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ASYMPTOTIC BEHAVIOR OF A DELAY PREDATOR-PREY SYSTEM WITH STAGE STRUCTURE AND VARIABLE COEFFICIENTS

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Abstract. In this paper, we establish a global attractor for a Lotka-Volterra type reaction-diffusion predator-prey model with stage structure for the predator, delay due to maturity and variable coefficients. This attractor is found by the method of upper and lower solutions and is given in terms of bounds for the coefficients.

1. Introduction

Almost all animals have the stage structure of immature and then mature, and in each stage they show different characteristics. For instance, immature predators are not able to hunt, while mature animals have more powerful survival capacities; likewise, rates of birth or death vary on each stage. Therefore, considering stage-structured models could lead to more accurate results.

In 2006, Xu, Chaplain and Davidson [7], considered the following Lotka-Volterra type reaction-diffusion predator-prey model with stage structure for the predator and delay due to maturity

$$\begin{aligned}\frac{\partial u_1}{\partial t} &= D_1\Delta u_1(t,x) + u_1(t,x)[r_1 - a_{11}u_1(t,x) - a_{12}u_2(t,x)],\\ &\quad (t,x)\in(0,\infty)\times\Omega\end{aligned} \tag{1.1}$$

$$\begin{aligned}\frac{\partial u_2}{\partial t} &= D_2\Delta u_2(t,x) + \alpha\int_0^\tau f(s)e^{-\gamma s}u_1(t-s,x)u_2(t-s,x)ds\\ &\quad - r_2u_2(t,x) - a_{22}u_2^2(t,x)\quad (t,x)\in(0,\infty)\times\Omega\end{aligned} \tag{1.2}$$

$$\frac{\partial u_i}{\partial \nu} = 0 \quad (i=1,2),\ t>0,\ x\in\partial\Omega \tag{1.3}$$

$$u_i(t,x) = \phi_i(t,x) \quad (i=1,2),\ t\in[-\tau,0],x\in\overline{\Omega} \tag{1.4}$$

In this problem, Ω is a bounded domain in $\mathbb{R}^n$ with smooth boundary $\partial\Omega$, where $\frac{\partial}{\partial\nu}$ denotes the outward normal derivative on $\partial\Omega$. The boundary conditions in (1.3) imply that the populations do not move across the boundary $\partial\Omega$. The parameters $r_1, r_2, a_{11}, a_{12}, a_{22}, \alpha, \gamma$ are positive constants. $u_1(t,x)$ represents the density of the prey population at time t and location x, $u_2(t,x)$ denotes the density of the

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mature predator population at time t and location x, respectively. The data $\phi_i(t,x)$ $(i=1,2)$ are nonnegative and Hölder continuous and satisfy $\frac{\partial\phi_i}{\partial\nu}=0$ in $(-\tau,0)\times\partial\Omega$. The model is derived under the following assumptions.

• The prey population: The growth of the species is of Lotka - Volterra nature. The parameters r_1, a_{11} and D_1 are the intrinsic growth rate, intra-specific competition rate and diffusion rate, respectively.

• The predator population: $a_{12}, \frac{\alpha}{a_{12}}, r_2$ and a_{22} are the capturing rate, conversion rate,death rate and intra-specific competition rate of the mature predator, respectively; $\gamma>0$ is the death rate of the immature predator population, D_2 is the diffusion rate of the mature population. The term $\alpha u_1(t-s,x)u_2(t-s,x)$ is the number born at time $t-s$ and location x per unit time, and is taken as proportional to the number of the prey and mature predator the around. $f(s)$ denotes the probability that the maturation time is between s and $s+ds$ with ds infinitesimal,and $\int_0^\infty f(s)ds=1$. $e^{-\gamma s}$ is the probability of an individual born at time $t-s$ still alive at time t. Individuals becoming mature at time t could have been born at any time prior to this, and the integral totals up the contributions from all previous times.

