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H_∞ CONTROL OF SWITCHED LINEAR PARABOLIC SYSTEMS

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Abstract. The H_∞ control problem of a class of switched linear parabolic systems is considered. By applying the multiple Lyapunov function method and the average dwell time scheme, sufficient conditions for exponential stability and the H_∞ control synthesis are established in terms of LMIs and a family of switching signals. The advantage in this work lies in the fact that sufficient conditions completely depend on the system parameters and the system can be analyzed by using numerical softwares. At the end of the paper, an example is given to illustrate the obtained result.

1. Introduction

During the previous decade, the study of switched systems has attracted considerable attention because of its significance in both theoretical research and practical applications [1, 3, 6, 10, 11, 15, 18, 20, 22, 23]. A switched system is a dynamical system described by a family of continuous-time subsystems and a rule that governs the switching among them. In many realistic cases, switched systems can be described by partial differential equations (PDE) or a combination of ordinary differential equations (ODE) and PDE such as in chemical industry process and biomedical engineering. We refer to these switched systems as distributed parameter switched systems (DPSS) or infinite dimensional switched systems [5, 13]. However, there are very few works concerning DPSS (see, eg. [1, 7, 9, 14, 16, 19, 21] and the references cited therein). For example, Sasane generalized the results presented in [15] to infinite dimensional Hilbert spaces [21], and showed when all subsystems are stable and commutative pairwise, the switched linear system is stable under an arbitrary switching via the common Lyapunov function. Michel and Sun provided the stability conditions for switched nonlinear systems on Banach spaces under constrained switching [14]. Hante and Sigalotti gave necessary and sufficient conditions in term of the existence of common Lyapunov functions for DPSS [1, 7]. It seems that the majority of works deal with the stability of DPSS.

The H_∞ control is an interesting research topic in the field of switched systems. Up to now, most of results in the literature are regarding the H_∞ control of switched

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systems which are described by ODEs [6, 12, 23]. For example, the stability, the L_2-gain analysis and the H_∞ control for switched systems via the multiple Lyapunov-like function methods is considered in [23]. The dynamic output feedback in H_∞ control design of switched linear systems are studied in terms of linear matrix inequalities (LMIs) in [6]. To the best of our knowledge, the H_∞ control has not been investigated for DPSS.

Motivated by the above consideration, we study the H_∞ control synthesis for switched linear parabolic systems in this paper. The main contributions of the present paper can be summarized as follows: Firstly, the concept of H_∞ control is extended to DPSS. Secondly, sufficient conditions for the exponential stabilization and the H_∞ control synthesis of DPSS are developed in terms of LMIs and a class of switching signals. Compared with the work in [7], our sufficient conditions completely depend on the system parameters.

In this article, $L_2(\Omega, R^n)$ denotes the Hilbert space of square integrable n dimensional vector-valued functions $\nu(x), x \in \Omega$ with the norm $\|\nu\|_{L_2} = \int_\Omega \nu^T \nu dx$. $L_2[t_0, \infty; L_2(\Omega, R^n))$ is the Hilbert space of square integrable functions $\nu(t, \cdot) \in L_2[t_0, \infty)$ with values $\nu(\cdot, x) \in L_2(\Omega, R^n)$. $H^2(\Omega, R^n)$ and $H_0^2(\Omega, R^n)$ denote the classical Sobolev spaces defined by $H^2(\Omega, R^n) = \{\nu \in L_2(\Omega, R^n) : \frac{\partial^2 \nu}{\partial x^2} \in L_2(\Omega, R^n)\}$ and $H_0^1(\Omega, R^n) = \{\nu \in L_2(\Omega, R^n) : \frac{\partial \nu}{\partial x} \in L_2(\Omega, R^n), \nu(\partial\Omega, t) = 0\}$ respectively. $\gamma_M(P)(\gamma_m(P))$ denotes the largest (smallest) eigenvalue of P. The symmetric elements of the matrix will be denoted by T.

