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PROPERTIES OF SOLUTIONS TO NEUMANN-TRICOMI PROBLEMS FOR LAVRENT'EV-BITSADZE EQUATIONS AT CORNER POINTS

MAKHMUD A. SADYBEKOV, NURGISSA A. YESSIRKEGENOV

Abstract. We consider the Neumann-Tricomi problem for the Lavrent'ev-Bitsadze equation for the case in which the elliptic part of the boundary is part of a circle. For the homogeneous equation, we introduce a new class of solutions that are not continuous at the corner points of the domain and construct nontrivial solutions in this class in closed form. For the nonhomogeneous equation, we introduce the notion of an n-regular solution and prove a criterion for the existence of such a solution.

1. Introduction

Let $\Omega \subset R^2$ be a finite domain bounded for $y < 0$ by the characteristics AC : $x + y = 0$ and $BC : x - y = 1$ of the Lavrent'ev-Bitsadze equation

$$\operatorname{sgn}(y)u_{xx} + u_{yy} = f(x, y) \tag{1.1}$$

and for $y > 0$ by the circular arc $\sigma_\delta = \{(x, y) : (x-1/2)^2+(y+\delta)^2 = 1/4+\delta^2,\ y > 0\}$.

Neumann-Tricomi problem (problem N-T). Find a solution of (1.1) with the boundary condition

$$u|_{AC} = 0, \tag{1.2}$$

$$\frac{\partial u}{\partial n}\Big|_{\sigma_\delta} = 0, \tag{1.3}$$

where $\frac{\partial}{\partial n} = (x - 1/2)\partial_x + (y + \delta)\partial_y$. We assume that the classical transmission conditions

$$u(x, +0) = u(x, -0), \quad u_y(x, +0) = u_y(x, -0), \quad 0 < x < 1, \tag{1.4}$$

hold for the solution on the line $y = 0$, of type change of the equation. Along with problem N–T, consider the adjoint problem.

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Problem N-T*. Find a solution of the equation

$$\operatorname{sgn}(y)\upsilon_{xx} + \upsilon_{yy} = g(x,y) \tag{1.5}$$

with the boundary condition

$$\upsilon|_{BC} = 0, \tag{1.6}$$

$$\frac{\partial \upsilon}{\partial n}|_{\sigma_\delta} = 0. \tag{1.7}$$

Here we also assume that the transmission conditions

$$\upsilon(x,+0) = \upsilon(x,-0), \quad \upsilon_y(x,+0) = \upsilon_y(x,-0), \quad 0 < x < 1, \tag{1.8}$$

are satisfied.

Bitsadze [1, p. 34-37] proved the existence and uniqueness of regular solution of Neumann-Tricomi problem. The completeness of eigenfunctions of the Neumann-Tricomi problem for a degenerate equation of mixed type in the elliptic part of the domain was investigated by Moiseev and Mogimi [7]. Also, they showed that a system of functions consisting of sums of Legendre functions is complete. The existence and uniqueness of a strong solution of the Tricomi problem (where instead of (1.3), it was given by condition $u|_\sigma = 0$) for the Lavrent'ev-Bitsadze equation were studied in [3, 4, 5].

In [8] the spectral methods of solving boundary value problems for mixed-type differential equations of second order in a 3D domain were studied. Existence and uniqueness of a solution of the Lavrent'ev–Bitsadze problem was proved.

In [11] we proved a criterion for the strong solvability of the Neumann-Tricomi problem in L_2. It was shown that if the elliptic part of the domain coincides with the semi-circle, then the Neumann-Tricomi problem in the classical domain is not strongly solvable in L_2.

In [10] for the Tricomi problem it was studied properties of solutions at corner points. Also, it was given a criterion for the existence of n-regular solution.

Relation between the uniqueness of solution of the problem and the order of smoothness (or features) of solutions is well-known and it is a particularly evident in problems for degenerate equations [9] and mixed-type equations [1].

In this paper, we introduce new classes of solutions of the Neumann-Tricomi problem depending on the behavior at the corner points and study their properties.

2. Main results

We say that a function $h(x,y)$ belongs to the class $C^{\alpha,\beta}_{A,B}(\overline{\Omega})$ if and only if $|x|^\alpha|1-x|^\beta h(x,y) \in C(\overline{\Omega})$. As usual, $\Omega_1 = \Omega \cap \{y > 0\}$ and $\Omega_2 = \Omega \cap \{y < 0\}$. A solution of the problem is understood as a function in the class $C^2(\Omega_1) \cap C^2(\Omega_2) \cap C^1(\Omega) \cap C^1(\sigma_\delta)$.

