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BLOW-UP OF SOLUTIONS FOR A NONLINEAR WAVE EQUATION WITH NONNEGATIVE INITIAL ENERGY

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Abstract. In this article, we study a wave equation with nonlinear boundary damping and interior source term. We prove two blow-up results with nonnegative initial energy; thus we extend the blow-up results by Feng et al [5].

1. Introduction

In this article, we study the following wave equation with nonlinear boundary damping and interior source term

$$\begin{gathered} y_{tt}(x,t) - y_{xx}(x,t) = |y(x,t)|^{p-1}y(x,t), \quad (x,t) \in (0,L)\times(0,T), \\ y(0,t) = 0, \quad y_x(L,t) = -|y_t(L,t)|^{m-1}y_t(L,t), \quad t\in[0,T), \\ y(x,0) = y^0(x), \quad y_t(x,0) = y^1(x), \quad x\in[0,L], \end{gathered} \tag{1.1}$$

where $(0,L)$ is a bounded open interval in $\mathbb{R}$, $m>1$, $p>1$. The wave equation with interior damping term has been extensively studied and several results concerning existence, asymptotic behavior and blow-up have been established. When $m=1$, Levine [7, 8] proved that the solution blows up in finite time with negative initial energy. When $m>1$, Georgiev and Todorova [6] extended this result and established a global existence result if $m \ge p$ and a blow-up result if $m<p$ for sufficiently large initial data. Later Messaoudi [12] improved [6] by considering only negative initial energy. The wave equation with boundary source term has also been extensively studied. Vitillaro [15] proved the existence of a global solution when $p \le m$ or the initial data are inside the potential well. In [19], Zhang and Hu proved the decay result when the initial data are inside a stable set, and the blow-up result when $p>m$ and the initial data is inside an unstable set. For other wave equations with nonlinear source and damping terms, we can also refer the reader to [1, 2, 3, 4, 10, 11, 13, 16, 17, 18] and references therein.

Recently, Feng et al [5] considered (1.1) and obtained the blow-up results with one of the following conditions: (A) $2m<p+1$ and $E(0)<0$; (B) $2m\ge p+1$, $E(0)<0$, and $L>\frac{4p}{(p-1)(p+1)}$. Later, Li et al [9] studied the interaction between the interior damping $y_t(x,t)$ and the boundary source $|y(L,t)|^{p-1}y(L,t)+by(L,t)$ and

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established three sufficient conditions for the blow-up results with some necessary restriction on b when the initial energy is positive or negative.

Motivated by [9], we intend to extend the results in [5] with nonnegative initial energy. For this purpose, we use an improved relationship between E_1 and $\|y\|_{p+1}^{p+1}$ which is given in Lemma 3.1 below. This article is organized as follows. In Section 2, we present some notation needed for our work and state our main results. In Section 3, we give the proof of Theorem 2.2. Section 4 is devoted to the proof Theorem 2.3.

2. Notation and main results

We define the following functionals

$$E(t) = \frac{1}{2}\|y_t(t)\|_2^2 + \frac{1}{2}\|y_x(t)\|_2^2 - \frac{1}{p+1}\|y(t)\|_{p+1}^{p+1}, \tag{2.1}$$

$$I(t) = \|y_x(t)\|_2^2 - \|y(t)\|_{p+1}^{p+1}, \tag{2.2}$$

and as in [5] we introduce the notation: $\|\cdot\|_q = \|\cdot\|_{L^q(0,L)}$ and the Hilbert space

$$H^1_{\rm left}(0,L) := \{u \in H^1(0,L) : u(0) = 0\}. \tag{2.3}$$

Set

$$E_1 := \Big(\frac{1}{2} - \frac{1}{p+1}\Big)\alpha_0, \quad \alpha_0 := C_*^{-\frac{2(p+1)}{p-1}}, \tag{2.4}$$

where C_* is the optimal constant of the Sobolev embedding $\|y\|_{p+1} \le C_*\|y_x\|_2$, for any $y \in H^1_{\rm left}(0,L)$.

Next, we give a the existence of a local solution.

