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EXISTENCE OF SOLUTIONS FOR p-LAPLACIAN FUNCTIONAL DYNAMIC EQUATIONS ON TIME SCALES

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Abstract. In this paper, the author studies boundary-value problems for p-Laplacian functional dynamic equations on a time scale. By using the fixed point theorem, sufficient conditions are established for the existence of positive solutions.

1. Introduction

Let $\mathbb{T}$ be a closed nonempty subset of $\mathbb{R}$, and let subspace have the topology inherited from the Euclidean topology on $\mathbb{R}$. In some of the current literature, $\mathbb{T}$ is called a time scale (or measure chain). For notation, we shall use the convention that, for each interval of J of $\mathbb{R}$, J will denote time scales interval, that is, $J := J \cap \mathbb{T}$.

In this paper, let $\mathbb{T}$ be a time scale such that $-r$, 0, $T \in \mathbb{T}$. We are concerned with the existence of positive solutions of the p-Laplacian dynamic equation, on a time scale,

$$\begin{gathered} [\phi_p(x^{\triangle}(t))]^{\nabla} + \lambda a(t) f(x(t), x(\mu(t))) = 0, \quad t \in (0,T), \\ x_0(t) = \psi(t), \quad t \in [-r,0], \quad x(0) - B_0(x^{\triangle}(0)) = 0, \quad x^{\triangle}(T) = 0, \end{gathered} \tag{1.1}$$

where $\lambda > 0$ and $\phi_p(u)$ is the p-Laplacian operator, i.e., $\phi_p(u) = |u|^{p-2}u$, $p > 1$, $(\phi_p)^{-1}(u) = \phi_q(u)$, $\frac{1}{p} + \frac{1}{q} = 1$. Also we assume the following:

(A) The function $f : (\mathbb{R}^+)^2 \to \mathbb{R}^+$ is continuous;
(B) the function $a : \mathbb{T} \to \mathbb{R}^+$ is left dense continuous (i.e., $a \in C_{ld}(\mathbb{T}, \mathbb{R}^+)$). Here $C_{ld}(\mathbb{T}, \mathbb{R}^+)$ denotes the set of all left dense continuous functions from $\mathbb{T}$ to $\mathbb{R}^+$;
(C) $\psi : [-r, 0] \to \mathbb{R}^+$ is continuous and $r > 0$;
(D) $\mu : [0,T] \to [-r,T]$ is continuous, $\mu(t) \leq t$ for all t;
(E) $B_0 : \mathbb{R} \to \mathbb{R}$ is continuous and satisfies that there are $\beta \geq \delta > 0$ such that
$$\delta s \leq B_0(s) \leq \beta s \text{ for } s \in \mathbb{R}^+.$$

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p-Laplacian problems with two-, three-, m-point boundary conditions for ordinary differential equations and finite difference equations have been studied extensively, for example see [3, 5, 9, 11] and references therein. However, there are not many concerning the p-Laplacian problems on time scales, especially for p-Laplacian functional dynamic equations on time scales.

The motivations for the present work stems from many recent investigations in [2, 7, 10] and references therein. Especially, Kaufmann and Raffoul [7] considered a nonlinear functional dynamic equation on a time scale and obtained sufficient conditions for the existence of positive solutions. In this paper, we apply the fixed point theorem to obtain at least one positive solution of boundary value problem (BVP for short) (1.1). We do not need the condition that $f(x_1, x_2)$ is increasing in each x_i, for $x_i > 0, i = 1, 2$. And we claim the condition $\psi \equiv 0$ is not essential in our results.

For convenience, we list the following well-known definitions which can be found in [1, 4, 6] and the references therein.