2. Problem formulation and preliminaries

Consider the switched linear parabolic systems

$$\begin{gathered}
\frac{\partial \nu(x,t)}{\partial t} = D_{\sigma(t)} \Delta \nu(x,t) + A_{\sigma(t)} \nu(x,t) + B_{\sigma(t)} u(x,t) + C_{\sigma(t)} \omega(x,t) \\
y(x,t) = E_{\sigma(t)} \nu(x,t) + F_{\sigma(t)} \omega(x,t) \\
\nu(t_0) = \nu_0 \\
\nu(x,t) = 0, \quad (x,t) \in \partial\Omega \times [t_0, +\infty)
\end{gathered} \tag{2.1}$$

with the static state feedback control

$$u(x,t) = K_{\sigma(t)} \nu(x,t) \tag{2.2}$$

where $\nu(x,t) \in L_2[t_0, \infty; L_2(\Omega, R^n))$ is a vector-valued function representing the state of the process, $u(x,t) \in L_2[t_0, \infty; L_2(\Omega, R^s))$ denotes the manipulated input, $\omega(x,t) \in L_2[t_0, \infty; L_2(\Omega, R^p))$ is the disturbance, and $y(x,t) \in L_2[t_0, \infty; L_2(\Omega, R^q))$ denotes the measured output with $(x,t) \in \Omega \times [t_0, +\infty)$. $\Omega = [0, \sqrt{2}] \times [0, \sqrt{2}] \subset R^2$ is a bounded domain with the smooth boundary. Δ denotes the Laplace operator; i.e., $\Delta = \sum_{k=1}^2 \frac{\partial^2}{\partial x_k^2}$. $D_i = \operatorname{diag}(d_{i1}, d_{i2}, \dots, d_{in})$ represent positive diagonal matrices, and A_i, B_i, C_i, E_i, F_i $(i = 1, 2, \dots, m)$ represent constant matrices of compatible dimensions. $\sigma(t) : [t_0, \infty) \to \Theta$ is the switching signal mapping time to some finite index set $\Theta = \{1, 2, \dots, m\}$, and the switching signal $\sigma(t)$ is a piecewise continuous (from the right) function depending on time or state or both. The discontinuities of $\sigma(t)$ are called switching times or switching instants. The integer m is the number of models (called subsystems) of the switched system.

The objective of this article is to establish sufficient conditions of the H_∞ control for the system (2.1)–(2.2). That is, we look for controller gain matrices K_i $(i \in \Theta)$ and a class of switching signals $\sigma(t)$, such that

1. When $\omega = 0$, the system (2.1) is exponentially stabilized by the state feedback control (2.2).
2. The system (2.1) is exponentially stabilized by (2.2) with the H_∞ disturbance level $\gamma > 0$, i.e., for a prescribed scalar $\gamma > 0$, the performance index is
$$J(\omega) = \int_{t_0}^{\infty} \int_\Omega [y^T(x,s)y(x,s) - \gamma^2 \omega^T(x,s)\omega(x,s)]\, dx\, ds \le 0$$
for all non-zero $\omega(x,t) \in L_2[t_0, \infty; L_2(\Omega, R^p))$ under the zero initial condition.

The following is the definition of average dwell time (ADT) [10].

Definition 2.1. Given some family of switching signals $\sigma(t) \in \Theta$, for each $\sigma(t)$ and each $t > s \ge t_0$, let $N_\sigma(s,t)$ denote the number of switching of $\sigma(t)$ in the open interval (s,t). If $N_\sigma(s,t) \le N_0 + \frac{t-s}{\tau_a}$ holds for $\tau_a > 0$ and $N_0 > 0$, then the positive constant τ_a is called the ADT and N_0 is the chatter bound.