They also use the following assumptions:

(H1) $f(t)$ is piecewise continuous in $[0,\tau]$ and has the property: $f(t)\geq 0$, $\int_0^\tau f(t)dt=1$; i.e., $f(t)$ is a probability diet on $[0,\tau]$

System (1.1)–(1.4) possesses a trivial uniform equilibrium $E_0(0,0)$ and a semi-trivial uniform equilibrium $E_1\big(\frac{r_1}{a_{11}},0\big)$. If the following holds:

(H2) $r_1\alpha I > r_2 a_{11}$.

Then (1.1)–(1.4) also has a unique positive uniform equilibrium $E^\star(u_1^\star,u_2^\star)$ where:

$$u_1^\star=\frac{r_1a_{22}+r_2a_{12}}{a_{11}a_{22}+\alpha a_{12}I},\quad u_2^\star=\frac{r_1\alpha I-r_2a_{11}}{a_{11}a_{22}+\alpha a_{12}I} \tag{1.5}$$

where $I=\int_0^\tau f(s)e^{-\gamma s}ds$. The main result in [7] is as follows:

Theorem 1.1. *Let the initial functions ϕ_i $(i=1,2)$ be Hölder continuous in $[-\tau,0]\times\overline{\Omega}$, with $\phi_i(t,x)\geq 0$, $\phi_i(0,x)\not\equiv 0$. Let $(u_1(t,x),u_2(t,x))$ satisfy* (1.1)–(1.4). *In addition to* (H1)–(H2), *assume that*

(H3) $a_{11}a_{22} > a_{12}\alpha\int_0^\tau f(s)e^{-\gamma s}ds$

Then $\lim_{t\to\infty}u_i(t,x)=u_i^\star$ $(i=1,2)$ uniformly for $x\in\overline{\Omega}$

Xu, Chaplain and Davidson considered both, the coefficients of inter-species interaction and their birth and death rates to be constant. However, it must not be forgotten that the variability implicit in the environment means that these coefficients may depend on variables such as time, temperature, light flux, etc. Therefore, whenever possible, it is convenient to introduce these factors as functions of these variables even though this may complicate the resolution of the system of differential equations [4].

In this work, we extend Theorem 3.3 to the case where the coefficients are functions of space and time as follows

$$\frac{\partial u_1}{\partial t} = D_1 \Delta u_1(t,x) + u_1(t,x)\big[r_1(t,x) - a_{11}(t,x)u_1(t,x) - a_{12}(t,x)u_2(t,x)\big] \tag{1.6}$$

$$\begin{aligned}\frac{\partial u_2}{\partial t} &= D_2 \Delta u_2(t,x) + \alpha(t,x)\int_0^\tau f(s)e^{-\gamma s}u_1(t-s,x)u_2(t-s,x)ds \\ &\quad - r_2(t,x)u_2(t,x) - a_{22}(t,x)u_2^2(t,x); \quad (t,x)\in(0,\infty)\times\Omega\end{aligned} \tag{1.7}$$

$$\frac{\partial u_i}{\partial \nu} = 0 \quad (i=1,2),\ t>0,\ x\in\partial\Omega \tag{1.8}$$

$$u_i(t,x) = \phi_i(t,x) \quad (i=1,2),\ t\in[-\tau,0],\ x\in\overline{\Omega} \tag{1.9}$$

In our case the asymptotic behavior of time-dependent solution will be determined since we will be able to a obtain a prior upper an lower bounds for the system (1.6)–(1.9).

Similar problems with constant coefficients are considered in [2, 8], where systems of equations with diffusion are studied. One equation with diffusion and variable coefficients is analyzed in [9]. The competition case with diffusion and variable coefficients is studied in [6]. Some cases of variable coefficients with no diffusion are studied in [1, 5, 10].

The rest of the paper is organized as follows. In section 2 we state the definition of upper and lower solutions, we also discuss the existence and uniqueness of positive solution of our system. In section 3 we find a global attractor for (1.6)–(1.9). Finally, we present a brief discussion in the last section.