Definition 1.1. For $t < \sup \mathbb{T}$ and $r > \inf \mathbb{T}$, define the forward jump operator σ and the backward jump operator ρ:

$$\sigma(t) = \inf\{\tau \in \mathbb{T} | \tau > t\} \in \mathbb{T}, \quad \rho(r) = \sup\{\tau \in \mathbb{T} | \tau < r\} \in \mathbb{T}$$

for all $t, r \in \mathbb{T}$. If $\sigma(t) > t$, t is said to be right scattered, and if $\rho(r) < r$, r is said to be left scattered. If $\sigma(t) = t$, t is said to be right dense, and if $\rho(r) = r$, r is said to be left dense. If $\mathbb{T}$ has a right scattered minimum m, define $\mathbb{T}_\kappa = \mathbb{T} - \{m\}$; otherwise set $\mathbb{T}_\kappa = \mathbb{T}$. If $\mathbb{T}$ has a left scattered maximum M, define $\mathbb{T}^\kappa = \mathbb{T} - \{M\}$; otherwise set $\mathbb{T}^\kappa = \mathbb{T}$.

Definition 1.2. For $x : \mathbb{T} \to \mathbb{R}$ and $t \in \mathbb{T}^\kappa$, we define the delta derivative of $x(t)$, $x^\triangle(t)$, to be the number (when it exists), with the property that, for any $\varepsilon > 0$, there is a neighborhood U of t such that

$$|[x(\sigma(t)) - x(s)] - x^\triangle(t)[\sigma(t) - s]| < \varepsilon|\sigma(t) - s|$$

for all $s \in U$. For $x : \mathbb{T} \to \mathbb{R}$ and $t \in \mathbb{T}_\kappa$, we define the nabla derivative of $x(t)$, $x^\nabla(t)$, to be the number (when it exists), with the property that, for any $\varepsilon > 0$, there is a neighborhood V of t such that

$$|[x(\rho(t)) - x(s)] - x^\nabla(t)[\rho(t) - s]| < \varepsilon|\rho(t) - s|$$

for all $s \in V$. If $\mathbb{T} = \mathbb{R}$, then $x^\triangle(t) = x^\nabla(t) = x'(t)$. If $\mathbb{T} = \mathbb{Z}$, then $x^\triangle(t) = x(t+1) - x(t)$ is forward difference operator while $x^\nabla(t) = x(t) - x(t-1)$ is the backward difference operator.

Definition 1.3. If $F^\triangle(t) = f(t)$, then we define the delta integral by $\int_a^t f(s)\triangle s = F(t) - F(a)$. If $\Phi^\nabla(t) = f(t)$, then we define the nabla integral by $\int_a^t f(s)\nabla s = \Phi(t) - \Phi(a)$.

In the following, we provide the definition of cones in Banach spaces, and we then state the fixed-point theorem for a cone preserving operator.

Definition 1.4. Let X be a real Banach space. A nonempty, closed, convex set $K \in X$ is called a cone, if it satisfies the following two conditions:

(i) $x \in K$, $\lambda \geq 0$ implies $\lambda x \in K$;
(ii) x and $-x$ in K implies $x = 0$.

Every cone $K \subset X$ induces an ordering in X given by $x \le y$ if and only if $y - x \in K$.

Lemma 1.5 ([8]). *Assume that X is a Banach space and $K \subset X$ is a cone in X; Ω_1, Ω_2 are open subsets of X, and $0 \in \overline{\Omega}_1 \subset \Omega_2$. Furthermore, let $F : K \cap (\overline{\Omega}_2 \setminus \Omega_1) \to K$ be a completely continuous operator satisfying one of the following conditions:*

(i) *$\|F(x)\| \le \|x\|$ for all $x \in K \cap \partial\Omega_1$, $\|F(x)\| \ge \|x\|$ for all $x \in K \cap \partial\Omega_2$;*

(i) *$\|F(x)\| \le \|x\|$ for all $x \in K \cap \partial\Omega_2$, $\|F(x)\| \ge \|x\|$ for all $x \in K \cap \partial\Omega_1$.*

Then there is a fixed point of F in $K \cap (\overline{\Omega}_2 \setminus \Omega_1)$.