Lemma 2.2 (Poincare's inequality [4]). *Let the scalar function $u \in H_0^1(\bar{\Omega}, R)$ with $\Omega \subseteq \Omega_1$, then we have*
$$\int_\Omega u^2 dx \le \gamma^2 \int_\Omega \sum_{i=1}^{n} (\frac{\partial u}{\partial x_i})^2 dx = \gamma^2 \int_\Omega \mid \nabla u \mid^2 dx \tag{2.3}$$
where $\Omega_1 : 0 \le x_i \le \delta (i = 1, 2, \dots, n)$, $\gamma = \frac{\delta}{\sqrt{n}}$, and $\nabla = (\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \dots, \frac{\partial}{\partial x_n})$.

3. Stabilization analysis of switched parabolic systems

In this section, we consider the exponential stabilization problem of the switched linear parabolic system
$$\begin{gathered} \frac{\partial \nu(x,t)}{\partial t} = D_{\sigma(t)} \Delta \nu(x,t) + A_{\sigma(t)} \nu(x,t) + B_{\sigma(t)} u(x,t) \\ \nu(t_0) = \nu_0 \\ \nu(x,t) = 0, \quad (x,t) \in \partial\Omega \times [t_0, +\infty) \end{gathered} \tag{3.1}$$
with the static state feedback control (2.2).

We assume that the state of system does not jump at switching instants; i.e., the state trajectory is continuous and the switching signal $\sigma(t)$ has the finite switching number in any finite time interval [10].

We start with the well-posedness problem of the closed-loop system (2.2)-(3.1). Define the state function $z(t)$ as $z(t) = \nu(\cdot, t)$ on the Hilbert space $H = L_2(\Omega, R^n)$ with the norm $\| \cdot \|_{L_2}$, then the equation of closed-loop (2.2) and (3.1) can be rewritten as
$$\dot{z}(t) = \widetilde{A}_{\sigma(t)} z(t) + f_{\sigma(t)}(t), \quad t \ge t_0 \tag{3.2}$$
in H, where the infinitesimal operators $\widetilde{A}_i$ ($\widetilde{A}_i x = D_i \Delta x$) have the dense domain $W = D(\widetilde{A}_i) = \{\nu \in H^2(\Omega, R^n) \bigcap H_0^1(\Omega, R^n) : \nu(x) = 0, x \in \partial\Omega\}$, $f_{\sigma(t)}(t) = [A_{\sigma(t)} + B_{\sigma(t)} K_{\sigma(t)}] z(t)$.

As we know, the infinitesimal operators $\widetilde{A}_i$ generate analytical semigroups $T_i(t)$ [17]. Because the state of system (2.2)-(3.1) does not jump at switching instants, for every initial value $\nu_0 \in W$, there exists a unique solution for system (2.2)-(3.1). Thus, the initial problem (2.2)-(3.1) turns out to be well-posed on the time interval $[t_0, \infty)$.

Lemma 3.1. *For the given scalar $\lambda > 0$, if there exist positive constants α_i, β_i, a constant $\mu \geq 1$, and continuous functions $V_i \in C(H \times [t_0, +\infty), R^+)$ such that the functions $V_i(t) = V_i(\nu, t)$ are absolutely continuous along the solutions ν of system* (2.2)-(3.1) *and satisfy*

$$\alpha_i \|\nu(t)\|_{L_2} \leq V_i(t) \leq \beta_i \|\nu(t)\|_{L_2} \tag{3.3}$$

$$\dot{V}_i(t) + \lambda V_i(t) \leq 0. \tag{3.4}$$

Furthermore, the Lyapunov function of the system satisfies

$$V_i(t) \leq \mu V_j(t), \quad \forall i, j \in \Theta. \tag{3.5}$$

Then the closed-loop system (2.2)-(3.1) *is exponentially stable for the arbitrary switching signal $\sigma(t)$ with the ADT $\tau_a > \frac{ln\mu}{\lambda}$.*

