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EXISTENCE OF POSITIVE SOLUTIONS FOR NONLINEAR DYNAMIC SYSTEMS WITH A PARAMETER ON A MEASURE CHAIN

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Abstract. In this paper, we consider the following dynamic system with parameter on a measure chain $\mathbb{T}$,

$$u_i^{\Delta\Delta}(t) + \lambda h_i(t) f_i(u_1(\sigma(t)), u_2(\sigma(t)), \dots, u_n(\sigma(t))) = 0, \quad t \in [a, b],$$
$$\alpha u_i(a) - \beta u_i^{\Delta}(a) = 0, \quad \gamma u_i(\sigma(b)) + \delta u_i^{\Delta}(\sigma(b)) = 0,$$

where $i = 1, 2, \dots, n$. Using fixed-point index theory, we find sufficient conditions the existence of positive solutions.

1. Introduction

The theory of dynamic equations on time scales has become a new important mathematical branch (see, for example, [1, 3, 8, 9]) since it was initiated by Hilger [14]. At the same time, boundary-value problems (BVPs) for scalar dynamic equations on time scales have received considerable attention [4, 5, 6, 7, 10, 11, 13, 15, 16]. However, to the best of our knowledge, only a few papers can be found in the literature for systems of BVPs for dynamic equations on time scales [16].

Sun, Zhao and Li [17] considered the following discrete system with parameter

$$\Delta^2 u_i(k) + \lambda h_i(k) f_i(u_1(k), u_2(k), \dots, u_n(k)) = 0, \quad k \in [0, T],$$
$$u_i(0) = u_i(T + 2) = 0,$$

where $i = 1, 2, \dots, n$, $\lambda > 0$ is a constant, T and $n \geq 2$ are two fixed positive integers. They established the existence of one positive solution by using the theory of fixed-point index [12].

Motivated by [17], the purpose of this paper is to study the following more general dynamic system with parameter on a measure chain $\mathbb{T}$,

$$u_i^{\Delta\Delta}(t) + \lambda h_i(t) f_i(u_1(\sigma(t)), u_2(\sigma(t)), \dots, u_n(\sigma(t))) = 0, \quad t \in [a, b], \tag{1.1}$$
$$\alpha u_i(a) - \beta u_i^{\Delta}(a) = 0, \quad \gamma u_i(\sigma(b)) + \delta u_i^{\Delta}(\sigma(b)) = 0, \tag{1.2}$$

where, $i = 1, 2, \dots, n$, $\lambda > 0$ is constant, $a, b \in \mathbb{T}$, α, β, γ, $\delta \geq 0$, $\gamma(\sigma(b) - \sigma^2(b)) + \delta \geq 0$, $r = \gamma\beta + \alpha\delta + \alpha\gamma(\sigma(b) - a) > 0$, and the function $\sigma(t)$ and $[a, b]$ is defined as

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in Section 2 below. Let $\mathbb{R}$ be the set of real numbers, and $\mathbb{R}_+ = [0, \infty)$. For $u = (u_1, u_2, \dots, u_n) \in \mathbb{R}_+^n$, let $\|u\| = \sum_{i=1}^n u_i$.

We make the following assumptions for $i = 1, 2, \dots, n$:

(H1) $h_i : [a, b] \to (0, \infty)$ is continuous.
(H2) $f_i : \mathbb{R}_+^n \to \mathbb{R}_+$ is continuous.

For convenience, we introduce the following notation

$$f_i^0 = \lim_{\|u\| \to 0} \frac{f_i(u)}{\|u\|}, \quad f_i^\infty = \lim_{\|u\| \to \infty} \frac{f_i(u)}{\|u\|}, \quad u \in \mathbb{R}_+^n,$$

$$f^0 = \sum_{i=1}^n f_i^0 \quad \text{and} \quad f^\infty = \sum_{i=1}^n f_i^\infty.$$

2. Preliminaries

In this section, we introduce several definitions on measure chains and some notation. Also we give some lemmas which are useful in proving our main result.