2. Main results

We note that $x(t)$ is a solution of (1.1) if and only if

$$x(t) = \begin{cases} B_0\Big(\phi_q\Big(\int_0^T \lambda a(r) f(x(r), x(\mu(r)))\nabla r\Big)\Big) \\ + \int_0^t \phi_q\Big(\int_s^T \lambda a(r) f(x(r), x(\mu(r)))\nabla r\Big)\triangle s, & t \in [0,T], \\ \psi(t), & t \in [-r, 0]. \end{cases} \tag{2.1}$$

Let $X = C_{ld}([0,T], \mathbb{R})$ be endowed with the norm $\|x\| = \max_{t\in[0,T]} |x(t)|$ and

$$K = \{x \in X : x(t) \ge \frac{\delta}{T+\beta}\|x\| \text{ for } t \in [0,T]\}.$$

Clearly, X is a Banach space with the norm $\|x\|$ and K is a cone in X. For each $x \in X$, extend $x(t)$ to $[-r, T]$ with $x(t) = \psi(t)$ for $t \in [-r, 0]$.

For $t \in [0,T]$, define $F : P \to X$ as

$$\begin{aligned} Fx(t) = B_0\Big(\phi_q\Big(\int_0^T \lambda a(r) f(x(r), x(\mu(r)))\nabla r\Big)\Big) \\ + \int_0^t \phi_q\Big(\int_s^T \lambda a(r) f(x(r), x(\mu(r)))\nabla r\Big)\triangle s, \end{aligned} \tag{2.2}$$

We seek a fixed point, x_1, of F in the cone P. Define

$$x(t) = \begin{cases} x_1(t), & t \in [0,T], \\ \psi(t), & t \in [-r, 0]. \end{cases}$$

Then $x(t)$ denotes a positive solution of (1.1). It follows from (2.2) that

$$\begin{aligned} \|Fx\| &= (Fx)(T) \\ &= B_0\Big(\phi_q\Big(\int_0^T \lambda a(r) f(x(r), x(\mu(r)))\nabla r\Big)\Big) \\ &\quad + \int_0^T \phi_q\Big(\int_s^T \lambda a(r) f(x(r), x(\mu(r)))\nabla r\Big)\triangle s \\ &\le (T+\beta)\lambda^{q-1}\phi_q\Big(\int_0^T a(r) f(x(r), x(\mu(r)))\Big). \end{aligned} \tag{2.3}$$

From (2.2) and (2.3), we have the following lemma.

Lemma 2.1. *Let F be defined by (2.2). If $x \in K$, then*

(i) *$F(K) \subset K$.*

(ii) *$F : K \to K$ is completely continuous.*

(iii) $x(t) \geq \frac{\delta}{T+\beta}\|x\|$, $t \in [0,T]$.

We need to define subsets of $[0,T]$ with respect to the delay μ. Set

$$Y_1 := \{t \in [0,T] : \mu(t) < 0\}; \quad Y_2 := \{t \in [0,T] : \mu(t) \geq 0\}.$$

Throughout this paper, we assume Y_1 is nonempty and $\int_{Y_1} a(r)\nabla r > 0$. Let

$$l =: \frac{\lambda^{1-q}}{(T+\beta)\phi_q\big(\int_0^T a(r)\nabla r\big)}, \quad m =: \frac{(T+\beta)\lambda^{1-q}}{\delta^2\phi_q\big(\int_{Y_1} a(r)\nabla r\big)},$$
$$\widetilde{l} =: \frac{1}{(T+\beta)\phi_q\big(\int_0^T a(r)\nabla r\big)}.$$

In additions to Conditions (A)–(E), we shall also consider the following:

(H1) $\lim_{x\to 0^+} \frac{f(x,\psi(s))}{x^{p-1}} < l^{p-1}$, uniformly in $s \in [-r,0]$;
(H2) $\lim_{x_1\to 0^+;x_2\to 0^+} \frac{f(x_1,x_2)}{\max\{x_1^{p-1},x_2^{p-1}\}} < l^{p-1}$;
(H3) $\lim_{x\to\infty} \frac{f(x,\psi(s))}{x^{p-1}} > m^{p-1}$, uniformly in $s \in [-r,0]$.

