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NONEXISTENCE OF SOLUTIONS TO SOME BOUNDARY-VALUE PROBLEMS FOR SECOND-ORDER ORDINARY DIFFERENTIAL EQUATIONS

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Abstract. We present a method to obtain concrete sufficient conditions which guarantee non-existence of solutions lying into a prescribed domain of six two- or three-point boundary value problems for second-order ordinary differential equations.

1. Introduction

Let I be the interval [0,1] of the real line $\mathbb{R}$ and let $C(I)$ be the Banach space of all continuous functions $x : I \to \mathbb{R}$, endowed with the sup-norm $\|\cdot\|$. In this article, we investigate the non-existence of a solution x of the equation

$$x''(t) + (Fx)(t) = 0, \quad \text{a. a. } t \in I, \tag{1.1}$$

satisfying one of the following conditions:

$$x(0) = 0, \quad x'(1) = \alpha x'(0), \tag{1.2}$$

$$x(0) = 0, \quad x(1) = \alpha x'(0), \tag{1.3}$$

$$x'(0) = 0, \quad x(1) = \alpha x(\eta), \tag{1.4}$$

$$x'(0) = 0, \quad x'(1) = \alpha x(0), \tag{1.5}$$

$$x(0) = 0, \quad x(1) = \alpha x(\eta), \tag{1.6}$$

$$x(0) = \alpha x'(0), \quad x(1) = 0, \tag{1.7}$$

and lying into a prescribed domain of the space $C(I)$. Here assume that $\alpha \geq 0$ and $\eta \in [0,1]$. The dependence of the response F on the function x might be in a moment or a functional way.

Some publications which deal with the existence of positive solutions of equations of the form (1.1) lying in a certain domain, associated with the conditions (or the multi-point version of them) respectively, are, for instance, [3, 4, 14, 16, 17] for (1.2); [18] for (1.3), [1, 13] for (1.4); [7] for (1.5); [2, 5, 6, 8, 9, 10, 11, 15, 19, 20, 21, 22, 27, 23, 25, 24, 26, 28] for (1.6); [12] for (1.7). (See, also, the references therein)

In most of these cases the response factor $(Fx)(t)$ has the Hammerstein form $q(t)f(x(t))$ and the quotient $f(u)/u$ plays a central role. In particular, its least

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and upper limiting values at 0+ and at $+\infty$ are combined with some arguments related to the Green's function, in order to ensure the applicability of a method leading to the existence of a fixed point of an appropriate integral operator. But, as the sufficient conditions are important for the existence of solutions, the necessary conditions are (more or less) equally important.

Working in this direction, in this paper we provide some rather simple sufficient conditions for the nonexistence of (positive) solutions, which lie in an angular domain of the origin. Our discussion refers to the existence of a real number $\rho > 0$ such that no solution of the problem exists satisfying the inequality

$$|(Fx)(t)| < \rho\|x\|,$$

for almost all $t \in I$. This fact implies that, if we have the response $q(t)f(x(t))$, then we can proceed further to obtain a more concrete domain not containing solutions. Indeed, assume that $(Fx)(t)$ is of the form $q(t)f(x(t))$, where, in the simplest case, q is measurable and essentially bounded and f is nondecreasing. Then, the general result is formulated as follows: *There is no solution x of equation* (1.1) *satisfying the conditions* $(1, j)$ *such that*

$$f(\|x\|) < \frac{\rho_j}{\|q\|_\infty}\|x\|,$$

where $j = 2, \dots, 7$.

2. Problem (1.2)-(1.1)

We start with the following theorem.

Theorem 2.1. *Assume that F is a function defined in a domain $\mathcal{D}(F)$ of the space $C(I)$ such that for each $x \in \mathcal{D}(F)$ the value $Fx : I \to \mathbb{R}$ is a Lebesgue measurable function.*

(i) *If $\alpha > 1$, then there is no positive solution x of the problem* (1.2)-(1.1) *lying in $\mathcal{D}(F)$ and such that $(Fx)(t) \geq 0$, a.e. on I.*