2. Preliminaries

Definition 2.1. A pair of functions

$$\tilde{u}(t,x) = (\tilde{u}_1(t,x), \tilde{u}_2(t,x)), \quad \hat{u}(t,x) = (\hat{u}_1(t,x), \hat{u}_2(t,x))$$

defined for $t \ge 0, x \in \overline{\Omega}$ are called coupled upper and lower solutions of systems (1.6)–(1.9) if $\tilde{u}_i \ge \hat{u}_i$ in $[-\tau \times \overline{\Omega})$ and if for all ψ_i such that $\hat{u}_i \le \psi_i \le \tilde{u}_i$ the following differential inequalities hold:

$$\begin{gathered}\frac{\partial \tilde{u}_1}{\partial t} \ge D_1\Delta\tilde{u}_1 + \tilde{u}_1(t,x)[r_1(t,x) - a_{11}(t,x)\tilde{u}_1(t,x) - a_{12}(t,x)\hat{u}_2(t,x)] \\ \frac{\partial \tilde{u}_2}{\partial t} \ge D_2\Delta\tilde{u}_2(t,x) + \alpha(t,x)\int_0^\tau f(s)e^{-\gamma s}\psi_1\psi_2 ds - r_2(t,x)\tilde{u}_2 - a_{22}(t,x)\tilde{u}_2^2 \\ \frac{\partial \hat{u}_1}{\partial t} \le D_1\Delta\hat{u}_1 + \hat{u}_1(t,x)[r_1(t,x) - a_{11}(t,x)\hat{u}_1 - a_{12}(t,x)\tilde{u}_2] \\ \frac{\partial \hat{u}_2}{\partial t} \le D_2\Delta\hat{u}_2(t,x) + \alpha(t,x)\int_0^\tau f(s)e^{-\gamma s}\psi_1\psi_2 ds \\ - r_2(t,x)\tilde{u}_2(t,x) - a_{22}(t,x)(\tilde{u}_2(t,x))^2\end{gathered}$$

for $(t,x)\in(0,\infty)\times\Omega$, and

$$\begin{gathered}\frac{\partial \hat{u}_i}{\partial \nu} \le 0 \le \frac{\partial \tilde{u}_i}{\partial \nu} \quad (i=1,2),\ (t,x)\in(0,\infty)\times\partial\Omega \\ \hat{u}_i(t,x) \le \phi_i(t,x) \le \tilde{u}_i(t,x) \quad (i=1,2),\ (t,x)\in[-\tau,0]\times\overline{\Omega}\end{gathered}$$

It is easy to see that $(0,0)$ and (k_1, k_2), with

$$k_1 = \max\{\frac{r_1}{A_{11}}, \sup_{-\tau\le\theta\le 0} \|\phi(\theta,\cdot)\|\},$$
$$k_2 = \max\{\frac{r_2}{\alpha_2 k_1 \int_0^\tau f(s)e^{-\gamma\tau}d\tau}, \sup_{-\tau\le\theta\le 0} \|\phi(\theta,\cdot)\|\},$$

are pairs of coupled lower-upper solutions of problem (1.6)–(1.9).

The existence of solutions of problem (1.6)–(1.9) is guaranteed by a result established by Redlinger in [3] if the reaction part of the equations satisfy the Lipschitz condition, which turns to be true in this case.

Proposition 2.2. *Let the initial function ϕ be Hölder continuous in $[-\tau, 0]\times\overline{\Omega}$. Assume that $A_1 \ge 0$, $B > 0$, $A_2 > 0$, and $f(s)$ is defined as in* (H1)*. Let $u(x,t)$ be a nonnegative nontrivial solution of the scalar problem*

$$\frac{\partial u}{\partial t} = D\Delta u + B\int_0^\tau f(s)e^{-\gamma s}u(t-s,x)ds - A_1u(t,x) - A_2u^2(t,x), \quad (t,x)\times\Omega,$$
$$\frac{\partial u}{\partial \nu} = 0, \quad (t,x)\times\partial\Omega,$$
$$u(t,x) = \phi(t,x) \ge 0, \quad \phi(0,x) \ne 0, \quad (t,x) \in [-\tau,0]\times\overline{\Omega}.$$

