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OSCILLATION OF SOLUTIONS TO SECOND-ORDER HALF-LINEAR DIFFERENTIAL EQUATIONS WITH NEUTRAL TERMS

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Abstract. This article studies the oscillatory behavior of second-order half-linear differential equations with several neutral terms. Some criteria are presented that include those reported in [22].

1. Introduction

Neutral differential equations are used for modeling many problems arising in astrophysics, atomic physics, gas and fluid mechanics, etc. Therefore, analysis of qualitative behavior of solutions to such equations is important for applications. In particular, oscillatory and nonoscillatory behavior of solutions to various classes of neutral differential equations has always attracted attention of researchers; see, e.g., the references in this article and their references. In this article, we are concerned with the oscillation of the second-order neutral differential equation

$$\big(r(t)|z'(t)|^{\alpha-1}z'(t)\big)' + q(t)|x(\sigma(t))|^{\alpha-1}x(\sigma(t)) = 0, \tag{1.1}$$

where $z(t) := x(t) + \sum_{i=1}^{m} p_i(t)x(\tau_i(t))$, and

- (H1) $m > 0$ is an integer, $q \in \mathrm{C}[t_0,\infty)$, $r, p_i, \tau_i, \sigma \in \mathrm{C}^1[t_0,\infty)$;
- (H2) $\alpha \geq 1$, $r(t) > 0$, $q(t) > 0$, $0 \leq p_i(t) \leq a_i < \infty$ for $i = 1, 2, \dots, m$;
- (H3) $\lim_{t\to\infty} \sigma(t) = \infty$, $\tau_i \circ \sigma = \sigma \circ \tau_i$, $\tau_i'(t) \geq \lambda_i > 0$ for $i = 1, 2, \dots, m$.

Also we assume that

$$\lim_{t\to\infty} R(t) < \infty, \quad R(t) := \int_{t_0}^{t} \frac{1}{r^{1/\alpha}(s)}\,\mathrm{d}s. \tag{1.2}$$

By a solution to (1.1), we mean a function $x \in \mathrm{C}[T_x,\infty)$, $T_x \geq t_0$, which has the property $r|z'|^{\alpha-1}z' \in \mathrm{C}^1[T_x,\infty)$ and satisfies (1.1) on $[T_x,\infty)$. We consider only those solutions x of (1.1) which satisfy $\sup\{|x(t)| : t \geq T\} > 0$ for all $T \geq T_x$ and tacitly assume that (1.1) possesses such solutions. A solution of (1.1) is said to be oscillatory if it does not have the largest zero on $[T_x,\infty)$; otherwise, it is called nonoscillatory. Equation (1.1) is said to be oscillatory if all its solutions are oscillatory.

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Below, we present some background details that motivate our study. Baculíková and Džurina [3], Li et al. [21, 22], and Zhang et al. [24] studied the neutral differential equation

$$(r(t)(x(t)+p(t)x(\tau(t)))')'+q(t)x(\sigma(t))=0,$$

and established some oscillation criteria under the conditions that

$$0\le p(t)\le p_0<\infty \quad \text{and} \quad \tau\circ\sigma=\sigma\circ\tau.$$

Agarwal et al. [2], Baculíková and Džurina [4], Baculíková et al. [5], and Li et al. [18, 19] considered oscillation of (1.1) in the case where $m=1$. Assuming

$$\lim_{t\to\infty}\int_{t_0}^{t}\frac{1}{r^{1/\alpha}(s)}\,\mathrm{d}s=\infty,$$

Zhang et al. [25] extended results of [3] to equation (1.1).

We stress that results in [2, 4, 5, 18, 19, 25] cannot be applied to (1.1) in the case where (1.2) holds and $m\neq 1$. Our objective in this work is to define a method for the analysis of oscillatory properties of (1.1) via the comparison principles suggested by Zhang et al. [25], under assumption (1.2).

In what follows, all functional inequalities are assumed to hold eventually, that is, for all t large enough.

