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EXISTENCE OF SOLUTIONS FOR SOME HAMMERSTEIN TYPE INTEGRO-DIFFERENTIAL INCLUSIONS

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Abstract. In the present work we obtain existence results for Hammerstein type integro-differential inclusions in a finite dimensional space for the cases when the integral multifunction satisfies upper Caratheodory conditions and when it is almost lower semicontinuous.

1. Introduction

Integral inclusions of the Hammerstein type have been studied in the articles [1, 2, 4, 5, 6, 9, 11, 14, 15, 16, 17, 18] and others. In a finite-dimentional space the inclusion was been studied in [1, 4, 9, 14, 16, 17]. O'Regan [17] investigated solvability of the inclusions in $\mathbb{R}$; Glashoff and Sprekels [9] considered the integral inclusions, arising in the theory of thermostats; Bulgakov and Lyapin [4] studied the properties of the set of solutions of the inclusions of Vollterra-Hammerstein type. The existence of solutions of the Hammerstein's integral inclusions in $\mathbb{R}^n$ was established in [1, 14]. For the inclusions in Banach space the problem of existence of solutions was considered in [5, 11, 15].

In the present work, applying the fixed point principle of Bohnenblust - Karlin we give existence results for the Hammerstein type integro-differential inclusion

$$u(t) \in \int_a^b K(t,s)F(s,u(s),u'(s))ds. \tag{1.1}$$

in $\mathbb{R}^n$.

Let X, Y be normed spaces; $P(Y)$ $[C(Y), K(Y), Cv(Y), Kv(Y)]$ denote the collections of all nonempty [respectively, nonempty: closed, compact, closed convex and compact convex] subsets of Y. Recall (see, e.g. [3, 10, 12]) that, a multimap $F: X \to P(Y)$ is said to be upper semicontinuous (u.s.c.) [lower semicontinuous (l.s.c.)] if the set $F_+^{-1}(V) = \{x \in X \mid F(x) \subset V\}$ is open [closed] for every open [respectively, closed] subset $V \subset Y$. A multimap F is said to be compact if the set $F(X)$ is relatively compact in Y.

Let $C([a,b],\mathbb{R}^n)$ $[C^1([a,b],\mathbb{R}^n), L^1([a,b],\mathbb{R}^n)]$ denote the collections of all continuous [respectively, continuously differentiable, integrable] functions on $[a,b]$ with values in $\mathbb{R}^n$.

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Let $F : [a,b] \times \mathbb{R}^n \times \mathbb{R}^n \to Kv(\mathbb{R}^n)$ be a multimap, satisfying the following assumptions:

(F1) For every $x \in \mathbb{R}^n \times \mathbb{R}^n$ the multifunction $F(\cdot, x) : [a,b] \to Kv(\mathbb{R}^n)$ has a measurable selection; i.e., there exists a measurable function $f(\cdot) \in L^1([a,b],\mathbb{R}^n)$ such that $f(t) \in F(t,x)$ for a.e. $t \in [a,b]$;

(F2) For a.e. $t \in [a,b]$ the multimap $F(t,\cdot) : \mathbb{R}^n \times \mathbb{R}^n \to Kv(\mathbb{R}^n)$ is u.s.c.;

(F3) For every bounded subset $\Omega \subset \mathbb{R}^n \times \mathbb{R}^n$ there exists a positive function $\vartheta_\Omega(\cdot) \in L^1([a,b],\mathbb{R})$ such that
$$\|F(t,x)\|_{\mathbb{R}^n} \leq \vartheta_\Omega(t),$$
for all $x \in \Omega$ and a.e. $t \in [a,b]$, where $\|F(t,x)\|_{\mathbb{R}^n} = \max\{\|y\|_{\mathbb{R}^n} : y \in F(t,x)\}$.