Theorem 2.1 ([5, Theorem 2.1])**.** *Assume that $(y^0, y^1) \in H^1_{\rm left}(0,L) \times L^2(0,L)$. Then (1.1) has a unique local solution $y(x,t)$ satisfying*

$$y(x,t) \in C(0,T_m; H^1_{\rm left}(0,L)), \quad y_t(x,t) \in C(0,T_m; L^2(0,L)),$$
$$y_t(L,t) \in L^{m+1}(0,T_m)$$

for some $T_m > 0$, and the energy equality

$$E(t) + \int_0^t |y_t(L,\tau)|^{m+1} d\tau = E(0) \tag{2.5}$$

holds for $0 \le t < T_m$.

Our main results are as follows.

Theorem 2.2. *Let $y(x,t)$ be a solution of problem (1.1). Assume that $2m < p+1$, $I(0) < 0$ and for any fixed $0 < \theta < 1$, $0 \le E(0) < \theta E_1$. Then the solution blows up in finite time.*

Theorem 2.3. *Let $y(x,t)$ be a solution of problem (1.1). Assume that $2m \ge p+1$, $I(0) < 0$ and for any fixed $0 < \theta < 1$, $0 \le E(0) < \theta E_1$. Furthermore, we assume that*

$$L > \frac{4p + \frac{2p(p-1)}{p+1-\theta(p-1)}}{(p-1)(p+1)[1 - \frac{2}{p+1-\theta(p-1)}]}, \tag{2.6}$$

then the solution blows up in finite time.

Remark 2.4. When $E(0) < 0$, the blow-up results have been proved in [5]. So, we consider here only the case $E(0) \ge 0$.

Remark 2.5. In the case $2m \geq p+1$, we note that the similar restriction on L as (2.6) has been used in [5], which means that the larger the interval $(0, L)$ is, the less the boundary damping effect. It is still the case when $I(0) < 0$ and $0 \leq E(0) < \theta E_1$.

3. Proof of Theorem 2.2

In this section, we consider the blow-up result in the case $2m < p+1$. For this purpose, we give the following lemmas first.

Lemma 3.1. *Let $y(x,t)$ be a solution of problem (1.1) with $0 \leq E(0) < \theta E_1$ and $I(0) < 0$. Then there exists a positive constant $0 < \beta < 1$ such that*

$$E_1 < \beta \frac{p-1}{2(p+1)} \|y(x,t)\|_{p+1}^{p+1}, \quad \forall t > 0. \tag{3.1}$$

Proof. We adopt the manner which was first introduced in [14]. From (2.1) and Sobolev embedding, we have

$$E(t) \geq \frac{1}{2}\|y_x\|_2^2 - \frac{1}{p+1}\|y\|_{p+1}^{p+1} \geq \frac{1}{2}\|y_x\|_2^2 - \frac{C_*^{p+1}}{p+1}\|y_x\|_2^{p+1}.$$

Let $h(\xi) = \frac{1}{2}\xi - \frac{C_*^{p+1}}{p+1}\xi^{\frac{p+1}{2}}$, then

$$E(t) \geq h(\xi) \quad \text{with } \xi = \|y_x\|_2^2.$$

It is easy to see that $h(\xi)$ is strictly increasing on $[0, \alpha_0)$, strictly decreasing on $(\alpha_0, +\infty)$ and takes its maximum value E_1 at α_0.

Since $I(0) < 0$, we have

$$\|y_x^0\|_2^2 < \|y^0\|_{p+1}^{p+1} \leq C_*^{p+1}\|y_x^0\|_2^{p+1},$$

which leads to

$$\|y_x^0\|_2^2 > \alpha_0, \quad \text{for } \alpha_0 \text{ defined by (2.4)}.$$

Furthermore, since

$$E_1 > E(0) \geq E(t) \geq h(\|y_x\|_2^2), \quad \forall t \geq 0,$$

there exists no time t^* such that $\|y_x(t^*)\|_2^2 = \alpha_0$. By the continuity of $\|y_x\|_2^2$, we obtain

$$\|y_x\|_2^2 > \alpha_0, \quad \forall t \geq 0.$$

On the other hand, we have

$$\frac{1}{p+1}\|y\|_{p+1}^{p+1} \geq -E(0) + \frac{1}{2}\|y_t\|_2^2 + \frac{1}{2}\|y_x\|_2^2 > -\theta E_1 + \frac{1}{2}\alpha_0 = \Big(\frac{p+1}{p-1} - \theta\Big)E_1,$$

which gives

$$E_1 < \frac{p-1}{2(p+1)} \frac{2}{(p+1) - \theta(p-1)}\|y\|_{p+1}^{p+1}.$$

Taking $\beta = \frac{2}{(p+1)-\theta(p-1)} \in (0,1)$, inequality (3.1) follows. □