2. Oscillation criteria

In what follows, we use the notation

$$Q(t):=\min\{q(t),q(\tau_1(t)),q(\tau_2(t)),\dots,q(\tau_m(t))\}$$

and

$$\tilde{Q}(t,t_1):=Q(t)(R(\eta(t))-R(t_1))^{\alpha},\quad \delta(t):=\int_t^{\infty}r^{-1/\alpha}(s)\,\mathrm{d}s,$$

for $t\ge t_1$, $t_1\ge t_0$ is sufficiently large, where η will be specified later.

Theorem 2.1. *Assume* (H1)–(H3) *and* (1.2). *Suppose that there exist two functions* $\eta,\xi\in \mathrm{C}[t_0,\infty)$ *such that* $\eta(t)\le\sigma(t)\le\xi(t)$ *and* $\lim_{t\to\infty}\eta(t)=\infty$. *If the first-order neutral differential inequalities*

$$\Big(y(t)+\sum_{i=1}^{m}\frac{{a_i}^{\alpha}}{\lambda_i}\,y(\tau_i(t))\Big)'+\frac{\tilde{Q}(t,t_1)}{(m+1)^{\alpha-1}}y(\eta(t))\le 0 \tag{2.1}$$

and

$$\Big(u(t)+\sum_{i=1}^{m}\frac{{a_i}^{\alpha}}{\lambda_i}\,u(\tau_i(t))\Big)'-\frac{Q(t)\delta^{\alpha}(\xi(t))}{(m+1)^{\alpha-1}}u(\xi(t))\ge 0 \tag{2.2}$$

have no positive solutions, then (1.1) *is oscillatory.*

Proof. Let x be an eventually positive solution of (1.1). Using that $x\mapsto x^{\alpha}$ is convex for $\alpha\ge 1$ and $x>0$, we obtain the following inequality that corresponds to (2.7) in [25, Theorem 1],

$$\begin{aligned}&(r(t)|z'(t)|^{\alpha-1}z'(t))'+\sum_{i=1}^{m}\frac{{a_i}^{\alpha}}{\lambda_i}(r(\tau_i(t))|z'(\tau_i(t))|^{\alpha-1}z'(\tau_i(t)))'\\&+\frac{Q(t)}{(m+1)^{\alpha-1}}z^{\alpha}(\sigma(t))\le 0.\end{aligned}\tag{2.3}$$

Assuming that $x(t) > 0$, Equation (1.1) implies that, for some t_1 large enough and for all $t \geq t_1$, either

$$z'(t) > 0, \quad (r(t)|z'(t)|^{\alpha-1}z'(t))' < 0, \tag{2.4}$$

or

$$z'(t) < 0, \quad (r(t)|z'(t)|^{\alpha-1}z'(t))' < 0. \tag{2.5}$$

Assume first that (2.4) holds. Inequality (2.3) reduces to

$$(r(t)(z'(t))^\alpha)' + \sum_{i=1}^{m} \frac{{a_i}^\alpha}{\lambda_i}(r(\tau_i(t))(z'(\tau_i(t)))^\alpha)' + \frac{Q(t)}{(m+1)^{\alpha-1}} z^\alpha(\sigma(t)) \leq 0. \tag{2.6}$$

The fact that $z'(t) > 0$ and $z(t) > 0$ imply that z^α is increasing. Then, (2.6) and $\eta(t) \leq \sigma(t)$ yield

$$(r(t)(z'(t))^\alpha)' + \sum_{i=1}^{m} \frac{{a_i}^\alpha}{\lambda_i}(r(\tau_i(t))(z'(\tau_i(t)))^\alpha)' + \frac{Q(t)}{(m+1)^{\alpha-1}} z^\alpha(\eta(t)) \leq 0. \tag{2.7}$$

It follows from (2.4) that $y := (z')^\alpha r$ is positive and decreasing. Using that $z(t) > 0$, we have

$$\begin{aligned} z(t) &\geq \int_{t_1}^{t} \frac{(r(s)(z'(s))^\alpha)^{1/\alpha}}{r^{1/\alpha}(s)} \,\mathrm{d}s \\ &\geq y^{1/\alpha}(t) \int_{t_1}^{t} \frac{\mathrm{d}s}{r^{1/\alpha}(s)} \\ &= y^{1/\alpha}(t)(R(t) - R(t_1)). \end{aligned} \tag{2.8}$$

Therefore, setting $y := (z')^\alpha r$ in (2.7) and using (2.8), one can see that y is a positive solution of (2.1). This contradicts our assumption that inequality (2.1) has no positive solutions.