Then we have

(i) *if $B\int_0^\tau f(s)e^{-\gamma s}ds > A_1$, then*
$$\lim_{t\to\infty} u(t,x) = \frac{B\int_0^\tau f(s)e^{-\gamma s}ds - A_1}{A_2} \quad \textit{uniformly for } x\in\overline{\Omega}$$

(ii) *if $B\int_0^\tau f(s)e^{-\gamma s}ds < A_1$, then*
$$\lim_{t\to\infty} u(t,x) = 0 \quad \textit{uniformly for } x\in\overline{\Omega}$$

A proof of the above proposition can be found in [7].

3. Global Attractor

We assume that system (1.6)–(1.9) has bounded variable coefficients with the following properties:

$$0 < a_{11} \le a_{11}(t,x) \le A_{11}, \quad 0 < a_{12} \le a_{12}(t,x) \le A_{12},$$
$$0 < r_1 \le r_1(t,x) \le R_1, \quad 0 < r_2 \le r_2(t,x) \le R_2,$$
$$0 \le \alpha_1 \le \alpha(t,x) \le \alpha_2, \quad 0 < a_{22} \le a_{22}(t,x) \le A_{22}\,.$$

We have the following results.

Proposition 3.1. *Let u_1, u_2 be solutions of* (1.6)–(1.9)*,*

$$\underline{M}_i = \liminf_{t\to\infty}[\min_{x\in\overline{\Omega}} u_i(t,x)], \quad \overline{M}_i = \limsup_{t\to\infty}[\max_{x\in\overline{\Omega}} u_i(t,x)]$$

and $I = \int_0^\tau f(s)e^{-\gamma s}ds$. Assume (H1)*, and that the initial conditions $\phi_i \ge 0$ for $i = 1,2$.*

(a) *If $\underline{M}_1 > \frac{R_2}{\alpha_1 I}$, then*

$$\frac{\alpha_1\underline{M}_1 I - R_2}{A_{22}} \le \underline{M}_2 \le \overline{M}_2 \le \frac{\alpha_2\overline{M}_1 I - r_2}{a_{22}} \tag{3.1}$$

(b) *If* $a_{11}r_2 \geq \alpha_2 R_1 I$, *then* $\underline{M}_2 = 0 = \overline{M_2}$.

Proof. Let $\bar{u}$ and $\hat{u}$ be solutions of

$$\begin{gathered} L_2\bar{u} = \alpha_2\overline{M_1}I(\bar{u}) - r_2\bar{u} - a_{22}\bar{u}^2, \\ L_2\hat{u} = \alpha_1\underline{M}_1 I(\hat{u}) - R_2\hat{u} - A_{22}\hat{u}^2, \end{gathered} \tag{3.2}$$

with boundary conditions and initial values as for u_2, where $L_i u = u_t - D_i\Delta u$ for $i = 1, 2$, and

$$I(u) = \int_0^\tau f(s)e^{-\gamma s}u(t-s,x)ds,$$

then $\bar{u}$ and $\hat{u}$ are upper and lower solutions of u_2; therefore

$$\hat{u} \leq u_2 \leq \bar{u}. \tag{3.3}$$

If $\underline{M}_1 > \frac{R_2}{\alpha_1 I}$ then $\overline{M}_1 > \frac{r_2}{\alpha_2 I}$, thus applying (a) of Proposition 2.2 we obtain

$$\lim_{t\to\infty} \hat{u} = \frac{\alpha_1\underline{M}_1 I - R_2}{A_{22}}, \quad \lim_{t\to\infty} \bar{u} = \frac{\alpha_2\overline{M}_1 I - r_2}{a_{22}},$$

from this and (3.3) we obtain (3.1). The proof of de second part is similar using (b) of Proposition 2.2. □

With a similar idea and taking the appropriate upper and lower solutions of (1.6) we note that

$$\frac{r_1 - \overline{M}_2 A_{12}}{A_{11}} \leq \underline{M}_1 \leq \overline{M}_1 \leq \frac{R_1 - \underline{M}_2 a_{12}}{a_{11}}. \tag{3.4}$$

With the hypothesis and notation of the previous proposition and its proof, we have the following result.