(ii) *If $\alpha \in [0, 1]$, then there is no solution x of problem* (1.2)-(1.1) *lying in $\mathcal{D}(F)$ and such that*

$$\operatorname{ess\,sup} \frac{(Fx)(t)}{x^2(t)} < +\infty. \tag{2.1}$$

Proof. (i) Assume that $\alpha > 1$ and let x be a positive solution of the problem in $\mathcal{D}(F)$. Consider a real number $\lambda \neq 0$ and write equation (1.1) in the form

$$x''(t) + \lambda x'(t) = \lambda x'(t) - (Fx)(t).$$

Multiply both sides with $e^{\lambda t}$ and take

$$\big(x'(t)e^{\lambda t}\big)' = [x''(t) + \lambda x'(t)]e^{\lambda t} = [\lambda x'(t) - (Fx)(t)]e^{\lambda t}.$$

Integrating from 0 to t, we obtain

$$\begin{aligned} x'(t)e^{\lambda t} &= x'(0) + \lambda \int_0^t x'(s)e^{\lambda s}ds - \int_0^t (Fx)(s)e^{\lambda s}ds. \\ &= x'(0) + \lambda x(t)e^{\lambda t} - \lambda x(0) - \int_0^t z(s)e^{\lambda s}ds, \end{aligned}$$

where $z(s) := \lambda^2 x(s) + (Fx)(s)$. Thus we have

$$x'(t) - \lambda x(t) = x'(0)e^{-\lambda t} - \int_0^t z(s)e^{-\lambda(t-s)}ds. \tag{2.2}$$

Multiplying both sides by $e^{-\lambda t}$, we obtain

$$\big(x(t)e^{-\lambda t}\big)' = [x'(t) - \lambda x(t)]e^{-\lambda t} = x'(0)e^{-2\lambda t} - \int_0^t z(s)e^{-\lambda(2t-s)}ds.$$

Integrate both sides from 0 to t and take

$$\begin{aligned} x(t)e^{-\lambda t} &= x'(0)\int_0^t e^{-2\lambda s}ds - \int_0^t\int_0^r z(s)e^{-\lambda(2r-s)}dsdr \\ &= \frac{x'(0)}{2\lambda}(1-e^{-2\lambda t}) - \int_0^t z(s)e^{\lambda s}\int_s^t e^{-2\lambda r}drds \\ &= \frac{x'(0)}{2\lambda}(1-e^{-2\lambda t}) - \frac{1}{2\lambda}\int_0^t z(s)e^{\lambda s}(e^{-2\lambda s} - e^{-2\lambda t}), \end{aligned}$$

because of (1.2). Finally we obtain

$$x(t) = \frac{x'(0)}{\lambda}\sinh(\lambda t) - \frac{1}{\lambda}\int_0^t z(s)\sinh(\lambda(t-s))ds. \tag{2.3}$$

By using (1.2) and (2.2) it follows that

$$\begin{aligned} \alpha x'(0) = x'(1) &= \lambda x(1) + x'(0)e^{-\lambda} - \int_0^1 z(s)e^{-\lambda(1-s)}ds \\ &= x'(0)\cosh(\lambda) - \int_0^1 z(s)\cosh(\lambda(1-s))ds \end{aligned}$$

and therefore the solution x must satisfy

$$x'(0)(\cosh(\lambda) - \alpha) = \int_0^1 z(s)\cosh(\lambda(1-s))ds, \tag{2.4}$$

for all $\lambda \neq 0$. Observe that the right side is a positive quantity, while the left side depends on λ. Hence, if it holds $x'(0) > 0$, choose λ such that $\cosh(\lambda) < \alpha$ and if $x'(0) < 0$, choose λ such that $\cosh(\lambda) > \alpha$, to get a contradiction.

(ii) Next assume that $\alpha \in [0,1]$. Let x be a solution satisfying relation (2.1). Choose λ negative large enough such that

$$z(t) := (Fx)(t) + \lambda x^2(t) < 0,$$

for a.a. $t \in I$. This and (2.4) imply that $x'(0) < 0$ and so, due to the fact that $x(0) = 0$, the solution x can not be positive. The proof is complete. $\square$

In the sequel we shall assume that $0 < \alpha < 1$ and moreover the function F satisfies the following condition:

(C) F is a function defined in a domain $\mathcal{D}(F)$ of the space $C(I)$ and for each $x \in \mathcal{D}(F)$ the value $Fx : I \to \mathbb{R}$ is an element of $\mathcal{L}_\infty$.