Consider now the second case. It follows from (2.5) that $(r(t)(-z'(t))^\alpha)' > 0$, and hence

$$z'(s) \leq \frac{r^{1/\alpha}(t)z'(t)}{r^{1/\alpha}(s)} \quad \text{for all} \quad s \geq t,$$

which, upon integration, leads to

$$z(l) \leq z(t) + r^{1/\alpha}(t)z'(t) \int_{t}^{l} \frac{\mathrm{d}s}{r^{1/\alpha}(s)}.$$

Since $z(t) > 0$, passing to the limit as $l \to \infty$,

$$0 \leq z(t) + r^{1/\alpha}(t)z'(t) \int_{t}^{\infty} \frac{\mathrm{d}s}{r^{1/\alpha}(s)}.$$

Therefore,

$$z(t) \geq -r^{1/\alpha}(t)z'(t)\delta(t). \tag{2.9}$$

Since $z'(t) < 0$, inequality (2.3) reduces to

$$(r(t)(-z'(t))^\alpha)' + \sum_{i=1}^{m} \frac{{a_i}^\alpha}{\lambda_i}(r(\tau_i(t))(-z'(\tau_i(t)))^\alpha)' - \frac{Q(t)}{(m+1)^{\alpha-1}} z^\alpha(\sigma(t)) \geq 0. \tag{2.10}$$

Note that $z(t) > 0$ and $z'(t) < 0$, thus z^α is decreasing. It follows from $\sigma(t) \leq \xi(t)$ that $z^\alpha(\sigma(t)) \geq z^\alpha(\xi(t))$ and

$$(r(t)(-z'(t))^\alpha)' + \sum_{i=1}^{m} \frac{{a_i}^\alpha}{\lambda_i}(r(\tau_i(t))(-z'(\tau_i(t)))^\alpha)' - \frac{Q(t)}{(m+1)^{\alpha-1}} z^\alpha(\xi(t)) \geq 0. \tag{2.11}$$

Therefore, setting $u := (-z')^\alpha r$ and using (2.9), we have that u is a positive solution of (2.2). This contradicts our assumption and the proof is complete. $\square$

Remark 2.2. When $m = 1$ and $\alpha = 1$, Theorem 2.1 reduces to [22, Theorem 1].

Using additional assumptions on the coefficients of (1.1), one can deduce from Theorem 2.1 a number of oscillation criteria applicable to different classes of equations. In what follows, we use the notation $\tau_*(t) := \max\{\tau_i(t) : i = 1, 2, \dots, m\}$ and $\tau(t) := \min\{\tau_i(t) : i = 1, 2, \dots, m\}$, the notation τ_*^{-1} and τ^{-1} stand for the inverse of the functions τ_* and τ, respectively.

Theorem 2.3. *Assume* (H1)–(H3) *and* (1.2). *Suppose that there exist two functions $\eta, \xi \in \mathrm{C}[t_0, \infty)$ such that $\eta(t) \le \sigma(t) \le \xi(t)$ and $\lim_{t\to\infty} \eta(t) = \infty$. Assume also that*

$$\tau_i(t) \ge t \quad \text{for } i = 1, 2, \dots, m. \tag{2.12}$$

If the first-order functional differential inequalities

$$w'(t) + \frac{\tilde{Q}(t, t_1)}{(m+1)^{\alpha-1}(1 + \sum_{i=1}^{m} \frac{{a_i}^\alpha}{\lambda_i})} w(\eta(t)) \le 0 \tag{2.13}$$