Proof. For $t \in [t_k, t_{k+1}) (k = 0, 1 \dots)$, from (3.4) it follows that

$$V_{\sigma(t)}(t) \leq e^{-\lambda(t-t_k)} V_{\sigma(t_k)}(t_k).$$

This, together with (3.5) gives

$$V_{\sigma(t)}(t) \leq \mu V_{\sigma(t_k^-)}(t_k^-) e^{-\lambda(t-t_k)}.$$

It is easy to show that

$$\begin{aligned} V_{\sigma(t)}(t) &\leq \mu V_{\sigma(t_k^-)}(t_k^-) e^{-\lambda(t-t_k)} \leq \mu V_{\sigma(t_{k-1})}(t_{k-1}) e^{-\lambda(t-t_k)} e^{-\lambda(t_k - t_{k-1})} \\ &\leq \mu^2 V_{\sigma(t_{k-1}^-)}(t_{k-1}^-) e^{-\lambda(t-t_{k-1})} \leq \dots \dots \leq \mu^k V_{\sigma(t_0)}(t_0) e^{-\lambda(t-t_0)} \end{aligned}$$

for all $t \geq t_0$ and a constant $\mu \geq 1$.

Note that when $k\tau_a \leq t - t_0$, we have

$$V_{\sigma(t)}(t) \leq e^{-\lambda(t-t_0)} e^{k ln\mu} V_{\sigma(t_0)}(t_0) \leq e^{-(\lambda - \frac{ln\mu}{\tau_a})(t-t_0)} V_{\sigma(t_0)}(t_0). \tag{3.6}$$

Combing (3.3) and (3.6), we obtain

$$\|\nu(t)\|_{L_2} \leq \frac{V_{\sigma(t)}(t)}{\alpha} \leq \frac{1}{\alpha} e^{-(\lambda - \frac{ln\mu}{\tau_a})(t-t_0)} V_{\sigma(t_0)}(t_0).$$

Thus we have

$$\|\nu(t)\|_{L_2} \leq \frac{1}{\alpha} e^{-(\lambda - \frac{ln\mu}{\tau_a})(t-t_0)} \cdot \gamma \|\nu_0\|_{L_2} \leq \frac{\gamma}{\alpha} e^{-(\lambda - \frac{ln\mu}{\tau_a})(t-t_0)} \|\nu_0\|_{L_2}$$

where $\gamma = \max_{i \in \Theta}\{\beta_i\}$ and $\alpha = \min_{i \in \Theta}\{\alpha_i\}$. Let $h = \lambda - \frac{ln\mu}{\tau_a} > 0$, and we have $\tau_a > \frac{ln\mu}{\lambda}$. It is obvious that the system (2.2)-(3.1) is exponentially stable for the arbitrary switching signal $\sigma(t)$ with the ADT $\tau_a > \frac{ln\mu}{\lambda}$. □

Theorem 3.2. *For the given scalar $\lambda > 0$, if there exist diagonal matrices $X_i > 0$, and matrices $Y_i > 0$ such that*

$$\Pi_i = -2D_i X_i + A_i X_i + X_i^T A_i^T + B_i Y_i + Y_i^T B_i^T + \lambda X_i < 0. \tag{3.7}$$

Then system (3.1) *can be exponentially stabilized by the state feedback control* (2.2) *with $K_i = Y_i X_i^{-1}$ for the arbitrary switching signal $\sigma(t)$ with the ADT $\tau_a > \ln(\mu)/\lambda$, where μ is determined by*

$$\mu = \max_{\forall k, l \in \Theta} \{\frac{\gamma_M(X_k)}{\gamma_m(X_l)}\}. \tag{3.8}$$

Proof. Choose the multiple Lyapunov function for the system (2.2)-(3.1)

$$V(t) = V_{\sigma(t)}(t) = \int_\Omega \nu^T(x,t) P_{\sigma(t)} \nu(x,t)\, dx \tag{3.9}$$

with constant diagonal matrices $P_i > 0$. It is not difficult to see that there exist positive numbers α_i, β_i and a constant $\mu \geq 1$ such that inequalities (3.3) and (3.5) hold. For inequalities (3.5), we can choose $\mu = \max_{\forall k,l\in\Theta}\{\frac{\gamma_M(P_k)}{\gamma_m(P_l)}\}$ [22].