For each $\rho > 0$ we shall denote by $\mathcal{A}_\rho$ the set of all functions $x \in \mathcal{D}(F)$ satisfying the inequality

$$\|Fx\|_\infty < \rho\|x\|.$$

Theorem 2.2. *Assume that F satisfies condition (C). Then there is none $x \in \mathcal{A}_{\rho_1}$ which solves problem* (1.2)-(1.1)*, where*
$$\rho_1 := 1 - \alpha.$$

Proof. Assume that a solution exists satisfying the requirements of the theorem and take ρ' such that $\rho' < \rho_1$ and $\|Fx\|_\infty < \rho'\|x\|$. Since $1-\alpha$ is the maximum of the function
$$\psi(\lambda) := \frac{\cosh(\lambda) - \sinh^2(\lambda) - \alpha}{\sinh^2(\lambda)}\lambda^2, \quad \lambda \geq 0,$$
we can choose $\lambda \geq 0$ such that
$$\rho' < \psi(\lambda). \tag{2.5}$$

Next, as in Theorem 2.1, we obtain relation (2.3). By using relation (2.2) and the boundary condition (1.2) we obtain
$$\alpha x'(0) = x'(1) = \lambda x(1) + x'(0)e^{-\lambda} - \int_0^1 z(s)e^{-\lambda(1-s)}ds,$$
and due to (2.3) it follows that
$$x'(0) = \frac{1}{\cosh(\lambda) - \alpha}\int_0^1 z(s)\cosh(\lambda(1-s))ds.$$
Hence we have
$$x(t) = \frac{\sinh(\lambda t)}{\lambda[\cosh(\lambda) - \lambda\alpha]}\int_0^1 z(s)\cosh(\lambda(1-s))ds - \frac{1}{\lambda}\int_0^t z(s)\sinh(\lambda(t-s))ds. \tag{2.6}$$

Assume that $\|x\| = x(t_0)$, for some $t_0 \in [0,1]$. Dividing both sides of (2.6) by $x(t_0)$, we obtain
$$\lambda[\cosh(\lambda) - \alpha] \leq \sinh(\lambda)\int_0^1 \Big[\lambda^2\frac{x(s)}{\|x\|} + \frac{|(Fx)(s)|}{\|x\|}\Big]\cosh(\lambda(1-s))ds.$$
From this relation we obtain
$$\lambda(\cosh(\lambda) - \alpha) < \sinh(\lambda)\int_0^1 [\lambda^2 + \rho']\cosh(\lambda(1-s))ds$$
and so
$$\lambda^2(\cosh(\lambda) - \alpha) < \sinh^2(\lambda)[\lambda^2 + \rho'].$$
The latter contradicts to (2.5) and so there is no solution of the problem. $\square$

3. Problem (1.3)-(1.1)

Before we will discuss the problem, we need some information about the function defined by
$$\phi(\lambda) := \frac{\sinh(\lambda)}{\lambda}(2 - \cosh(\lambda)), \quad \lambda \in [0,1].$$
Observe that $2 - \cosh(\lambda) > 0$ for all $\lambda \in [0,1]$. Also, since $\phi(0) = 1$ we conclude that for each $\alpha \in (0,1)$ there is a $\lambda \in (0,1)$ such that $\alpha < \phi(\lambda)$. Thus the set
$$D_\alpha := \{\lambda \in [0,1) : \phi(\lambda) > \alpha\}$$
is nonempty and it contains a right neighborhood of 0.

Theorem 3.1. *Assume that F satisfies condition* (C). *Then there is none $x \in \mathcal{A}_{\rho_2}$ which solves problem* (1.3)-(1.1), *where*

$$\rho_2 := 2(1-\alpha).$$

Proof. Assume that a solution exists satisfying the requirements of the theorem and take ρ' such that $\rho' < \rho_2$ and $\|Fx\|_\infty < \rho'\|x\|$. Since the number $2(1-\alpha)$ is the maximum of the quantity

$$\psi_1(\lambda) := \frac{\phi(\lambda) - \alpha}{[\cosh(\lambda) - 1]\sinh(\lambda)}\lambda^3, \quad \lambda \geq 0,$$

we can choose $\lambda > 0$ such that

$$\rho' < \psi_1(\lambda). \tag{3.1}$$

Next, following the same method as in Theorem 2.1, we obtain relation (2.3). By using the boundary condition (1.3) we obtain

$$\alpha x'(0) = x(1) = \frac{x'(0)}{\lambda}\sinh(\lambda) - \frac{1}{\lambda}\int_0^1 z(s)\sinh(\lambda(1-s))ds,$$

from which it follows that

$$x'(0) = \frac{1}{\sinh(\lambda) - \lambda\alpha}\int_0^1 z(s)\sinh(\lambda(1-s))ds.$$