and

$$h'(t) - \frac{Q(t)\delta^\alpha(\xi(t))}{(m+1)^{\alpha-1}(1 + \sum_{i=1}^{m} \frac{{a_i}^\alpha}{\lambda_i})} h(\tau_*^{-1}(\xi(t))) \ge 0 \tag{2.14}$$

have no positive solutions, then equation (1.1) *is oscillatory.*

Proof. Let x be an eventually positive solution of (1.1). As in the proof of Theorem 2.1, there exist two possible cases (2.4) and (2.5) for all $t \ge t_1$. Assume first that (2.4) holds. Along the same lines as in [25, Theorem 2], we deduce that the inequality (2.13) has a positive solution, which contradicts our assumption. Consider now the second case. It has been established in Theorem 2.1 that the function $u := (-z')^\alpha r$ is positive, increasing, and satisfies the inequality (2.2). We now define

$$h(t) := u(t) + \sum_{i=1}^{m} \frac{{a_i}^\alpha}{\lambda_i} u(\tau_i(t)). \tag{2.15}$$

Then, we have

$$h(t) \le u(\tau_*(t))\Big(1 + \sum_{i=1}^{m} \frac{{a_i}^\alpha}{\lambda_i}\Big).$$

Substituting the latter inequality into (2.2), we see that h is a positive solution of (2.14). This contradiction completes the proof. $\square$

Remark 2.4. Theorem 2.3 includes [22, Theorem 2] in the case where $m = 1$ and $\alpha = 1$.

Combining Theorem 2.3 with the oscillation criteria presented in Ladde et al. [16, Theorems 2.1.1 and 2.4.1], we obtain the following result.

Corollary 2.5. *Assume* (H1)–(H3), (1.2), *and* (2.12). *Suppose further that there exist two functions $\eta, \xi \in \mathrm{C}[t_0, \infty)$ such that $\eta(t) < t$, $\xi(t) > \tau_*(t)$, $\eta(t) \le \sigma(t) \le \xi(t)$, and $\lim_{t\to\infty} \eta(t) = \infty$. If*

$$\liminf_{t\to\infty} \int_{\eta(t)}^{t} Q(s) R^\alpha(\eta(s))\, \mathrm{d}s > \frac{(m+1)^{\alpha-1}(1 + \sum_{i=1}^{m} \frac{{a_i}^\alpha}{\lambda_i})}{\mathrm{e}} \tag{2.16}$$

and

$$\liminf_{t\to\infty}\int_t^{\tau_*^{-1}(\xi(t))} Q(s)\delta^\alpha(\xi(s))\,\mathrm{d}s > \frac{(m+1)^{\alpha-1}(1+\sum_{i=1}^m \frac{{a_i}^\alpha}{\lambda_i})}{\mathrm{e}}, \tag{2.17}$$

then (1.1) *is oscillatory.*

Proof. By [16, Theorem 2.1.1], assumption (2.16) ensures that the differential inequality (2.13) has no positive solutions. On the other hand, by [16, Theorem 2.4.1], condition (2.17) guarantees that the differential inequality (2.14) has no positive solutions. Application of Theorem 2.3 yields the result. □

Remark 2.6. When $\alpha = 1$ and $m = 1$, Corollary 2.5 includes [22, Corollary 3].

Theorem 2.7. *Assume* (H1)–(H3), (1.2), *and let* $\tau_*(t) \le t$. *Suppose that there exist two functions* $\eta, \xi \in \mathrm{C}[t_0,\infty)$ *such that* $\eta(t) \le \sigma(t) \le \xi(t)$ *and* $\lim_{t\to\infty}\eta(t) = \infty$. *If the first-order functional differential inequalities*

$$w'(t) + \frac{\tilde{Q}(t,t_1)}{(m+1)^{\alpha-1}(1+\sum_{i=1}^m \frac{{a_i}^\alpha}{\lambda_i})}\, w(\tau^{-1}(\eta(t))) \le 0 \tag{2.18}$$