Differentiating $V(t)$ with respect to t along the trajectory of the closed-loop system (2.2)-(3.1), we have

$$\begin{aligned}
&\dot V_i(t) + \lambda V_i(t)\\
&= \int_\Omega [\Delta\nu(x,t)]^T D_i P_i \nu(x,t) dx + \int_\Omega \nu^T(x,t) P_i D_i \Delta\nu(x,t) dx\\
&\quad + \int_\Omega \nu^T(x,t)[P_i A_i + P_i B_i K_i]\nu(x,t) dx\\
&\quad + \int_\Omega \nu^T(x,t)[A_i^T P_i + K_i^T B_i^T P_i]\nu(x,t) dx + \int_\Omega \nu^T(x,t)\lambda P_i \nu(x,t) dx.
\end{aligned} \tag{3.10}$$

Because D_i and P_i are positive diagonal matrices, we find that $P_i D_i = D_i P_i$. Thus we have

$$\begin{aligned}
&\int_\Omega [\Delta\nu(x,t)]^T D_i P_i \nu(x,t) dx + \int_\Omega \nu^T(x,t) P_i D_i \Delta\nu(x,t) dx\\
&\leq 2\lambda_{\max}(P_i D_i)\int_\Omega \Delta\nu^T(x,t) I \nu(x,t) dx\\
&\leq 2\lambda_{\max}(P_i D_i)\int_\Omega [\Delta\nu_1(x,t), \dots, \Delta\nu_n(x,t)] \begin{bmatrix}\nu_1(x,t)\\ \dots\\ \nu_n(x,t)\end{bmatrix} dx\\
&\leq -2\lambda_{\max}(P_i D_i)\int_\Omega [\nu_1(x,t)\Delta\nu_1(x,t) + \dots + \nu_n(x,t)\Delta\nu_n(x,t)] dx.
\end{aligned}$$

According to Gauss's divergence theorem, Poincare's inequality (2.3) and taking into account the boundary condition in (3.1), we obtain

$$\begin{aligned}
&\int_\Omega [\Delta\nu(x,t)]^T D_i P_i \nu(x,t) dx + \int_\Omega \nu^T(x,t) P_i D_i \Delta\nu(x,t) dx\\
&\leq -2\lambda_{\max}(P_i D_i)\int_\Omega [\nu_1^2(x,t) + \dots + \nu_n(^2x,t)] dx\\
&\leq -2\lambda_{\max}(P_i D_i)\int_\Omega \nu^T(x,t) I \nu(x,t) dx\\
&\leq \int_\Omega \nu^T(x,t)(-2P_i D_i)\nu(x,t) dx < 0.
\end{aligned} \tag{3.11}$$

Substituting (3.11) into (3.10) yields

$$\dot V_i(t) + \lambda V_i(t) \leq \int_\Omega \nu^T(x,t)\Gamma_i \nu(x,t)\, dx$$

where

$$\Gamma_i = -2P_i D_i + P_i A_i + A_i^T P_i + P_i B_i K_i + K_i^T B_i^T P_i + \lambda P_i.$$

When

$$\Gamma_i = -2P_i D_i + P_i A_i + A_i^T P_i + P_i B_i K_i + K_i^T B_i^T P_i + \lambda P_i < 0, \tag{3.12}$$

we have $\dot{V}_i(t) + \lambda V_i(t) < 0$ (for all $\nu(x,t) \neq 0$). If the switching signal $\sigma(t)$ satisfies the ADT $\tau_a > \ln(\mu)/\lambda$, all conditions in Lemma 3.1 hold. Hence, the closed-loop system (2.2)-(3.1) is exponentially stable.