Set

$$H(t) = \theta E_1 - E(t),$$

then it is clear that $H(t)$ is increasing, $H(t) \geq H(0) > 0$ and

$$H(t) \leq \frac{\theta\beta(p-1) + 2}{2(p+1)}\|y\|_{p+1}^{p+1}. \tag{3.2}$$

Lemma 3.2. *Under the assumptions of Lemma 3.1, there exists a positive constant C such that*

$$\|y\|_{p+1}^{s} \le C\|y\|_{p+1}^{p+1}, \tag{3.3}$$

for any $2 \le s \le p+1$.

Proof. If $\|y\|_{p+1}^{p+1} \ge 1$, then $\|y\|_{p+1}^{s} \le C\|y\|_{p+1}^{p+1}$, since $s \le p+1$. If $\|y\|_{p+1}^{p+1} < 1$, then $\|y\|_{p+1}^{s} \le \|y\|_{p+1}^{2}$, since $2 \le s$. Using the Sobolev embedding inequality, (2.1), and Lemma 3.1, we have

$$\|y\|_{p+1}^{2} \le C_*\|y_x\|_2^2 \le 2C_*\Big(E(t)+\|y\|_{p+1}^{p+1}\Big) \le 2C_*\Big(E_1+\|y\|_{p+1}^{p+1}\Big) \le C\|y\|_{p+1}^{p+1}. \tag{3.4}$$

This completes the proof. $\square$

As in [5], we choose a constant r such that

$$0 < \max\Big\{\frac{2}{p+1}, \frac{m}{p+1-m}\Big\} < r < 1. \tag{3.5}$$

Then we infer that

$$2 \le m+1, m\frac{r+1}{r}, \frac{p+1}{2}(1+r) < p+1. \tag{3.6}$$

Lemma 3.3. *Under the assumptions of Lemma 3.1, there exists a positive constant C such that*

$$|y(L,t)|^{m+1} \le C\big[\|y\|_{p+1}^{m+1} + \|y\|_{p+1}^{m\frac{r+1}{r}} + \|y\|_{p+1}^{\frac{p+1}{2}(1+r)}\big]. \tag{3.7}$$

Proof. By using Lemme 3.2 and the proof of [5, Lemma 3.2], we obtain (3.7). $\square$

Set

$$L(t) = H^{1-\sigma}(t) + \varepsilon\int_0^L y(t)y_t\mathrm{d}x, \tag{3.8}$$

for ε small to be chosen later and

$$0 < \sigma < \min\Big\{\frac{p-1}{2(p+1)}, \frac{p-m}{m(p+1)}, \frac{1}{m} - \frac{1+r}{r(p+1)}, \frac{1}{m} - \frac{1+r}{2m}\Big\}. \tag{3.9}$$

Then we have the following lemma.

Lemma 3.4. *Under the assumptions of Lemma 3.1, there exists a positive constant C such that*

$$H^{\sigma m}(t)|y(L,t)|^{m+1} \le C\|y\|_{p+1}^{p+1}, \tag{3.10}$$

for any $2m < p+1$.

Proof. By using (3.2), Lemma 3.3, Lemma 3.2 and the proof of [5, Lemma 3.3], we complete the proof. $\square$

Now, we are ready to proof our first main result.

Proof of Theorem 2.2. Computing a derivative of (3.8) yields

$$\begin{aligned} L'(t) \ge{}& (1-\sigma)H^{-\sigma}(t)|y_t(L,t)|^{m+1} + 2\varepsilon\|y_t(t)\|_2^2 + 2\varepsilon H(t) - 2\varepsilon\theta E_1 \\ & - \varepsilon|y_t(L,t)|^m|y(L,t)| + \varepsilon\frac{p-1}{p+1}\|y(t)\|_{p+1}^{p+1}. \end{aligned} \tag{3.11}$$

Using Young's inequality and (3.1), we have

$$
\begin{aligned}
L'(t) \geq\ & (1-\sigma)H^{-\sigma}(t)|y_t(L,t)|^{m+1} + 2\varepsilon\|y_t(t)\|_2^2 + 2\varepsilon H(t) \\
& + \frac{\varepsilon(p-1)(1-\theta\beta)}{p+1}\|y(t)\|_{p+1}^{p+1} - \frac{m\varepsilon}{m+1}\delta^{-\frac{m+1}{m}}|y_t(L,t)|^{m+1} \\
& - \frac{\varepsilon}{m+1}\delta^{m+1}|y(L,t)|^{m+1}.
\end{aligned} \tag{3.12}
$$

