \). In this case it is easy to show that the energy of global minimizer \( J(u_\varepsilon) \) is estimated as follows
\begin{equation}
J_{\varepsilon_j}(u_{\varepsilon_j}) \geq J_{\varepsilon_j}(u_{\varepsilon_j}, B_{r_2-\varepsilon}(0)) + \varepsilon_j \omega N - 1 H(\tau_2) \beta_1 + B + O(\varepsilon_j \eta) + o(\varepsilon_j), \quad (3.16)
\end{equation}
where \( B = -\int_{\tau_{2,\varepsilon}}^{\tau_{1,\varepsilon}} r^{N-1} F(r, 1) dr \).

Let us assume the result does not hold. Then \( H(\tau_1) > \min_{s \in I_2} H(s) \) or \( H(\tau_2) > \min_{s \in I_2} H(s) \) hold. We assume \( H(\tau_1) = \min_{s \in I_2} H(s) \) and \( H(\tau_2) > \min_{s \in I_2} H(s) \). We also assume \( \tau_1 = \tau_1 \). We note that if \( H(\tau_1) > \min_{s \in I_1} H(s) \) or \( \tau_1 \in \text{int} I_1 \), the proof is more easy.

Let we take \( \tilde{\tau}_2 \in \text{int} I_2 \) such that \( H(\tau_2) > H(\tilde{\tau}_2) > \min_{s \in I_2} H(s) \) and consider the function
\begin{equation}
\hat{u}_\varepsilon(r) := \begin{cases}
  u_\varepsilon(r) & \text{on } [0, \tau_2 - \varepsilon) \\
  1 + \frac{\varepsilon}{2}(r - \tau_2) & \text{on } [\tau_2 - \varepsilon, \tau_2] \\
  1 & \text{on } [\tau_2, \tilde{\tau}_2 - \varepsilon] \\
  1 - \frac{\varepsilon}{2}(r - \tilde{\tau}_2) & \text{on } [\tilde{\tau}_2 - \varepsilon, \tilde{\tau}_2] \\
  V(h(\tilde{\tau}_2)) & \text{on } [\tilde{\tau}_2, \tilde{\tau}_2 + \varepsilon R''] \\
  -1 - \frac{\varepsilon}{2}(r - \tilde{\tau}_2 - \varepsilon R' - \varepsilon) & \text{on } [\tilde{\tau}_2 + \varepsilon R', \tilde{\tau}_2 + \varepsilon R' + \varepsilon] \\
  -1 & \text{on } [\tilde{\tau}_2 + \varepsilon R' + \varepsilon, 1],
\end{cases}
\end{equation}
where \( V \) is the unique solution of \( (3.2) \) and \( R'' \) is the unique value such that \( V(h(\tau_1)R'') = -1 + \eta \).

Since \( u_\varepsilon \) is global minimizer, we can estimate the energy of \( J_\varepsilon(\hat{u}_\varepsilon) \) as follows
\begin{equation}
J_\varepsilon(u_\varepsilon) \leq J_\varepsilon(\hat{u}_\varepsilon) \leq J_\varepsilon(u_\varepsilon, B_{r_2-\varepsilon}(0)) + \varepsilon \omega N - 1 H(\tilde{\tau}_2) \beta_1 + B + O(\varepsilon \eta) + o(\varepsilon), \quad (3.17)
\end{equation}
Then we have a contradiction from (3.16) and (3.17) by taking \( \varepsilon = \varepsilon_j \) and sufficiently large \( j \).

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**References**


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