Left- and right- multiplying (3.12) by P_i^{-1} and letting $X_i = P_i^{-1}$ and $Y_i = K_i P_i^{-1}$, it is not difficult to see that equality (3.12) is equivalent to (3.7) and $\mu = \max_{\forall k,l \in \Theta}\{\frac{\gamma_M(P_k)}{\gamma_m(P_l)}\}$ leads to (3.8) immediately. Consequently, the proof is completed. $\square$

4. H_∞ control synthesis

In the section, we consider the H_∞ control problem for system (2.1)–(2.2).

Lemma 4.1. *For given scalars $\lambda > 0$ and $\gamma > 0$, if there exist diagonal matrices $P_i > 0$ such that*

$$\begin{bmatrix} \Gamma_i + E_i^T E_i & P_i C_i + E_i^T F_i \\ C_i^T P_i + F_i^T E_i & -\gamma^2 I + F_i^T F_i \end{bmatrix} < 0, \tag{4.1}$$

then for any $t \in [t_k, t_{k+1})$, along the trajectory of system (2.1)-(2.2), we have

$$V_i(t) \le e^{-\lambda(t-t_k)} V_i(t_k) - \int_{t_k}^{t} \int_\Omega e^{-\lambda(t-s)} \Upsilon(x,s)\,dx\,ds \tag{4.2}$$

where

$$\begin{gathered} \Gamma_i = -2P_i D_i + P_i A_i + A_i^T P_i + P_i B_i K_i + K_i^T B_i^T P_i + \lambda P_i, \\ \Upsilon(x,s) = y^T(x,s) y(x,s) - \gamma^2 \omega^T(x,s)\omega(x,s). \end{gathered}$$

Proof. Differentiating $V_i(t)$ with respect to t along the trajectory of the closed-loop system (2.1)–(2.2) and using the similar argument described in the previous section, we have

$$\dot{V}_i(t) + \lambda V_i(t) \le \int_\Omega \eta^T(x,t) \begin{bmatrix} \Gamma_i & P_i C_i \\ C_i^T P_i & 0 \end{bmatrix} \eta(x,t) dx, \tag{4.3}$$

where $\eta^T(x,t) = [\nu(x,t), \omega(x,t)]^T$. It follows from (4.1) and (4.3) that

$$\dot{V}_i(t) + \lambda V_i(t) < -\int_\Omega \eta^T(x,t) \begin{bmatrix} E_i^T E_i & E_i^T F_i \\ F_i^T E_i & -\gamma^2 I + F_i^T F_i \end{bmatrix} \eta(x,t) dx = -\int_\Omega \Upsilon(x,s) dx.$$

By calculation, we have

$$\frac{d}{dt}(e^{\lambda t} V_i(t)) < -e^{\lambda t} \int_\Omega \Upsilon(x,s) dx. \tag{4.4}$$

Integrating leads to (4.2). $\square$

Theorem 4.2. *For given scalars $\lambda > 0$ and $\gamma > 0$, if there exist diagonal matrices $P_i > 0$ such that*

$$\begin{bmatrix} \Gamma_i + E_i^T E_i & P_i C_i + E_i^T F_i \\ C_i^T P_i + F_i^T E_i & -\gamma^2 I + F_i^T F_i \end{bmatrix} < 0. \tag{4.5}$$

Then, the system (2.1) can be exponentially stabilized by the state feedback control (2.2) with the H_∞ disturbance level $\gamma > 0$ for the arbitrary switching signal $\sigma(t)$ with the ADT $\tau_a > \frac{ln\mu}{\lambda}$, where μ is determined by $\mu = \max_{\forall k,l \in \Theta}\{\frac{\gamma_M(P_k)}{\gamma_m(P_l)}\}$.