Let $\delta^{m+1} = k^{-m}H^{\sigma m}$ for $k > 0$ to be chosen later, then from (3.12) and Lemma 3.4 we obtain

$$
\begin{aligned}
L'(t) \geq\ & \Big(1-\sigma-\frac{km\varepsilon}{m+1}\Big)H^{-\sigma}(t)|y_t(L,t)|^{m+1} + 2\varepsilon\|y_t(t)\|_2^2 \\
& + \varepsilon\Big[\frac{(p-1)(1-\theta\beta)}{p+1} - \frac{Ck^{-m}}{m+1}\Big]\|y(t)\|_{p+1}^{p+1}.
\end{aligned} \tag{3.13}
$$

Choose k large enough so that

$$
\frac{(p-1)(1-\theta\beta)}{p+1} - \frac{Ck^{-m}}{m+1} > 0,
$$

then (3.13) reduces to

$$
L'(t) \geq \Big(1-\sigma-\frac{km\varepsilon}{m+1}\Big)H^{-\sigma}(t)|y_t(L,t)|^{m+1} + \varepsilon\gamma\big[\|y_t(t)\|_2^2 + \|y(t)\|_{p+1}^{p+1}\big].
$$

where $\gamma > 0$ is the minimum of coefficients of $\|y_t(t)\|_2^2$ and $\|y(t)\|_{p+1}^{p+1}$. We continue the remaining part as that of [5, Theorem 2.2] to finish the proof. □

4. Proof of Theorem 2.3

In this section, we consider the blow-up result in the case of $2m \geq p+1$. Set

$$
G(t) = E_1 - E(t) + \epsilon\int_0^L xy_x(t)y_t(t)\mathrm{d}x + \rho\epsilon\int_0^L y_t(t)y(t)\mathrm{d}x, \tag{4.1}
$$

with $\rho \in \Big(\frac{2+\frac{\beta(p-1)}{2}}{(p-1)(1-\beta)}, \frac{L(p+1)}{2p}\Big)$, where β is given in the proof of Lemma 3.1 and ϵ is a small and positive constant satisfying

$$
G(0) = E_1 - E(0) + \epsilon\int_0^L xy_x^0y^1\mathrm{d}x + \rho\epsilon\int_0^L y^1y^0\mathrm{d}x > 0. \tag{4.2}
$$

Lemma 4.1. *Under the assumptions of Theorem 2.3, we have $G(t) > 0$ for all $t \geq 0$. And there exists a positive constant $\eta > 0$ such that*

$$
G'(t) \geq \eta[|y_t(L,t)|^{2m} + |y_t(L,t)|^2 + |y_t(L,t)|^{p+1}]. \tag{4.3}
$$

Proof. As in [5], using (1.1), (2.1) and Lemma 3.1, we arrive at

$$
\begin{aligned}
G'(t) \geq & |y_t(L,t)|^{m+1} + \frac{L}{2}\epsilon|y_t(L,t)|^2 + \frac{L}{2}\epsilon|y_t(L,t)|^{2m} + \frac{L\epsilon}{p+1}|y(L,t)|^{p+1} \\
& - [\epsilon + 2\rho\epsilon](E(t) - E_1) + 2\rho\epsilon\|y_t(t)\|_2^2 - \rho\epsilon|y_t(L,t)|^m|y(L,t)| \\
& + \epsilon\Big[\frac{p-1}{p+1}\rho - \frac{2}{p+1}\Big]\|y(t)\|_{p+1}^{p+1} - [\epsilon + 2\rho\epsilon]E_1 \\
\geq & |y_t(L,t)|^{m+1} + \frac{L}{2}\epsilon|y_t(L,t)|^2 + \frac{L}{2}\epsilon|y_t(L,t)|^{2m} + \frac{L\epsilon}{p+1}|y(L,t)|^{p+1} \\
& + 2\rho\epsilon\|y_t(t)\|_2^2 - \rho\epsilon|y_t(L,t)|^m|y(L,t)| \\
& + \epsilon\Big[\frac{p-1}{p+1}\rho - \frac{2}{p+1} - \frac{\beta(1+2\rho)(p-1)}{2(p+1)}\Big]\|y(t)\|_{p+1}^{p+1}.
\end{aligned} \tag{4.4}
$$