It is known (see, e.g. [3]) that under these conditions the superposition multioperator
$$\wp_F : C([a,b],\mathbb{R}^n \times \mathbb{R}^n) \to Cv(L^1([a,b],\mathbb{R}^n)),$$
$$\wp_F(u) = \{f \in L^1([a,b],\mathbb{R}^n) : f(s) \in F(s,u(s)), \text{ for a.e. } s \in [a,b]\},$$
is well defined and closed; i.e., it has a closed graph.

For every function $u \in C^1([a,b],\mathbb{R}^n)$ the function
$$v : [a,b] \to \mathbb{R}^n \times \mathbb{R}^n,$$
$$v(s) = (u(s), u'(s)),$$
is continuous. And hence the multioperator
$$\wp_F^1 : C^1([a,b],\mathbb{R}^n) \to Cv(L^1([a,b],\mathbb{R}^n)),$$
$$\wp_F^1(u) = \wp_F(v),$$
is defined and closed. Consider the linear operator
$$A : L^1([a,b],\mathbb{R}^n) \to C^1([a,b],\mathbb{R}^n),$$
$$A(f)(t) = \int_a^b K(t,s)f(s)ds,$$
where $K : [a,b] \times [a,b] \to L(\mathbb{R}^n)$ and $L(\mathbb{R}^n)$ denotes the collection of all linear operators in $\mathbb{R}^n$. The following statement can be easily verified.

Theorem 1.1. *Let the kernel $K : [a,b] \times [a,b] \to L(\mathbb{R}^n)$ satisfy the following assumptions:*

(K1) *the function $K(\cdot,s)x : [a,b] \to \mathbb{R}^n$ is differentiable on $[a,b]$ for all $x \in \mathbb{R}^n$ and a.e. $s \in [a,b]$; i.e., there exists $K_t'(t,s) \in L(\mathbb{R}^n)$ such that:*
$$\lim_{\Delta t \to 0} \frac{K(t+\Delta t,s)x - K(t,s)x}{\Delta t} = K_t'(t,s)x,$$
for all $t \in [a,b], x \in \mathbb{R}^n$ and a.e. $s \in [a,b]$;

(K2) *there exists $K > 0$ such that*
$$\|K(t,s)\|_L \leq K, \quad \|K_t'(t,s)\|_L \leq K, \quad \Big\|\frac{K(t+\Delta t,s) - K(t,s)}{\Delta t}\Big\| \leq K,$$
for all $t, t+\Delta t \in [a,b]$ and a.e. $s \in [a,b]$;

(K3) *for every $t \in [a,b]$ the functions $s \mapsto K(t,s)x$ and $s \mapsto K_t'(t,s)x$ are integrable for all $x \in \mathbb{R}^n$;*

(K4) *there exist a positive function* $\omega(\cdot) \in L^1([a,b],\mathbb{R})$ *and a function* $\eta(\cdot) \in C([a,b],\mathbb{R})$ *such that*

$$\|K_t'(t_2,s) - K_t'(t_1,s)\|_L \le \omega(s)|\eta(t_2) - \eta(t_1)|,$$

for all $t_1, t_2 \in [a,b]$ *and a.e.* $s \in [a,b]$.

Then the operator A *is completely continuous.*

Following [3, Theorem 1.5.30] we obtain the following result.

Theorem 1.2. *Let multimap* $F : [a,b]\times\mathbb{R}^n\times\mathbb{R}^n \to Kv(\mathbb{R}^n)$ *satisfy the assumptions* (F1)–(F3) *and the operator* A *satisfy conditions* (K1)–(K4). *Then the multioperator* $A \circ \wp_F^1$ *is closed.*

Consider the integral multioperator

$$\Gamma = A \circ \wp_F^1 : \; C^1([a,b],\mathbb{R}^n) \to Kv(C^1([a,b],\mathbb{R}^n)),$$

$$\Gamma(u)(t) = \int_a^b K(t,s)F(s,u(s),u'(s))ds.$$

Applying [3, Theorem 1.2.48], Theorem 1.1 and Theorem 1.2 we obtain the following theorem.