Proof. It follows from (4.5) that

$$\begin{bmatrix} \Gamma_i + E_i^T E_i & P_i C_i + E_i^T F_i \\ C_i^T P_i + F_i^T E_i & -\gamma^2 I + F_i^T F_i \end{bmatrix} = \begin{bmatrix} \Gamma_i & P_i C_i \\ * & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} E_i^T E_i & E_i^T F_i \\ F_i^T E_i & F_i^T F_i \end{bmatrix} < 0.$$

Since

$$\begin{bmatrix} E_i^T E_i & E_i^T F_i \\ F_i^T E_i & F_i^T F_i \end{bmatrix} = \begin{bmatrix} E_i^T \\ F_i^T \end{bmatrix} \begin{bmatrix} E_i & F_i \end{bmatrix} \geq 0,$$

we have

$$\begin{bmatrix} \Gamma_i & P_i C_i \\ * & -\gamma^2 I \end{bmatrix} < 0.$$

According to the Schur complement [2], we have $\Gamma_i < 0$. By virtue of the proof of Theorem 3.1, the closed-loop system (2.1)–(2.2) is exponentially stable when $\omega = 0$.

On the other hand, combining (3.5) and (4.2), we obtain

$$V_i(t) \leq \mu e^{-\lambda(t-t_k)} V_i(t_k^-) - \int_{t_k}^t \int_\Omega e^{-\lambda(t-s)} \Upsilon(x,s)\, dx\, ds.$$

Following [22] and repeating the above procedure, we obtain

$$\begin{aligned} &V_i(t) \\ &\leq \mu^k e^{-\lambda t} V(\nu_0) - \mu^k \int_{t_0}^{t_1} \int_\Omega e^{-\lambda(t-s)} \Upsilon(x,s)\, dx\, ds \\ &\quad - \mu^{k-1} \int_{t_1}^{t_2} \int_\Omega e^{-\lambda(t-s)} \Upsilon(x,s)\, dx\, ds - \dots - \int_{t_k}^t \int_\Omega e^{-\lambda(t-s)} \Upsilon(x,s)\, dx\, ds \\ &= \mu^k e^{-\lambda t} V(\nu_0) - \int_{t_0}^t \int_\Omega e^{-\lambda(t-s) + N_\sigma(s,t) ln\mu} \Upsilon(x,s)\, dx\, ds. \end{aligned} \tag{4.6}$$

Because $V_i(t) > 0$, the zero initial condition implies $V(\nu_0) = 0$. Using (4.6) yields

$$\int_{t_0}^t \int_\Omega e^{-\lambda(t-s) + N_\sigma(s,t) ln\mu} \Upsilon(x,s)\, dx\, ds \leq 0.$$

It follows from $e^{N_\sigma(s,t) ln\mu} \geq 1 (\mu \geq 1, N_\sigma(s,t) > 0)$ that

$$\int_{t_0}^t \int_\Omega e^{-\lambda(t-s)} [y^T(x,s) y(x,s) - \gamma^2 \omega^T(x,s) \omega(x,s)]\, dx\, ds \leq 0. \tag{4.7}$$

Notice that since $N_\sigma(t_0,s) \leq N_0 + \frac{s-t_0}{\tau_a}$, $N_0 > 0$, and $\tau_a > \frac{ln\mu}{\lambda}$, we derive that

$$N_\sigma(t_0,s) ln\mu \leq N_0 ln\mu + \lambda(s - t_0).$$

Multiplying both sides of (4.7) by $e^{-[N_0 ln\mu + \lambda(s-t_0)]}$ gives

$$\begin{aligned} &\int_{t_0}^t \int_\Omega e^{-\lambda(t-s) - [N_0 ln\mu + \lambda(s-t_0)]} y^T(x,s) y(x,s)\, dx\, ds \\ &\leq \int_{t_0}^t \int_\Omega e^{-\lambda(t-s) - [N_0 ln\mu + \lambda(s-t_0)]} \gamma^2 \omega^T(x,s) \omega(x,s)\, dx\, ds. \end{aligned} \tag{4.8}$$