Hence we have

$$x(t) = \frac{\sinh(\lambda t)}{\lambda[\sinh(\lambda) - \lambda\alpha]}\int_0^1 z(s)\sinh(\lambda(1-s))ds - \frac{1}{\lambda}\int_0^t z(s)\sinh(\lambda(t-s))ds. \tag{3.2}$$

From here and our assumptions we conclude that $\sinh(\lambda) > \lambda\alpha$.

Next assume that $\|x\| = x(t_0)$, for some $t_0 \in [0,1]$. Dividing both sides of (3.2) by $x(t_0)$, we obtain

$$\lambda[\sinh(\lambda) - \lambda\alpha] \leq \sinh(\lambda)\int_0^1 \Big[\lambda^2\frac{x(s)}{\|x\|} + \frac{|(Fx)(s)|}{\|x\|}\Big]\sinh(\lambda(1-s))ds.$$

From this relation we obtain

$$\lambda(\sinh(\lambda) - \lambda\alpha) < \sinh(\lambda)\int_0^1 [\lambda^2 + \rho']\sinh(\lambda(1-s))ds$$

and so

$$\lambda^2(\sinh(\lambda) - \lambda\alpha) < \sinh(\lambda)[\lambda^2 + \rho'](\cosh(\lambda) - 1).$$

The latter contradicts to (3.1) and so there is no solution of the problem. $\square$

4. Problem (1.4)-(1.1)

Theorem 4.1. *Assume that F satisfies condition* (C). *Then there is no $x \in \mathcal{A}_{\rho_3}$ that solves problem* (6.2)-(1.2), *where*

$$\rho_3 := \sup_{\lambda>0}\frac{2 - \cosh(\lambda) - \alpha e^{\lambda(\eta-1)}}{\cosh(\lambda) - 1}\lambda^2.$$

Proof. Assume that a solution exists satisfying the requirements of the theorem and take ρ' such that $\rho' < \rho_3$ and $\|Fx\|_\infty < \rho'\|x\|$. Choose $\lambda > 0$ such that

$$\rho' < \frac{2 - \cosh(\lambda) - \alpha e^{\lambda(\eta-1)}}{\cosh(\lambda) - 1}\lambda^2. \tag{4.1}$$

Following the same method as in Theorem 2.1, we obtain relation (2.2), which due to (1.4) becomes

$$x'(t) - \lambda x(t) = -\int_0^t z(s)e^{-\lambda(t-s)}ds. \tag{4.2}$$

Multiplying both sides with $e^{\lambda t}$ we obtain

$$x(t) = x(0)e^{\lambda t} - \int_0^t z(s)\sinh(\lambda(t-s))ds. \tag{4.3}$$

From this relation and (1.4) we derive

$$x(0) = \frac{1}{\lambda(e^\lambda - \alpha e^{\lambda\eta})}\Big[\int_0^1 z(s)\sinh(\lambda(1-s))ds - \alpha\int_0^\eta z(s)\sinh(\lambda(\eta-s))ds\Big],$$

and therefore we have

$$\begin{aligned} x(t) = \frac{e^{\lambda t}}{\lambda(e^\lambda - \alpha e^{\lambda\eta})}\Big[&\int_0^1 z(s)\sinh(\lambda(1-s))ds \\ &- \alpha\int_0^\eta z(s)\sinh(\lambda(\eta-s))ds\Big] - \frac{1}{\lambda}\int_0^t z(s)\sinh(\lambda(t-s))ds. \end{aligned} \tag{4.4}$$

Assume that $\|x\| = x(t_0)$, for some $t_0 \in [0,1]$. Dividing both sides of (4.4) by $x(t_0)$, we obtain

$$\lambda^2[e^\lambda - \alpha e^{\lambda\eta}] \le e^\lambda[\lambda^2 + \rho'](\cosh(\lambda) - 1)).$$

The latter contradicts to (4.1) and so there is no solution of the problem. □

The following table shows the values of the bound ρ_3 for some values of the coefficient α and the argument η.