Theorem 1.3. *Let the conditions* (K1)–(K4) *and* (F1)–(F3) *hold. Then multioperator* Γ *is u.s.c. and the restriction of* Γ *to any bounded subset* $\Omega \subset C^1([a,b],\mathbb{R}^n)$ *is compact; i.e., the set* $\Gamma(\Omega)$ *is relatively compact.*

Consider now the multioperator Γ when the multimap $F : [a,b] \times \mathbb{R}^n \times \mathbb{R}^n \to K(\mathbb{R}^n)$ is almost lower semicontinuous (a.l.s.c.). Recall (see, e.g. [3, 12]) that F is said to be an a.l.s.c. multimap if there exists a sequence of disjoint compact sets $\{I_m\}, I_m \subset [a,b]$ such that:

(i) $\mathrm{meas}([a,b] \setminus \bigcup_m I_m) = 0$;
(ii) the restriction of F on each set $J_m = I_m \times \mathbb{R}^n \times \mathbb{R}^n$ is l.s.c.

We also assume that F satisfies the condition of boundedness (F3). Then the superposition multioperator

$$\wp_F^1 : C^1([a,b];\mathbb{R}^n) \to C(L^1([a,b];\mathbb{R}^n))$$

is l.s.c. (see [3, 7, 12]). Consider again the multioperator

$$A \circ \wp_F^1 : C^1([a,b];\mathbb{R}^n) \to P(C^1([a,b];\mathbb{R}^n)),$$

where the operator A is given by the above conditions (K1)–(K4). From [3, Theorem 1.3.11] it follows easily that the multioperator $\Gamma = A \circ \wp_F^1$ is l.s.c. The following statement can be easily verified.

Theorem 1.4. *Let* (F1), (F3) *and* (K1)–(K4) *hold. Then for any bounded subset* $\Omega \subset C^1([a,b];\mathbb{R}^n)$ *the set* $\Gamma(\Omega)$ *is relatively compact.*

Let E be a Banach space. A nonempty subset $M \subset L^1([a,b];E)$ is said to be decomposable provided for every $f,g \in M$ and each Lebesgue measurable subset $m \subset [a,b]$,

$$f \cdot k_m + h \cdot k_{([a,b]\setminus m)} \in M,$$

where k_m is the characteristic function of the set m (see, e.g. [3, 8, 12] for further details).

Theorem 1.5 ([8]). *Let X be a separable metric space and E a Banach space. Then every l.s.c. multimap $G : X \to P(L^1([a,b];E))$ with closed decomposable values has a continuous selection; i.e., there exists a continuous map $g : X \to L^1([a,b];E)$ such that $g(x) \in G(x)$ for all $x \in X$.*

It is clear that for every $u \in C^1([a,b];\mathbb{R}^n)$, the set $\wp_F^1(u)$ is closed and decomposable. Then the multioperator $\wp_F^1$ has a continuous selection

$$\ell : C^1([a,b];\mathbb{R}^n) \to L^1([a,b];\mathbb{R}^n), \quad \ell(u) \in \wp_F^1(u).$$

Therefore, the continuous operator $\gamma : C^1([a,b];\mathbb{R}^n) \to C^1([a,b];\mathbb{R}^n)$,

$$\gamma(u)(t) = \int_a^b K(t,s)\ell(u)(s)ds$$

is a continuous selection for the multioperator Γ. By virtue of Theorem 1.4, the operator γ is completely continuous and its fixed points are also fixed points of the multioperator Γ.

2. Main results

In this section, we give some existence results of solutions of the inclusion (1.1).