Definition 2.1. Let $\mathbb{T}$ be a closed subset of $\mathbb{R}$ with the properties

$$\sigma(t) = \inf\{\tau \in \mathbb{T} : \tau > t\} \in \mathbb{T}$$
$$\rho(t) = \sup\{\tau \in \mathbb{T} : \tau < t\} \in \mathbb{T}$$

for all $t \in \mathbb{T}$ with $t < \sup \mathbb{T}$ and $t > \inf \mathbb{T}$, respectively. We assume throughout that $\mathbb{T}$ has the topology that it inherits from the standard topology on $\mathbb{R}$. We say t is right-scattered, left-scattered, right-dense and left-dense if $\sigma(t) > t$, $\rho(t) < t$, $\sigma(t) = t$, $\rho(t) = t$, respectively.

Throughout this paper we assume that $a \le b$ are points in $\mathbb{T}$.

Definition 2.2. If $r, s \in \mathbb{T} \cup \{-\infty, +\infty\}$, $r < s$, then an open interval (r, s) in $\mathbb{T}$ is defined by

$$(r, s) = \{t \in \mathbb{T} : r < t < s\}.$$

Other types of intervals are defined similarly.

Definition 2.3. *Assume that $x : \mathbb{T} \to \mathbb{R}$ and fix $t \in \mathbb{T}$. Then, x is called differentiable at $t \in \mathbb{T}$ if there exists a $\theta \in \mathbb{R}$, such that, for any given $\varepsilon > 0$, there is an open neighborhood U of t, such that*

$$|x(\sigma(t)) - x(s) - \theta[\sigma(t) - s]| \le \varepsilon |\sigma(t) - s|, \quad s \in U.$$

In this case, θ is called the Δ-derivative of x at $t \in \mathbb{T}$ and we denote it by $\theta = x^\Delta(t)$. It can be shown that if $x : \mathbb{T} \to \mathbb{R}$ is continuous at $t \in \mathbb{T}$, then

$$x^\Delta(t) = \frac{x(\sigma(t)) - x(t)}{\sigma(t) - t}$$

if t is right-scattered, and

$$x^\Delta(t) = \lim_{s \to t} \frac{x(t) - x(s)}{t - s}$$

if t is right-dense.

In the rest of the paper, we assume that the set $[a, \sigma(b)]$ is, such that

$$\xi = \min\{t \in \mathbb{T} : t \geq \frac{\sigma(b)+3a}{4}\}, \quad \omega = \max\{t \in \mathbb{T} : t \leq \frac{3\sigma(b)+a}{4}\},$$

exist and satisfy

$$\frac{\sigma(b)+3a}{4} \leq \xi < \omega \leq \frac{3\sigma(b)+a}{4}.$$

We also assume that if $\sigma(\omega) = b$ and $\delta = 0$, then $\sigma(\omega) < \sigma(b)$.

We denote by $G(t,s)$ the Green function of the boundary-value problem

$$-u^{\Delta\Delta}(t) = 0, \quad t \in [a,b],$$
$$\alpha u(a) - \beta u^{\Delta}(a) = 0, \quad \gamma u(\sigma(b)) + \delta u^{\Delta}(\sigma(b)) = 0,$$

which is explicitly given in [11],

$$G(t,s) = \begin{cases} \frac{1}{r}\{\alpha(t-a)+\beta\}\{\gamma(\sigma(b)-\sigma(s))+\delta\}, & t \leq s, \\ \frac{1}{r}\{\alpha(\sigma(s)-a)+\beta\}\{\gamma(\sigma(b)-t)+\delta\}, & t \geq \sigma(s), \end{cases}$$

for $t \in [a, \sigma^2(b)]$ and $s \in [a,b]$, where $r = \gamma\beta + \alpha\delta + \alpha\gamma(\sigma(b)-a)$. For this Green function, we have the following lemmas [8, 9, 11].

Lemma 2.4. *Assume $\alpha, \beta, \gamma, \delta \geq 0$, $\gamma(\sigma(b) - \sigma^2(b)) + \delta \geq 0$, and*

$$r = \gamma\beta + \alpha\delta + \alpha\gamma(\sigma(b)-a) > 0\,.$$

Then, for $(t,s) \in [a, \sigma^2(b)] \times [a,b]$, $0 \leq G(t,s) \leq G(\sigma(s), s)$.