η \ α (ρ_3)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	1.808	1.628	1.464	1.307	1.160	1.020	0.888	0.761	0.639
0.1	1.806	1.625	1.453	1.291	1.137	0.990	0.849	0.714	0.585
0.2	1.805	1.620	1.444	1.275	1.114	0.959	0.811	0.667	0.529
0.3	1.804	1.615	1.434	1.260	1.092	0.929	0.772	0.620	0.472
0.4	1.803	1.611	1.426	1.246	1.071	0.900	0.735	0.573	0.416
0.5	1.802	1.608	1.418	1.233	1.051	0.873	0.699	0.528	0.361
0.6	1.801	1.605	1.412	1.221	1.034	0.849	0.667	0.487	0.310
0.7	1.800	1.603	1.407	1.212	1.019	0.828	0.639	0.451	0.265
0.8	1.800	1.599	1.403	1.205	1.009	0.813	0.618	0.423	0.230
0.9	1.800	1.600	1.400	1.201	1.002	0.803	0.604	0.379	0.207

5. Problem (1.5)-(1.1)

Theorem 5.1. *Assume that F satisfies condition* (C)*. Then there is none $x \in \mathcal{A}_{\rho_4}$ which solves problem* (1.5)-(1.1)*, where*

$$\rho_4 := \sup_{\lambda>0}\frac{\lambda[e^\lambda - \alpha]}{\lambda\cosh(\lambda) - \lambda + 1 - e^{-\lambda}} - \lambda^2.$$

Proof. Assume that a solution x exists satisfying the requirements of the theorem and take ρ' such that $\rho' < \rho_4$ and $\|Fx\|_\infty < \rho'\|x\|$. Choose $\lambda > 0$ such that

$$\rho' < \frac{\lambda[e^\lambda - \alpha]}{\lambda\cosh(\lambda) - \lambda + 1 - e^{-\lambda}} - \lambda^2. \tag{5.1}$$

Next, following the same method as in Theorem 4.1, we obtain relation (4.3), which because of (1.5) gives

$$\alpha x(0) = \frac{1}{\lambda e^\lambda - \alpha}\int_0^1 z(s)\Big[\sinh(\lambda(1-s)) + e^{-\lambda(1-s)}\Big]ds.$$

Therefore,

$$x(t) = \frac{e^{\lambda t}}{\lambda e^\lambda - \alpha)}\int_0^1 z(s)\Big[\sinh(\lambda(1-s)) + e^{-\lambda(1-s)}\Big]ds - \int_0^t z(s)e^{\lambda(t-s)}ds. \tag{5.2}$$

Next assume that $\|x\| = x(t_0)$, for some $t_0 \in [0,1]$. Dividing both sides of (5.2) by $x(t_0)$, we, finally, obtain

$$\lambda[\lambda - e^{-\lambda}\alpha] \le [\lambda^2 + \rho'](\lambda\cosh(\lambda) - \lambda + 1 - e^{-\lambda}).$$

The latter contradicts (5.1) and so there is no solution of the problem. □

The parameter ρ_4 for various values of the coefficient α is given in the following table.

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
ρ_4	1.480	1.375	1.271	1.167	1.063	0.961	0.859	0.757	0.657

6. Problem (1.6)-(1.1)

Theorem 6.1. *Assume that F satisfies condition* (C) *and, moreover, assume that $0 < \alpha < 1$ and $0 < \eta < 1$. Then there is none $x \in \mathcal{A}_{\rho_5}$ which solves the problem* (1.6)-(1.1)*, where*

$$\rho_5 := \sup_{\lambda>0}\frac{\lambda^2[\sinh(\lambda) - \alpha\sinh(\eta\lambda)]}{\sinh(\lambda)\big(\cosh(\lambda) - 1\big)} - \lambda^2.$$

Proof. Assume that a solution x exists satisfying the requirements of the theorem and take ρ' such that $\rho' < \rho_5$ and $\|Fx\|_\infty < \rho'\|x\|$. Choose $\lambda > 0$ such that

$$\rho' < \frac{\lambda^2[\sinh(\lambda) - \alpha\sinh(\eta\lambda)]}{\sinh(\lambda)\big(\cosh(\lambda) - 1\big)} - \lambda^2. \tag{6.1}$$

Next, following the same method as in Theorem 2.1, we obtain relation (2.3), which because of (1.6) gives

$$x'(0) = \frac{1}{\sinh(\lambda) - \alpha\sinh(\lambda\eta)}\Big[\int_0^1 z(s)\sinh(\lambda(1-s))ds - \int_0^\eta z(s)\sinh(\lambda(\eta-s))ds\Big].$$