Theorem 2.1. *Let the conditions* (K1)–(K4) *and* (F1)–(F2) *hold. Assume that:*

(F3') *there exists a positive function $\omega \in L^1([a,b],\mathbb{R})$ such that*

$$\|F(t,x,y)\|_{\mathbb{R}^n} \le \omega(t)(1 + \|x\|_{\mathbb{R}^n} + \|y\|_{\mathbb{R}^n}),$$

for all $x, y \in \mathbb{R}^n$ and a.e. $t \in [a,b]$;

(F4) $2K\int_a^b \omega(t)dt < 1$, *where K is the constant from the condition* (K2).

Then the inclusion (1.1) *has at least one solution.*

Proof. It is easy to see that from (F3') we obtain (F3). Consider the multioperator Γ on the ball $T = T(\|u\|_{C^1} \le \rho)$. We have

$$\|\Gamma(u)\|_{C^1} = \max\big\{\big\|\int_a^b K(t,s)f(s)ds\big\|_{C^1} : f \in \wp_F^1(u)\big\},$$

where

$$\begin{aligned}\big\|\int_a^b K(t,s)f(s)ds\big\|_{C^1} &= \max\big\{\big\|\int_a^b K(t,s)f(s)ds\big\|_{\mathbb{R}^n} : t \in [a,b]\big\}\\ &\quad + \max\big\{\big\|\int_a^b K'_t(t,s)f(s)ds\big\|_{\mathbb{R}^n} : t \in [a,b]\big\}.\end{aligned}$$

It is clear that

$$\big\|\int_a^b K(t,s)f(s)ds\big\|_{\mathbb{R}^n} \le \int_a^b \|K(t,s)\|_L\|f(s)\|_{\mathbb{R}^n}ds \le K\int_a^b \|f(s)\|_{\mathbb{R}^n}ds,$$

and

$$\big\|\int_a^b K'_t(t,s)f(s)ds\big\|_{\mathbb{R}^n} \le \int_a^b \|K'_t(t,s)\|_L\|f(s)\|_{\mathbb{R}^n}ds \le K\int_a^b \|f(s)\|_{\mathbb{R}^n}ds.$$

Since $f(s) \in F(s, u(s), u'(s))$ for a.e. $s \in [a,b]$ we have

$$\begin{aligned}\|f(s)\|_{\mathbb{R}^n} &\leq \|F(s,u(s),u'(s))\|_{\mathbb{R}^n}\\ &\leq \omega(s)(1+\|u(s)\|_{\mathbb{R}^n}+\|u'(s)\|_{\mathbb{R}^n})\\ &\leq \omega(s)(1+\|u\|_{C^1})\\ &\leq \omega(s)(1+\rho),\end{aligned}$$

for a.e. $s \in [a,b]$. Consequently,

$$K\int_a^b \|f(s)\|_{\mathbb{R}^n}\,ds \leq K(1+\rho)\int_a^b \omega(s)ds.$$

And hence

$$\Big\|\int_a^b K(t,s)f(s)ds\Big\|_{C^1} \leq 2K(1+\rho)\int_a^b \omega(s)ds.$$

The last inequality is true for all $f \in \wp_F^1(u)$, and so we obtain

$$\|\Gamma(u)\|_{C^1} \leq 2K(1+\rho)\int_a^b \omega(s)ds.$$

Choose ρ so that

$$\rho \geq \frac{2K\int_a^b \omega(s)ds}{1-2K\int_a^b \omega(s)ds}.$$

Then $\|\Gamma(u)\|_{C^1} \leq \rho$. Consider the upper semicontinuous multioperator $\Gamma : T \to Kv(T)$. By Theorem 1.3, the multioperator Γ is compact. From the Bohnenblust-Karlin Theorem (see, e.g. [3]) it follows that the multioperator Γ has at least one fixed point $u^* \in T$: $u^* \in \Gamma(u^*)$. The function u^* is a solution of the inclusion (1.1). $\square$

Theorem 2.2. *Let the conditions* (K1)–(K4) *and* (F_L) *hold. Assume that there exist two numbers* $\lambda, \beta \in \mathbb{R}; \beta > 0$ *and a positive function* $\omega \in L^1([a,b],\mathbb{R})$ *such that:*