Lemma 2.5. *(i) If $(t,s) \in [(\sigma(b)+3a)/4, (3\sigma(b)+a)/4] \times [a,b]$, then $G(t,s) \geq lG(\sigma(s), s)$, where*

$$l = \min\Big\{\frac{\alpha[\sigma(b)-a]+4\beta}{4\alpha[\sigma(b)-a]+4\beta}, \frac{\gamma[\sigma(b)-a]+4\delta}{4\gamma[\sigma(b)-\sigma(a)]+4\delta}\Big\};$$

(ii) If $(t,s) \in [\xi, \sigma(\omega)] \times [a,b]$, then $G(t,s) \geq kG(\sigma(s), s)$, where

$$k = \min\Big\{l, \min_{s\in[a,b]} \frac{G(\sigma(\omega), s)}{G(\sigma(s), s)}\Big\}.$$

The following well-known result of the fixed-point index is crucial in our arguments.

Lemma 2.6 ([12])**.** *Let E be a Banach space and K a cone in E. For $r > 0$, define $K_r = \{u \in K : \|u\| < r\}$. Assume that $A : \bar{K}_r \to K$ is completely continuous, such that $Ax \neq x$ for $x \in \partial K_r = \{u \in K : \|u\| = r\}$.*

(i) *If $\|Ax\| \geq \|x\|$, for $x \in \partial K_r$, then $i(A, K_r, K) = 0$.*
(ii) *If $\|Ax\| \leq \|x\|$, for $x \in \partial K_r$, then $i(A, K_r, K) = 1$.*

To apply Lemma 2.6 to (1.1) and (1.2), we define the Banach space $B = \{x | x : [a, \sigma^2(b)] \to \mathbb{R}$ is continuous $\}$, for $x \in B$, let $|x|_0 = \max_{t\in[a,\sigma^2(b)]} |x(t)|$ and $E = B^n$, for $u = (u_1, u_2, \dots, u_n) \in E$, $\|u\| = \sum_{i=1}^n |u_i|_0$.

For $u \in E$ or $\mathbb{R}^n_+$, $\|u\|$ denotes the norm of u in E and $\mathbb{R}^n_+$, respectively.

Define K to be a cone in E by

$$K = \big\{u = (u_1, u_2, \dots, u_n) \in E : u_i(t) \geq 0, t \in [a, \sigma^2(b)], i = 1, 2, \dots, n, \\ \text{and } \min_{t\in[\xi,\sigma(\omega)]} \sum_{i=1}^n u_i(t) \geq k\|u\|\big\}.$$

For $u = (u_1, u_2, \dots, u_n) \in K$, let

$$A(u) = (A_1(u), A_2(u), \dots, A_n(u)),$$

where

$$A_i(u) = \lambda \int_a^{\sigma(b)} G(t,s)h_i(s)f_i(u_1(\sigma(s)), \dots, u_n(\sigma(s)))\Delta s, \quad t \in [a, \sigma^2(b)].$$

Lemma 2.7. *Assume that (H1) and (H2) hold, then $A : K \to K$ is completely continuous.*

Proof. For $u = (u_1, u_2, \dots, u_n) \in K$, and $i = 1, 2, \dots, n$, it follows from Lemma 2.4 that

$$\begin{aligned} 0 \le A_i(u)(t) &= \lambda \int_a^{\sigma(b)} G(t,s)h_i(s)f_i(u_1(\sigma(s)), \dots, u_n(\sigma(s)))\Delta s \\ &\le \lambda \int_a^{\sigma(b)} G(\sigma(s), s)h_i(s)f_i(u_1(\sigma(s)), \dots, u_n(\sigma(s)))\Delta s, \quad t \in [a, \sigma^2(b)]. \end{aligned}$$

So, for $i = 1, 2, \dots, n$,

$$|A_i(u)|_0 \le \lambda \int_a^{\sigma(b)} G(\sigma(s), s)h_i(s)f_i(u_1(\sigma(s)), \dots, u_n(\sigma(s)))\Delta s,$$

For $t \in [\xi, \sigma(\omega)]$, from Lemma 2.5 and the above inequality, we have

$$\begin{aligned} A_i(u)(t) &= \lambda \int_a^{\sigma(b)} G(t,s)h_i(s)f_i(u_1(\sigma(s)), \dots, u_n(\sigma(s)))\Delta s \\ &\ge k\lambda \int_a^{\sigma(b)} G(\sigma(s), s)h_i(s)f_i(u_1(\sigma(s)), \dots, u_n(\sigma(s)))\Delta s \\ &\ge k|A_i(u)|_0, \quad i = 1, 2, \dots, n. \end{aligned}$$