Proposition 3.2. *If* $a_{11}r_2 \leq \alpha_2 R_1 I$, *then*

$$\overline{M}_2 \leq \frac{\alpha_2\frac{R_1}{a_{11}}I - r_2}{a_{22}} \tag{3.5}$$

Proof. ¿From equation (3.4) we have that $\overline{M}_1 < R_1/a_{11}$, therefore for each $\epsilon > 0$ there exists $T > 0$ such that $u_1(t-s,x) < (R_1/a_{11}) + \epsilon$ for all $t > T$, $s \in [0,\tau]$ and $x \in \Omega$. Let ω_2 be a solution of

$$L_2\omega_2 = \alpha_2\big(\frac{R_1}{a_{11}} + \varepsilon\big)I(\omega_2)(t,x) - r_2\omega_2(t,x) - a_{22}(\omega_2(t,x))^2; \quad t > T,\ x \in \Omega \tag{3.6}$$

$$\frac{\partial\omega_2}{\partial t} = 0 \quad t > T,\ x \in \partial\Omega \tag{3.7}$$

$$\omega_2(t,x) = k_2 \quad (t,x) \in [T-\tau, T] \times \overline{\Omega}. \tag{3.8}$$

where $I(w)(t,x) = \int_0^\infty f(s)e^{-\gamma s}w(t-s,x)ds$. Since

$$\alpha_2\big(\frac{R_1}{a_{11}} + \varepsilon\big)I \geq \alpha_2\frac{R_1}{a_{11}}I \geq r_2$$

then we can use Proposition 2.2 to obtain

$$\lim_{t\to\infty} \omega_2(t,x) = \frac{\alpha_2\big(\frac{R_1}{a_{11}} + \varepsilon\big)I - r_2}{a_{22}}.$$

Since ω_2 is an upper solution of u_2, $\omega_2(t,x) \leq u_2(t,x)$ for $t > T$ and $x \in \Omega$; therefore

$$\overline{M}_2 \leq \lim_{t\to\infty} \omega_2(t,x) = \frac{\alpha_2\big(\frac{R_1}{a_{11}} + \varepsilon\big)I - r_2}{a_{22}}$$

and we obtain (3.5) from the fact that ϵ is arbitrary. □

Now, we are able to state our two main results.

Theorem 3.3. *Let the initial functions ϕ_i be Hölder continuous in $[-\tau, 0] \times \overline{\Omega}$, with $\phi_i(t,x) \geq 0$, $\phi_i(0,x) \neq 0$ for $i = 1, 2$. Let $u_1(t,x)$, $u_2(t,x)$ satisfy* (1.6)–(1.9). *In addition to* (H1) *assume further that*

$$a_{11} r_2 \leq \alpha_2 R_1 I, \tag{3.9}$$

$$\alpha_2 A_{12} R_1 I \leq a_{11}(a_{22} r_1 + a_{12} r_2)\,. \tag{3.10}$$

Then

$$\alpha_1 I \underline{M}_1 - A_{22} \underline{M}_2 \leq R_2, \tag{3.11}$$

$$r_2 \leq \alpha_2 I \overline{M}_1 - a_{22} \overline{M}_2, \tag{3.12}$$

$$r_1 \leq A_{11} \underline{M}_1 + A_{12} \overline{M}_2, \tag{3.13}$$

$$a_{11} \overline{M}_1 + a_{12} \underline{M}_2 \leq R_1. \tag{3.14}$$

Proof. From (3.10) we obtain

$$\frac{\alpha_2 \frac{R_1}{a_{11}} I - r_2}{a_{22}} \leq \frac{r_1 - \frac{A_{11} R_2}{\alpha_1 I}}{A_{12}}\,.$$