Theorem 2.2. *Assume Conditions (A)–(E), (H1)–(H3) are satisfied. Then, for each* $0 < \lambda < \infty$, *BVP* (1.1) *has at least a positive solution.*

Proof. Apply Condition (H1) and set $\varepsilon_1 > 0$ such that if $0 < x \leq \varepsilon_1$, then

$$f(x,\psi(s)) < (lx)^{p-1}, \quad \text{for each } s \in [-r,0].$$

Apply Condition (H2) and set $\varepsilon_2 > 0$ such that if $0 < x_1 \leq \varepsilon_2, 0 < x_2 \leq \varepsilon_2$, then

$$f(x_1,x_2) < \max\{x_1^{p-1},x_2^{p-1}\}l^{p-1}.$$

Set $\rho_1 = \min\{\varepsilon_1,\varepsilon_2\}$. Then, for any $x \in K$ with $\|x\| = \rho_1$, from (2.3), we have

$$\begin{aligned}
&\|Fx\| \\
&\leq (T+\beta)\lambda^{q-1}\phi_q\Big(\int_0^T a(r)f(x(r),x(\mu(r)))\nabla r\Big) \\
&= (T+\beta)\lambda^{q-1}\Big[\phi_q\Big(\int_{Y_1} a(r)f(x(r),\psi(\mu(r)))\nabla r + \int_{Y_2} a(r)f(x(r),x(\mu(r)))\nabla r\Big)\Big] \\
&\leq l(T+\beta)\lambda^{q-1}\max_{t\in[0,T]}\{x(t)\}\phi_q\Big(\int_0^T a(r)\nabla r\Big) \\
&= l(T+\beta)\lambda^{q-1}\|x\|\phi_q\Big(\int_0^T a(r)\nabla r\Big) \\
&= \|x\| \quad \text{for } x \in K\cap\partial\Omega_1,
\end{aligned} \tag{2.4}$$

where $\Omega_1 = \{x \in K : \|x\| < \rho_1\}$. On the other hand, apply Condition (H3) and set $\rho_2 > \rho_1$ such that if $x \geq \frac{\rho_2}{T+1+\beta}$, then

$$f(x,\psi(s)) > (mx)^{p-1}, \quad \text{for each } s \in [-r,0].$$

Define $\Omega_2 = \{x \in K : \|x\| < \rho_2\}$. For $x \in K$ with $\|x\| = \rho_2$, we have

$$x(t) \geq \frac{\delta}{T+\beta}\|x\|, \quad t \in [0,T],$$

Thus, we have

$$
\begin{aligned}
\|Fx\| &= (Fx)(T) \\
&\geq \delta\phi_q\Big(\int_0^T \lambda a(r)f(x(r),x(\mu(r)))\nabla r\Big) \\
&\geq \delta\lambda^{q-1}\phi_q\Big(\int_{Y_1} a(r)f(x(r),\psi(\mu(r)))\nabla r\Big) \\
&\geq m\delta\lambda^{q-1}\min_{t\in Y_1}\{x(t)\}\phi_q\Big(\int_{Y_1} a(r)\nabla r\Big) \\
&\geq \frac{m\delta^2\lambda^{q-1}}{T+\beta}\|x\|\phi_q\Big(\int_{Y_1} a(r)\nabla r\Big) \\
&= \|x\| \quad \text{for } x\in K\cap\partial\Omega_2.
\end{aligned} \tag{2.5}
$$

Applying Condition (i) of Lemma 1.5, the proof is complete. □

Note that Theorem 2.2 is useful, but it does not apply if

$$f(x_1,x_2) = x_1^2 + x_2^2,$$

and if $\mu(t) < 0$ for some t satisfying $\psi(\mu(t)) > l$ and $p = 2$, for example. That is, Condition (H1) is not satisfied in this case.