So, for $t \in [\xi, \sigma(\omega)]$,

$$\sum_{i=1}^n A_i(u)(t) \ge k \sum_{i=1}^n |A_i(u)|_0 = k\|Au\|.$$

Hence,

$$\min_{t \in [\xi, \sigma(\omega)]} \sum_{i=1}^n A_i(u)(t) \ge k\|Au\|;$$

i.e., $A(u) \in K$. Further, it is easy to see that $A : K \to K$ is completely continuous. The proof is complete. $\square$

Now, it is not difficult to show that the problem (1.1) and(1.2) is equivalent to the fixed-point equation $A(u) = u$ in K. Let

$$\gamma_i = \max_{t \in [a, \sigma^2(b)]} \int_\xi^{\sigma(\omega)} G(t,s)h_i(s)\Delta s, \quad \text{and} \quad \Gamma = \min_{1 \le i \le n} \{\gamma_i\}.$$

Lemma 2.8. *Assume that (H1) and (H2) hold. Let $u = (u_1, u_2, \dots, u_n) \in K$ and $\eta > 0$. If there exists f_{i_0} such that*

$$f_{i_0}(u_1(\sigma(t)), u_2(\sigma(t)), \dots, u_n(\sigma(t))) \ge \eta \sum_{i=1}^n u_i(t), \quad t \in [\xi, \sigma(\omega)], \tag{2.1}$$

then $\|A(u)\| \ge \lambda k \eta \Gamma \|u\|$.

Proof. From the definition of K and (2.1), we have

$$\begin{aligned}
\|A(u)\| &= \sum_{i=1}^{n} |A_i(u)|_0 \\
&\geq |A_{i_0}|_0 = \lambda \max_{t\in[a,\sigma^2(b)]} \int_a^{\sigma(b)} G(t,s)h_{i_0}(s)f_{i_0}(u_1(\sigma(s)),\dots,u_n(\sigma(s)))\Delta s \\
&\geq \lambda \max_{t\in[a,\sigma^2(b)]} \int_\xi^{\sigma(\omega)} G(t,s)h_{i_0}(s)f_{i_0}(u_1(\sigma(s)),\dots,u_n(\sigma(s)))\Delta s \\
&\geq \lambda\eta \max_{t\in[a,\sigma^2(b)]} \int_\xi^{\sigma(\omega)} G(t,s)h_{i_0}(s)\sum_{i=1}^{n} u_i(s)\Delta s \\
&\geq k\lambda\eta\|u\|\gamma_{i_0} \\
&\geq k\lambda\eta\Gamma\|u\|.
\end{aligned}$$

The proof is complete. □

For each $i = 1, 2, \dots, n$, we define a new function $\tilde{f}_i : \mathbb{R}_+ \to \mathbb{R}_+$ by

$$\tilde{f}_i(t) = \max\{f_i(u) : u \in \mathbb{R}_+^n,\ \|u\| \leq t\}.$$

Denote

$$\tilde{f_i^0} = \lim_{t\to 0} \frac{\tilde{f}_i(t)}{t}, \quad \tilde{f_i^\infty} = \lim_{t\to\infty} \frac{\tilde{f}_i(t)}{t}.$$

As in [18, Lemma 2.8], we can obtain the following result.

Lemma 2.9. *Assume that (H2) holds. Then,* $\tilde{f_i^0} = f_i^0$ *and* $\tilde{f_i^\infty} = f_i^\infty$.

Lemma 2.10. *Assume that (H1) and (H2) hold. Let $h > 0$. If there exists $\varepsilon > 0$, such that*

$$\tilde{f}_i(h) \leq \varepsilon h, \quad i = 1, 2, \dots, n, \tag{2.2}$$

then $\|A(u)\| \leq \lambda\varepsilon C\|u\|$, for $u \in \partial K_h$, where

$$C = \sum_{i=1}^{n} [\max_{t\in[a,\sigma^2(b)]} \int_a^{\sigma(b)} G(t,s)h_i(s)\Delta s]\,.$$