and

$$h'(t) - \frac{Q(t)\delta^\alpha(\xi(t))}{(m+1)^{\alpha-1}(1+\sum_{i=1}^m \frac{{a_i}^\alpha}{\lambda_i})}\, h(\xi(t)) \le 0 \tag{2.19}$$

have no positive solutions, then (1.1) *is oscillatory.*

Proof. Let x be an eventually positive solution of (1.1). By the proof of Theorem 2.1, there exist two possible cases (2.4) and (2.5) for all $t \ge t_1$. Assume first that (2.4) holds. Following the same lines as in [25, Theorem 3], we conclude that the inequality (2.18) has a positive solution, which contradicts our assumption. Consider now the second case. We have shown in Theorem 2.1 that the function $u := r(-z')^\alpha$ is positive, increasing, and satisfies the inequality (2.2). By virtue of $\tau_*(t) \le t$, the inequality

$$h(t) \le u(t)\Big(1+\sum_{i=1}^m \frac{{a_i}^\alpha}{\lambda_i}\Big)$$

holds for the function h defined by (2.15). Substituting this inequality into (2.2), we conclude that h is a positive solution of (2.19). This contradiction completes the proof. □

Remark 2.8. Theorem 2.7 includes [22, Theorem 4] when $\alpha = 1$ and $m = 1$.

Corollary 2.9. *Assume* (H1)–(H3), (1.2), *and let* $\tau_*(t) \le t$. *Suppose also that there exist two functions* $\eta, \xi \in \mathrm{C}[t_0,\infty)$ *such that* $\eta(t) < \tau(t)$, $\xi(t) > t$, $\eta(t) \le \sigma(t) \le \xi(t)$, *and* $\lim_{t\to\infty}\eta(t) = \infty$. *If*

$$\liminf_{t\to\infty}\int_{\tau^{-1}(\eta(t))}^{t} Q(s)R^\alpha(\eta(s))\,\mathrm{d}s > \frac{(m+1)^{\alpha-1}(1+\sum_{i=1}^m \frac{{a_i}^\alpha}{\lambda_i})}{\mathrm{e}} \tag{2.20}$$

and

$$\liminf_{t\to\infty}\int_t^{\xi(t)} Q(s)\delta^\alpha(\xi(s))\,\mathrm{d}s > \frac{(m+1)^{\alpha-1}(1+\sum_{i=1}^m \frac{{a_i}^\alpha}{\lambda_i})}{\mathrm{e}}, \tag{2.21}$$

then (1.1) *is oscillatory.*

Proof. As in [16, Theorem 2.1.1], condition (2.20) ensures that the differential inequality (2.18) has no positive solutions. On the other hand, it follows from [16, Theorem 2.4.1] that condition (2.21) guarantees that the differential inequality (2.19) has no positive solutions. Application of Theorem 2.7 completes the proof. □

Remark 2.10. When $m = 1$ and $\alpha = 1$, Corollary 2.9 reduces to [22, Corollary 5].

The following example illustrates possible applications of the theoretical results obtained.

Example 2.11. For $t \geq 1$, consider a second-order neutral differential equation

$$\begin{aligned} &\Big(\mathrm{e}^t\Big(x(t) + \frac{1}{4}x\big(t - \frac{\pi}{4}\big) + \frac{1}{4}x\big(t - \frac{\pi}{2}\big)\Big)'\Big)' \\ &+ 12\sqrt{65}\mathrm{e}^t x\Big(t - \frac{1}{8}\arcsin\frac{\sqrt{65}}{65}\Big) = 0. \end{aligned} \tag{2.22}$$

Let $\eta(t) = t - \pi$ and $\xi(t) = t + \pi/4$. It is not difficult to verify that all assumptions of Corollary 2.9 are satisfied. Hence, equation (2.22) is oscillatory. As a matter of fact, one such solution is $x(t) = \sin(8t)$.

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