- (F3") $\|F(t,x,y)-\lambda(x+y)\|_{\mathbb{R}^n} \leq \beta(\|x\|_{\mathbb{R}^n}+\|y\|_{\mathbb{R}^n})+\omega(t)$, *for all* $x,y \in \mathbb{R}^n$ *and a.e.* $t \in [a,b]$*;*
- (F5) $2K(b-a)(\beta+|\lambda|) < 1$, *where K is the constant from (K2).*

Then inclusion (1.1) *has at least one solution.*

For the proof we need the following result (see, e.g. [13]).

Lemma 2.3. *Let A be nonlinear and B be linear completely continuous operators in a Banach space E. If on the sphere $S = S(\|x\| = \rho)$ the following inequality holds*

$$\|Ax - Bx\| < \|x - Bx\|.$$

Then the equation $x = Ax$ in the ball $T(\|x\| \leq \rho)$ has at least one solution.

Proof of Theorem 2.2. It is easy to see that from (F3") we have

$$\|F(t,x,y)\|_{\mathbb{R}^n} \leq (\beta+|\lambda|)(\|x\|_{\mathbb{R}^n}+\|y\|_{\mathbb{R}^n})+\omega(t),$$

for all $x,y \in \mathbb{R}^n$ and a.e. $t \in [a,b]$. And hence we obtain (F3). Consider a linear operator $B : C^1([a,b];\mathbb{R}^n) \to C^1([a,b];\mathbb{R}^n)$,

$$Bu(t) = \lambda\int_a^b K(t,s)(u(s)+u'(s))ds.$$

It is clear that B is completely continuous. Consider the multioperator Γ and the operator B on $S = S(\|u\|_{C^1} = \rho)$. For each function $u \in S$ we have

$$\|\Gamma u - Bu\|_{C^1} = \sup\big\{\big\|\int_a^b K(t,s)[f(s) - \lambda(u(s)+u'(s))]ds\big\|_{C^1} : f \in \wp_F(u)\big\}.$$

On the other hand

$$\begin{aligned}
&\big\|\int_a^b K(t,s)[f(s) - \lambda(u(s)+u'(s)]ds\big\|_{C^1} \\
&= \max_{t\in[a,b]} \big\|\int_a^b K(t,s)[f(s) - \lambda(u(s)+u'(s))]ds\big\|_{\mathbb{R}^n} \\
&\quad + \max_{t\in[a,b]} \big\|\int_a^b K'_t(t,s)[f(s) - \lambda(u(s)+u'(s))]ds\big\|_{\mathbb{R}^n} \\
&\le \max_{t\in[a,b]} \int_a^b \|K(t,s)\|_L \|f(s) - \lambda(u(s)+u'(s))\|_{\mathbb{R}^n} ds \\
&\quad + \max_{t\in[a,b]} \int_a^b \|K'_t(t,s)\|_L \|f(s) - \lambda(u(s)+u'(s))\|_{\mathbb{R}^n} ds \\
&\le 2K \int_a^b \|f(s) - \lambda(u(s)+u'(s))\|_{\mathbb{R}^n} ds.
\end{aligned}$$

Since $f(s) \in F(s,u(s),u'(s))$ for a.e. $s \in [a,b]$ we have

$$\begin{aligned}
\|f(s) - \lambda(u(s)+u'(s))\|_{\mathbb{R}^n} &\le \|F(s,u(s),u'(s)) - \lambda(u(s)+u'(s))\|_{\mathbb{R}^n} \\
&\le \beta(\|u(s)\|_{\mathbb{R}^n} + \|u'(s)\|_{\mathbb{R}^n}) + \omega(s) \\
&\le \beta\|u\|_{C^1} + \omega(s) = \beta\rho + \omega(s),
\end{aligned}$$

for a.e. $s \in [a,b]$. Therefore,

$$\begin{aligned}
\big\|\int_a^b K(t,s)[f(s) - \lambda(u(s)+u'(s))]ds\big\|_{C^1} &\le 2K\int_a^b (\beta\rho + \omega(s))ds \\
&\le 2K\beta\rho(b-a) + 2K\int_a^b \omega(s)ds.
\end{aligned}$$