Proof. Suppose $u \in \partial K_h$; i.e., $u \in K$ and $\|u\| = h$, then it follows from (2.2) that

$$\begin{aligned}
A_i(u)(t) &= \lambda \int_a^{\sigma(b)} G(t,s)h_i(s)f_i(u_1(\sigma(s)),\dots,u_n(\sigma(s)))\Delta s \\
&\leq \lambda \int_a^{\sigma(b)} G(t,s)h_i(s)\tilde{f}_i(h)\Delta s \\
&\leq \lambda\varepsilon h \int_a^{\sigma(b)} G(t,s)h_i(s)\Delta s \\
&\leq \lambda\varepsilon h \max_{t\in[a,\sigma^2(b)]} \int_a^{\sigma(b)} G(t,s)h_i(s)\Delta s, \quad t \in [a, \sigma^2(b)],\ i = 1, 2, \dots, n.
\end{aligned}$$

So,

$$|A_i(u)|_0 \leq \lambda\varepsilon h \max_{t\in[a,\sigma^2(b)]} \int_a^{\sigma(b)} G(t,s)h_i(s)\Delta s, \quad i = 1, 2, \dots, n.$$

Therefore,

$$\|A(u)\| = \sum_{i=1}^{n} |A_i(u)|_0 \leq \lambda\varepsilon h \sum_{i=1}^{n} [\max_{t\in[a,\sigma^2(b)]} \int_a^{\sigma(b)} G(t,s)h_i(s)\Delta s] = \lambda\varepsilon C\|u\|.$$

The proof is complete. □

3. Main Result

Our main result is the following theorem.

Theorem 3.1. *Assume that (H1) and (H2) hold. Then, for all $\lambda > 0$, (1.1) and (1.2) has a positive solution if one of the following two conditions holds:*

(a) $f^0 = 0$ *and* $f^\infty = \infty$;
(b) $f^0 = \infty$ *and* $f^\infty = 0$.

Proof. First, we suppose that (a) holds. Since $f^0 = 0$ implies that $f_i^0 = 0$, $i = 1, 2, \dots, n$, it follows from Lemma 2.9 that $\tilde{f}_i^0 = 0$, $i = 1, 2, \dots, n$. Therefore, we can choose $r_1 > 0$, such that

$$\tilde{f}_i(r_1) \leq \varepsilon r_1, \quad i = 1, 2, \dots, n,$$

where the constant $\varepsilon > 0$ satisfies $\lambda\varepsilon C < 1$, and C is defined in Lemma 2.10. By Lemma 2.10, we have

$$\|A(u)\| \leq \lambda\varepsilon C\|u\| < \|u\|, \quad \text{for } u \in \partial K_{r_1}. \tag{3.1}$$

Now, since $f^\infty = \infty$, there exists f_{i_0} so that $f_{i_0}^\infty = \infty$. Therefore, there is $H > 0$, such that

$$f_{i_0}(u) \geq \eta\|u\|, \quad \text{for } u \in \mathbb{R}_+^n, \quad \text{and} \quad \|u\| \geq H,$$

where $\eta > 0$ is chosen so that $\lambda\eta k\Gamma > 1$. Let $r_2 = \max\{2r_1, \frac{1}{k} H\}$. If $u \in \partial K_{r_2}$, then

$$\|u\| = \sum_{i=1}^{n} |u_i|_0 \geq \sum_{i=1}^{n} u_i(t) \geq k\|u\| = kr_2 \geq H, \quad t \in [\xi, \sigma(\omega)],$$

which implies that

$$f_{i_0}(u_1(\sigma(t)), u_2(\sigma(t)), \dots, u_n(\sigma(t))) \geq \eta\|u\| \geq \eta \sum_{i=1}^{n} u_i(t), \quad t \in [\xi, \sigma(\omega)].$$

It follows from Lemma 2.8 that

$$\|A(u)\| \geq \lambda\eta\Gamma k\|u\| > \|u\|, \quad \text{for } u \in \partial K_{r_2}. \tag{3.2}$$

By (3.1), (3.2) and Lemma 2.6,

$$i(A, K_{r_1}, K) = 1 \quad \text{and} \quad i(A, K_{r_2}, K) = 0.$$

It follows from the additivity of the fixed-point index that

$$i(A, K_{r_2} \backslash \bar{K}_{r_1}, K) = -1,$$

which implies that A has a fixed point $u \in K_{r_2} \backslash \bar{K}_{r_1}$. The fixed point $u \in K_{r_2} \backslash \bar{K}_{r_1}$ is the desired positive solution of (1.1) and (1.2).