Thus we obtain

$$\int_{t_0}^t \int_\Omega e^{-\lambda t} y^T(x,s) y(x,s)\, dx\, ds \leq \int_{t_0}^t \int_\Omega e^{-\lambda t} \gamma^2 \omega^T(x,s) \omega(x,s)\, dx\, ds. \tag{4.9}$$

Integrating both sides from $t = t_0$ to ∞ gives

$$\int_{t_0}^{\infty} \int_{\Omega} y^T(x,s) y(x,s)\, dx\, ds \le \int_{t_0}^{\infty} \int_{\Omega} \gamma^2 \omega^T(x,s)\omega(x,s)\, dx\, ds \tag{4.10}$$

i.e., $J(\omega) \le 0$. This completes the proof. $\square$

The matrix inequalities (4.5) are not LMIs. Left- and right- multiplying (4.5) by $\operatorname{diag}\{P_i^{-1}, I\}$. Let $X_i = P_i^{-1}$, $Y_i = K_i P_i^{-1}$. It follows from the Schur complement that (4.5) is equivalent to the LMIs

$$\begin{bmatrix} \Pi_i & C_i + X_i E_i^T F_i & X_i & 0 \\ * & -\gamma^2 I + F_i^T F_i & 0 & 0 \\ * & * & (-E_i^T E_i)^{-1} & 0 \\ * & * & * & -I \end{bmatrix} < 0, \tag{4.11}$$

where $\Pi_i = -2D_i X_i + A_i X_i + X_i^T A_i^T + B_i Y_i + Y_i^T B_i^T + \lambda X_i$. Next, we show a result which can be obtained using matlab software.

Theorem 4.3. *For given scalars $\lambda > 0$ and $\gamma > 0$, if there exist diagonal matrices $X_i > 0$ and matrices $Y_i > 0$, such that the LMIs* (4.11) *are feasible. Then the system* (2.1) *can be exponentially stabilized by the state feedback control* (2.2) *with $K_i = Y_i X_i^{-1}$ with the H_∞ disturbance level $\gamma > 0$ for the arbitrary switching signal $\sigma(t)$ with the ADT $\tau_a > \frac{ln\mu}{\lambda}$, where μ is determined by* (3.8).

Example 4.4. *Consider the switched parabolic equations* (2.1) *under the state feedback* (2.2). *Suppose there are two subsystems with parameters*

$$\begin{gathered} D_1 = [1\ 0; 0\ 2], \quad D_2 = [2\ 0; 0\ 1], \quad A_1 = [1\ 3; 2\ 3], \quad A_2 = [1\ 2; 3\ 1], \\ B_1 = [6\ 7; 5\ 1], \quad B_2 = [5\ 2; 3\ 1], \quad C_1 = [3\ 5; 4\ 2], \quad C_2 = [5\ 2; 3\ 2], \\ E_1 = [1\ 0; 0\ 1], \quad E_2 = [1\ 0; 0\ 1], \quad F_1 = [1\ 2; 2\ 1], \quad F_2 = [2\ 1; 1\ 2]. \end{gathered}$$

Set $\lambda = 0.6$, $\gamma = 0.8$, using Theorem 4.3, by resolving LMIs (4.11), *we obtain*

$$X_1 = [489.9209\ 0;\ 0\ 490.7482], \quad X_2 = [195.4787\ 0;\ 0\ 193.5938].$$

The state feedback matrices are

$$K_1 = \begin{bmatrix} -0.1487 & -6.4763 \\ -4.0296 & 3.3322 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 1.8293 & -3.8292 \\ -5.6515 & 9.3622 \end{bmatrix}$$

From (3.8) *we obtain that $\mu = 2.5349$ and $\tau_a > \frac{ln\mu}{\lambda} = 1.5503$. So system* (2.1) *can be exponentially stabilized.*

Conclusion. By using multiple Lyapunov function and ADT method, we establish some new criteria for the exponential stabilization and the H_∞ control synthesis of switched linear parabolic systems via the state feedback. All the results are given in terms of LMIs and a class of signals which can be easily tested by matlab software.

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