We now provide a second theorem to address the above case. Firstly, we assume

(H2')
$$\lim_{x_1\to 0^+;x_2\to 0^+} \frac{f(x_1,x_2)}{\max\{x_1^{p-1},x_2^{p-1}\}} < \widetilde{l}^{p-1}.$$

Theorem 2.3. *Assume Conditions (A)–(E), (H2') and (H3) are satisfied. Then, there exists $L > 0$ such that for each $0 < \lambda < L$, BVP (1.1) has at least a positive solution.*

Proof. We outline the proof as a modification of the proof of Theorem 2.2. Only the argument in the construction of Ω_1 is modified. As in the proof of Theorem 2.2, apply Condition (H2'); this time set $\varepsilon_2 > 0$ such that if $0 < x_1 \leq \varepsilon_2, 0 < x_2 \leq \varepsilon_2$, then

$$f(x_1,x_2) < \max\big\{x_1^{p-1},x_2^{p-1}\big\}\widetilde{l}^{p-1}.$$

Now, set $\rho_1 = \varepsilon_2$ and define

$$\Omega_1 = \{x\in K : \|x\| < \rho_1\}$$

In particular, note that ρ_1 is independent of λ. Let

$$D = \Big\{\frac{\max_{x\in\overline{\Omega}_1}\int_{Y_1} f(x(r),\psi(\mu(r))\nabla r)}{(T+\beta)\phi_q\big(\int_0^T a(r)\nabla r\big)}\Big\}^{p-1}$$

Recall that f is continuous so D is well defined. Assume

$$\lambda^{q-1} \leq \min\Big\{1,\ \frac{\rho_1}{\max_{x\in\overline{\Omega}_1}\int_{Y_1} f(x(r),\psi(\mu(r))\nabla r)}\Big\} := L^{q-1}.$$

Then we have

$$
\begin{aligned}
&\|Fx\| \\
&\le (T+\beta)\lambda^{q-1}\phi_q\Big(\int_0^T a(r)f(x(r),x(\mu(r)))\nabla r\Big) \\
&= (T+\beta)\lambda^{q-1}\Big[\phi_q\Big(\int_{Y_1} a(r)f(x(r),\psi(\mu(r)))\nabla r + \int_{Y_2} a(r)f(x(r),x(\mu(r)))\nabla r\Big)\Big] \\
&\le (T+\beta)\lambda^{q-1}\max\big\{D^{q-1},\widetilde{l}\max_{t\in Y_2}\{x(t)\}\big\}\phi_q\Big(\int_0^T a(r)\nabla r\Big) \\
&\le \|x\| \quad \text{for } x\in K\cap\partial\Omega_1.
\end{aligned}
$$

The remainder of the proof of Theorem 2.2 carries over verbatim. □

We now consider analogous conditions:

(H4) $\lim_{x\to 0^+}\frac{f(x,\psi(s))}{x^{p-1}} > m^{p-1}$, uniformly in $s\in[-r,0]$;

(H5) $\lim_{x\to\infty}\frac{f(x,\psi(s))}{x^{p-1}} < l^{p-1}$, uniformly in $s\in[-r,0]$;

(H6) $\lim_{x_1\to\infty;x_2\to\infty}\frac{f(x_1,x_2)}{\max\{x_1^{p-1},x_2^{p-1}\}} < l^{p-1}$.