Next, we suppose that (b) holds. Since $f^0 = \infty$, there exists f_{i_0} so that $f_{i_0}^0 = \infty$. Therefore, there is $r_1 > 0$, such that

$$f_{i_0}(u) \geq \eta\|u\|, \quad \text{for } u \in \mathbb{R}_+^n, \quad \text{and} \quad \|u\| \leq r_1,$$

where $\eta > 0$ is chosen so that $\lambda\eta k\Gamma > 1$. If $u \in \partial K_{r_1}$, then

$$f_{i_0}(u_1(\sigma(t)), u_2(\sigma(t)), \dots, u_n(\sigma(t))) \geq \eta\|u\| \geq \eta \sum_{i=1}^{n} u_i(t), \quad t \in [\xi, \sigma(\omega)].$$

It follows from Lemma 2.8 that

$$\|A(u)\| \geq \lambda\eta\Gamma k\|u\| > \|u\|, \quad \text{for } u \in \partial K_{r_1}. \tag{3.3}$$

In view of $f^\infty = 0$ implies that $f_i^\infty = 0$, $i = 1, 2, \dots, n$, it follows from Lemma 2.9 that $f_i^{\tilde{\infty}} = 0$, $i = 1, 2, \dots, n$. Therefore, we can choose $r_2 > 2r_1$, such that

$$\tilde{f}_i(r_2) \leq \varepsilon r_2, \quad i = 1, 2, \dots, n,$$

where the constant $\varepsilon > 0$ satisfies

$$\lambda\varepsilon C < 1,$$

and C is defined in Lemma 2.10. We have by Lemma 2.10 that

$$\|A(u)\| \leq \lambda\varepsilon C\|u\| < \|u\|, \quad \text{for } u \in \partial K_{r_2}. \tag{3.4}$$

By (3.3), (3.4) and Lemma 2.6,

$$i(A, K_{r_1}, K) = 0 \quad \text{and} \quad i(A, K_{r_2}, K) = 1.$$

It follows from the additivity of the fixed-point index that

$$i(A, K_{r_2} \backslash \bar{K}_{r_1}, K) = 1,$$

which implies that A has a fixed point $u \in K_{r_2} \backslash \bar{K}_{r_1}$, which is the desired positive solution of (1.1) and (1.2). $\square$

Remark 3.2. It is worth noting that these techniques can be extended to the following multi-point system based in [6],

$$(p_i y_i^\Delta)^\Delta(t) - q_i(t)y_i(t) + \lambda h_i(t) f_i(y_1(\sigma(t)), y_2(\sigma(t)), \dots, y_m(\sigma(t))) = 0, \quad t \in (t_1, t_n),$$

$$\alpha y_i(t_1) - \beta p_i(t_1) y_i^\Delta(t_1) = \sum_{k=2}^{n-1} a_{ki} y_i(t_k), \quad \gamma y_i(t_n) + \delta p_i(t_n) y_i^\Delta(t_n) = \sum_{k=2}^{n-1} b_{ki} y_i(t_k),$$

for $i = 1, 2, \dots, m$.

Example 3.3. Let $\mathbb{T} = \{1 - (\frac{1}{2})^{\mathbb{N}_0}\} \cup [1, 2]$. We consider the dynamic system

$$u_i^{\Delta\Delta}(t) + \lambda f_i(u_1(\sigma(t)), u_2(\sigma(t)), \dots, u_n(\sigma(t))) = 0, \quad t \in [0, 1], \tag{3.5}$$

$$u_i(0) - u_i^\Delta(0) = 0, \quad u_i(1) + u_i^\Delta(1) = 0, \tag{3.6}$$

$i = 1, 2, \dots, n$, where $f_i : \mathbb{R}_+^n \to \mathbb{R}_+$ is define by

$$f_i(u_1, u_2, \dots, u_n) = (u_1 + u_2 + \dots + u_n)^{i+1}, \quad i = 1, 2, \dots, n.$$

It is easy to see that

$$f^0 = 0 \quad \text{and} \quad f^\infty = \infty.$$

So, it follows from Theorem 3.1 that for all $\lambda > 0$, (3.5)-(3.6) has at least one positive solution.

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