The above inequality holds for all $f \in \wp^1_F(u)$, and so we obtain

$$\|\Gamma u - Bu\|_{C^1} \le 2K\beta\rho(b-a) + 2K\int_a^b \omega(s)ds, \quad \forall u \in S.$$

On the other hand for each $t \in [a,b]$ we have

$$\begin{aligned}
\|u(t) - B(u)(t)\|_{\mathbb{R}^n} &= \big\|u(t) - \lambda\int_a^b K(t,s)(u(s)+u'(s))ds\big\|_{\mathbb{R}^n} \\
&\ge \|u(t)\|_{\mathbb{R}^n} - \|\lambda\int_a^b K(t,s)(u(s)+u'(s))ds\|_{\mathbb{R}^n} \\
&\ge \|u(t)\|_{\mathbb{R}^n} - |\lambda|\int_a^b \|K(t,s)\|_L\|u(s)+u'(s)\|_{\mathbb{R}^n} ds \\
&\ge \|u(t)\|_{\mathbb{R}^n} - |\lambda|\int_a^b K\|u\|_{C^1} ds \\
&\ge \|u(t)\|_{\mathbb{R}^n} - K\rho|\lambda|(b-a).
\end{aligned}$$

Analogously,

$$\|u'(t) - (B(u))'(t)\|_{\mathbb{R}^n} \geq \|u'(t)\|_{\mathbb{R}^n} - K\rho|\lambda|(b-a).$$

And hence we obtain

$$\begin{aligned}\|u - Bu\|_{C^1} &= \max_{t\in[a,b]} \|u(t) - Bu(t)\|_{\mathbb{R}^n} + \max_{t\in[a,b]} \|u'(t) - (Bu)'(t)\|_{\mathbb{R}^n} \\ &\geq \|u\|_{C^1} - 2K\rho|\lambda|(b-a) \\ &= \rho(1 - 2K|\lambda|(b-a)).\end{aligned}$$

Choose ρ so that $\rho > \frac{2K\int_a^b \omega(s)ds}{1-2K(b-a)(\beta+|\rho|)}$. Then $\|\Gamma u - Bu\|_{C^1} < \|u - Bu\|_{C^1}$, for all $u \in S$. Let γ be an arbitrary continuous selection of the multioperator Γ. Then on the sphere S we have

$$\|\gamma u - Bu\|_{C^1} \leq \|\Gamma u - Bu\|_{C^1} < \|u - Bu\|_{C^1}\,.$$

By Lemma 2.3, the operator γ has at least one fixed point in the ball $T(\|u\|_{C^1} < \rho)$: $u_* = \gamma(u_*)$. The function u_* is a solution of the inclusion (1.1). □

Theorem 2.4. *Let the conditions* (K1)–(K4), (F1), (F3'), (F4) *hold. Then inclusion* (1.1) *has at least one solution.*

Proof. From the proof of Theorem 2.1 it follows that with the conditions (F3') and (F4) we can choose a number $\rho > 0$ such that the multioperator Γ maps the ball $T(\|u\|_{C^1} \leq \rho)$ into itself. Let γ be an arbitrary continuous selection of the multioperator Γ. Then the operator γ maps the ball $T(\|u\|_{C^1} \leq \rho)$ into itself. Consider the completely continuous operator $\gamma : T \to T$. From the Schauder fixed point theorem, the operator γ has at least one fixed point on T, i.e. there exists a function $u_* \in T$ such that: $u_* = \gamma(u_*)$. The function u_* is a solution of the inclusion (1.1). □

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