Now, we can use Proposition 3.2, because of (3.9), the above inequality leads to

$$\overline{M}_2 \leq \frac{r_1 - \frac{A_{11} R_2}{\alpha_1 I}}{A_{12}}$$

which implies

$$\frac{R_2}{\alpha_1 I} \leq \frac{r_1 - \overline{M}_2 A_{12}}{A_{11}}$$

and with (3.4), we obtain

$$\frac{R_2}{\alpha_1 I} \leq \underline{M}_1,$$

thus, we can use Proposition 3.1 to obtain (3.11)–(3.12), and (3.13)–(3.14) follow from (3.4). □

Theorem 3.4. *Let $\delta = a_{11} a_{22} A_{11} A_{22} - a_{12} A_{12} \alpha_1 \alpha_2 I^2$, and assume hypothesis* (H1) *and* (3.9)–(3.10). *If $\delta > 0$, then*

$$s_1 \leq \underline{M}_1 \leq \overline{M}_1 \leq S_1, \quad s_2 \leq \underline{M}_2 \leq \overline{M}_2 \leq S_2,$$

where

$$\begin{aligned}
s_1 &= \frac{a_{11} A_{22}(A_{12} r_2 + a_{22} r_1) - \alpha_2 A_{12} I (a_{12} R_2 + A_{22} R_1)}{\delta}, \\
S_1 &= \frac{A_{11} a_{22}(a_{12} R_2 + A_{22} R_1) - \alpha_1 a_{12} I (A_{12} r_2 + a_{22} r_1)}{\delta}, \\
s_2 &= \frac{a_{11} a_{22}(\alpha_1 r_1 I - A_{11} R_2) - \alpha_1 A_{12} I (\alpha_2 R_1 I - a_{11} r_2)}{\delta}, \\
S_2 &= \frac{A_{11} A_{22}(\alpha_2 R_1 I - a_{11} r_2) - \alpha_2 a_{12} I (\alpha_1 r_1 I - A_{11} R_2)}{\delta}.
\end{aligned}$$

Proof. This is the solution of the set of inequalities (3.11)–(3.14) using the fact that $\delta > 0$. □

Theorem 3.5. *Let the initial functions ϕ_i (i=1,2) be Hölder continuous in $[-\tau,0]\times\overline{\Omega}$ with $\phi_i(t,x)\geq 0$. Let (u_1,u_2) satisfy* (1.6)–(1.9). *In addition to* (H1) *assume further that*

$$a_{11}r_2 \geq \alpha_2 R_1 I$$

then $\underline{M}_2 = 0 = \overline{M_2}$ and

$$\frac{r_1}{A_{11}} \leq \underline{M}_1 \leq \overline{M}_1 \leq \frac{R_1}{a_{11}}\,. \tag{3.15}$$

The above theorem is a consequence of the second part of the Proposition 3.1 and (3.4).

As a consequence of Theorem 3.4 and Theorem 3.5, when the coefficients are constants we obtain [7, Theorems 2.1 and 2.2]. However, here we provided another way to prove these two theorem.

3.1. **Discussion.** Motivated by the work on [6], in this paper we have incorporated variable coefficients in to a Lotka Volterra type predator-prey model with diffusion and stage structure. By using the coupled upper-lower solutions technique, we give sufficient conditions to guarantee the existence of a global attractor for the system. Biologically condition (15) says that lower bound of the death rate of the mature predator and the lower bound of the intra specific competition rate of the prey are sufficiently low. Condition (16) means that the inter specific growth rate of the prey and the inter specific interaction between the prey and the mature predator are low enough. Theorem 3.5 ecologically implies that the predator population will go to extinction but the prey population will persist and this occurs if the death rate of the mature predator population and the intra specific competition rate are high and the conversion rate of the predator and the intrinsic growth rate of the prey are sufficiently low. According to Theorem 3.4 and Theorem 3.5 we note that the bounds do not depend on the diffusion coefficients D_1 and D_2, that is the global attractors found depend only on the reaction terms.

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