Using the choice of ρ and Young's inequality, we obtain

$$
\begin{aligned}
G'(t) \geq & \frac{L}{2}\epsilon|y_t(L,t)|^2 + \frac{L}{2}\epsilon|y_t(L,t)|^{2m} + \frac{L\epsilon}{p+1}|y(L,t)|^{p+1} \\
& - \frac{p\rho\epsilon}{p+1}|y_t(L,t)|^{m\frac{p+1}{p}} - \frac{\epsilon\rho}{p+1}|y(L,t)|^{p+1}.
\end{aligned} \tag{4.5}
$$

Then by repeating similar computations as that of [5, Lemma 4.1], we complete the proof. $\square$

Set

$$
F(t) := G^{1-\alpha}(t) + \mu\int_0^L y_t(t)y(t)\mathrm{d}x \quad \text{with} \quad \alpha = \frac{p-1}{2(p+1)}, \tag{4.6}
$$

where μ is small enough to be chosen later.

Proof of Theorem 2.3. (Sketch) By repeating similar computations as that of [5, Theorem 2.3], from (3.1) we obtain

$$
\begin{aligned}
& F'(t) \\
& \geq (1-\alpha)G^{-\alpha}(t)\big(\eta - \mu CK^{\frac{1}{1-\alpha}}\big)[|y_t(L,t)|^{2m} + |y_t(L,t)|^2 + |y(L,t)|^{p+1}] \\
& \quad + 2\mu\|y_t\|_2^2 - 2\mu(E(t) - E_1) - 2\mu E_1 - \mu\alpha K^{-\frac{1}{\alpha}}G^{1-\alpha}(t) + \mu\frac{p-1}{p+1}\|y(t)\|_{p+1}^{p+1} \\
& \geq (1-\alpha)G^{-\alpha}(t)\big(\eta - \mu CK^{\frac{1}{1-\alpha}}\big)[|y_t(L,t)|^{2m} + |y_t(L,t)|^2 + |y(L,t)|^{p+1}] \\
& \quad + 2\mu\|y_t\|_2^2 - \mu\alpha K^{-\frac{1}{\alpha}}G^{1-\alpha}(t) + \frac{\mu(p-1)(1-\beta)}{p+1}\|y(t)\|_{p+1}^{p+1},
\end{aligned}
$$

where $K > 0$ to be chosen later. Applying the Cauchy-Schwarz inequality, the Sobolev embedding and Lemma 3.1 to (4.1), we obtain

$$
\begin{aligned}
G(t) & \leq E_1 - E(t) + L\epsilon\int_0^L |y_x(t)||y_t(t)|\mathrm{d}x + \rho\epsilon\int_0^L |y_t(t)||y(t)|\mathrm{d}x \\
& \leq \Big(\frac{L\epsilon}{2} + \frac{\rho\epsilon}{2} - \frac{1}{2}\Big)\|y_t(t)\|_2^2 + \Big(\frac{L\epsilon}{2} + \frac{\rho c_0^2\epsilon}{2} - \frac{1}{2}\Big)\|y_x(t)\|_2^2 \\
& \quad + \frac{2+\beta(p-1)}{2(p+1)}\|y(t)\|_{p+1}^{p+1},
\end{aligned} \tag{4.7}
$$

where c_0 is the Sobolev embedding constant of $\|y\|_2 \le c_0\|y_x\|_2$. From (2.1) and Lemma 3.1 it follows that

$$\begin{aligned} \|y_t(t)\|_2^2 + \|y_x(t)\|_2^2 &= 2E(t) + \frac{2}{p+1}\|y(t)\|_{p+1}^{p+1} \\ &< 2E_1 + \frac{2}{p+1}\|y(t)\|_{p+1}^{p+1} \\ &< \frac{2+\beta(p-1)}{p+1}\|y(t)\|_{p+1}^{p+1}. \end{aligned} \tag{4.8}$$

Combining (4.7) and (4.8), we obtain

$$G(t) \le C\|y(t)\|_{p+1}^{p+1}. \tag{4.9}$$

Continuing as in the proof of [5, Theorem 2.3] we can complete the proof. □

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