Theorem 2.4. *Assume Conditions (A)–(E), (H4)–(H6) are satisfied. Then, for each $0<\lambda<\infty$, BVP (1.1) has at least a positive solution.*

Proof. Apply Condition (H4) and set $\rho_1>0$ such that if $0<x\le\rho_1$, then

$$f(x,\psi(s)) > (mx)^{p-1}.$$

Define $\Omega_1=\{x\in K:\|x\|<\rho_1\}$. For $x\in K$ with $\|x\|=\rho_1$, we have

$$x(t)\ge\frac{\delta}{T+\beta}\|x\|,\quad t\in[0,T],$$

Thus,

$$
\begin{aligned}
\|Fx\| &= (Fx)(T) \\
&\ge \delta\phi_q\Big(\int_0^T \lambda a(r)f(x(r),x(\mu(r)))\nabla r\Big) \\
&\ge \delta\lambda^{q-1}\phi_q\Big(\int_{Y_1} a(r)f(x(r),\psi(\mu(r)))\nabla r\Big) \\
&\ge m\delta\lambda^{q-1}\min_{t\in Y_1}\{x(t)\}\phi_q\Big(\int_{Y_1} a(r)\nabla r\Big) \\
&\ge \frac{m\delta^2\lambda^{q-1}}{T+\beta}\|x\|\phi_q\Big(\int_{Y_1} a(r)\nabla r\Big) \\
&= \|x\| \quad \text{for } x\in K\cap\partial\Omega_1.
\end{aligned}
$$

To construct Ω_2, we consider two cases, f bounded and f unbounded: When f is bounded, the construction is straightforward. If $f(x_1,x_2)$ is bounded by $N^{p-1}>0$, set

$$\rho_2=\max\big\{2\rho_1, N(T+\beta)\phi_q\big(\int_0^T a(r)\nabla r\big)\big\}.$$

Then define $\Omega_2=\{x\in K:\|x\|<\rho_2\}$. For $x\in K$ with $\|x\|=\rho_2$, we have

$$\|Fx\|\le N(T+\beta)\phi_q\big(\int_0^T a(r)\nabla r\big)\le\rho_2.$$

Assume f is unbounded. Apply Condition (H5) and set $\varepsilon_1 > 0$ such that if $x > \varepsilon_1$, then
$$f(x, \psi(s)) < (lx)^{p-1}, \quad \text{for each } s \in [-r, 0].$$
Apply Condition (H6) and set $\varepsilon_2 > 0$ such that if $x_1 \geq \varepsilon_2, x_2 \geq \varepsilon_2$, then
$$f(x_1, x_2) < \max\{x_1^{p-1}, x_2^{p-1}\} l^{p-1}.$$
Set $\rho_2 = \max\{2\rho_1, \varepsilon_1, \varepsilon_2\}$. Then, for any $x \in K$ with $\|x\| = \rho_2$, from (2.3), we have
$$\begin{aligned}
&\|Fx\| \\
&\leq (T+\beta)\lambda^{q-1}\phi_q\Big(\int_0^T a(r) f(x(r), x(\mu(r)))\nabla r\Big) \\
&= (T+\beta)\lambda^{q-1}\Big[\phi_q\Big(\int_{Y_1} a(r) f(x(r), \psi(\mu(r)))\nabla r + \int_{Y_2} a(r) f(x(r), x(\mu(r)))\nabla r\Big)\Big] \\
&\leq l(T+\beta)\lambda^{q-1}\max_{t\in[0,T]}\{x(t)\}\phi_q\Big(\int_0^T a(r)\nabla r\Big) \\
&= l(T+\beta)\lambda^{q-1}\|x\|\phi_q\Big(\int_0^T a(r)\nabla r\Big) \\
&= \|x\| \quad \text{for } x \in K \cap \partial\Omega_2,
\end{aligned}$$
where $\Omega_2 = \{x \in K : \|x\| < \rho_2\}$. Apply Condition (ii) of Lemma 1.5, the proof is complete. $\square$

Similarly, assuming

(H6')
$$\lim_{x_1\to\infty; x_2\to\infty} \frac{f(x_1, x_2)}{\max\{x_1^{p-1}, x_2^{p-1}\}} < \tilde{l}^{p-1},$$
we have the following theorem which is analogous to Theorem 2.3.

Theorem 2.5. *Assume Conditions (A)–(E), (H5) and (H6') are satisfied. Then, there exists $L > 0$ such that for each $0 < \lambda < L$, BVP (1.1) has at least one positive solution.*

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