Therefore,

$$\begin{aligned} x(t) = \frac{\sinh(\lambda t)}{\lambda\big(\sinh(\lambda) - \alpha\sinh(\lambda\eta)\big)}\Big[\int_0^1 z(s)\sinh(\lambda(1-s))ds \\ - \int_0^\eta z(s)\sinh(\lambda(\eta-s))ds\Big] - \frac{1}{\lambda}\int_0^t z(s)\sinh(\lambda(t-s))ds. \end{aligned} \tag{6.2}$$

Next assume that $\|x\| = x(t_0)$, for some $t_0 \in [0,1]$. Dividing both sides of (6.2) by $x(t_0)$, we, finally, obtain

$$\lambda^2[\sinh(\lambda) - \alpha\sinh(\lambda\eta)] \le [\sinh(\lambda)[\lambda^2 + \rho'](\cosh(\lambda) - 1).$$

The latter contradicts (6.1) and so there is no solution of the problem. □

The following table shows the values of the bound ρ_5 for some values of the coefficient α and the argument η.

α \ η (ρ_5)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	1.978	1.959	1.939	1.919	1.899	0.878	0.858	0.839	0.818
0.2	1.958	1.919	1.878	1.839	1.798	1.759	1.718	1.679	1.637
0.3	1.938	1.919	1.878	1.839	1.798	1.759	1.718	1.518	1.458
0.4	1.917	1.838	1.758	1.679	1.598	1.519	1.439	1.355	1.267
0.5	1.897	1.797	1.697	1.599	1.498	1.398	1.298	1.195	1.097
0.6	1.879	1.759	1.638	1.519	1.399	1.279	1.159	1.037	0.918
0.7	1.858	1.719	1.578	1.439	1.296	1.159	1.017	0.879	0.736
0.8	1.839	1.678	1.518	1.359	1.195	1.039	0.879	0.719	0.559
0.9	1.819	1.637	1.458	1.278	1.099	0.917	0.738	0.559	0.379

7. Problem (1.7)-(1.1)

Theorem 7.1. *Assume that F satisfies condition* (C). *Then there is no $x \in \mathcal{A}_{\rho_6}$ which solves problem* (1.7)-(1.1), *where*

$$\rho_6 := \sup_{\lambda>0} \frac{\lambda^2[\alpha\lambda + \cosh(\lambda)]}{\sinh(\lambda)\big(\lambda\alpha e^{\lambda} + \cosh(\lambda)\big)} - \lambda^2.$$

Proof. Assume that a solution x exists satisfying the requirements of the theorem and take ρ' such that $\rho' < \rho_5$ and $\|Fx\|_\infty < \rho'\|x\|$. Choose $\lambda > 0$ such that

$$\rho' < \frac{\lambda^2[\sinh(\lambda) - \alpha\sinh(\eta\lambda)]}{\sinh(\lambda)\big(\cosh(\lambda) - 1\big)} - \lambda^2. \tag{7.1}$$

Next, following the same method as previously, we obtain

$$x'(0) = \frac{1}{\alpha\lambda + \cosh(\lambda)} \int_0^1 z(s)\sinh(\lambda(1-s))ds.$$

Therefore,

$$x(t) = \frac{\lambda\alpha e^{\lambda t} + \sinh(\lambda t)}{\lambda\big(\alpha\lambda + \cosh(\lambda)\big)} \int_0^1 z(s)\cosh(\lambda(1-s))ds - \frac{1}{\lambda}\int_0^t z(s)\cosh(\lambda(t-s))ds. \tag{7.2}$$

Next assume that $\|x\| = x(t_0)$, for some $t_0 \in [0,1]$. Dividing both sides of (7.2) by $x(t_0)$, we, finally, obtain

$$\lambda^2[\alpha\lambda + \cosh(\lambda)] \le \sinh(\lambda)[\lambda^2 + \rho'](\lambda\alpha e^{\lambda} + \sinh(\lambda)).$$

The latter contradicts (7.1) and so there is no solution of the problem. □

The following tableshows the values of the bound ρ_6, for some values of the coefficient α.

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
ρ_6	0.223	0.216	0.211	0.206	0.202	0.198	0.195	0.192	0.190

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