We denote the angle at which the curve σ_δ approaches the line of change of type satisfies

$$\gamma_\delta = \operatorname{arccot}(2\delta), \quad 0 < \gamma_\delta < \pi. \tag{2.1}$$

Theorem 2.1. *There are infinitely many solutions $u_k(x,y) \in C^{-\alpha_k,\alpha_k}_{A,B}(\overline{\Omega}) \cap C^1(\sigma_\delta)$ to the homogeneous problem N-T ($f \equiv 0$). These solutions are given by the relations*

$$u_k(x,y) = \begin{cases} \operatorname{Re}\big(\frac{1-x+iy}{(1-x)^2+y^2} - 1\big)^{\alpha_k} + \operatorname{Im}\big(\frac{1-x+iy}{(1-x)^2+y^2} - 1\big)^{\alpha_k} & \text{for } y > 0, \\ \big(\frac{1}{1-x-y} - 1\big)^{\alpha_k} & \text{for } y < 0; \end{cases} \tag{2.2}$$

where

$$\alpha_k = \pi(1+4k)/(4\gamma_\delta), \quad k = 0,1,\dots \tag{2.3}$$

In addition, $u_k(x,y) \notin L_2(\Omega)$ *for* $k \ge 1$, *and* $u_0(x,y) \in L_2(\Omega)$ *only if*

$$\gamma_\delta > \pi/4. \tag{2.4}$$

Theorem 2.2. *There are infinitely many solutions* $\upsilon_k(x,y) \in C^{\alpha_k,-\alpha_k}_{A,B}(\overline{\Omega}) \cap C^1(\sigma_\delta)$ *to the homogeneous problem N-T** ($g \equiv 0$), *where* α_k *is given by relation* (2.3). *These solutions are given by the formulas*

$$\upsilon_k(x,y) = \begin{cases} \operatorname{Re}\big(\frac{x+iy}{x^2+y^2} - 1\big)^{\alpha_k} + \operatorname{Im}\big(\frac{x+iy}{x^2+y^2} - 1\big)^{\alpha_k} & \text{for } y>0, \\ \big(\frac{1}{x-y} - 1\big)^{\alpha_k} & \text{for } y<0; \end{cases} \tag{2.5}$$

in addition, $\upsilon_k(x,y) \notin L_2(\Omega)$ *for* $k \ge 1$, *and* $\upsilon_0(x,y) \in L_2(\Omega)$ *only under condition* (2.4).

Proof of Theorem 2.1. **1.** We denote

$$w(x,y) = \Big(\frac{1-x+iy}{(1-x)^2+y^2} - 1\Big)^{\alpha_k},$$

then $u(x,y) = \operatorname{Re} w(x,y) + \operatorname{Im} w(x,y)$, $y>0$. For $y>0$ and $f=0$ equation (1.1) can be written as

$$u_{xx} + u_{yy} = 0.$$

By a direct calculation, we have

$$\begin{aligned} w_{xx} &= \alpha_k(\alpha_k-1)\Big(\frac{1-x+iy}{(1-x)^2+y^2} - 1\Big)^{\alpha_k-2} \\ &\quad\times \frac{(1-x)^4 + 4(1-x)^3 yi - 6(1-x)^2y^2 - 4(1-x)y^3 i + y^4}{((1-x)^2+y^2)^4} \\ &\quad+ \alpha_k\Big(\frac{1-x+iy}{(1-x)^2+y^2} - 1\Big)^{\alpha_k-1} \frac{2(1-x)^3 + 6(1-x)^2yi - 6(1-x)y^2 - 2y^3 i}{((1-x)^2+y^2)^3}, \end{aligned}$$

$$\begin{aligned} w_{yy} &= \alpha_k(\alpha_k-1)\Big(\frac{1-x+iy}{(1-x)^2+y^2} - 1\Big)^{\alpha_k-2} \\ &\quad\times \frac{-(1-x)^4 - 4(1-x)^3 yi + 6(1-x)^2y^2 + 4(1-x)y^3 i - y^4}{((1-x)^2+y^2)^4} \\ &\quad+ \alpha_k\Big(\frac{1-x+iy}{(1-x)^2+y^2} - 1\Big)^{\alpha-1} \frac{-2(1-x)^3 - 6(1-x)^2yi + 6(1-x)y^2 + 2y^3 i}{((1-x)^2+y^2)^3}. \end{aligned}$$

Thus, $w_{xx} + w_{yy} = 0$, since $(1-x)^2 + y^2 \ne 0$ in Ω_1, hence,

$$\operatorname{Re}(w_{xx} + w_{yy}) + \operatorname{Im}(w_{xx} + w_{yy}) = 0 \Rightarrow u_{xx} + u_{yy} = 0.$$

For $y<0$ and $f=0$ equation (1.1) can be written as

$$u_{xx} - u_{yy} = 0.$$

By a direct calculation, from (2.2), for $y<0$ we have

$$\begin{aligned} u_{xx}(x,y) &= \alpha_k(\alpha_k-1)\Big(\frac{1}{1-x-y} - 1\Big)^{\alpha_k-2} \frac{1}{(1-x-y)^4} \\ &\quad+ \alpha_k\Big(\frac{1}{1-x-y} - 1\Big)^{\alpha_k-1} \frac{2}{(1-x-y)^3}, \end{aligned}$$

$$\begin{aligned} u_{yy}(x,y) &= \alpha_k(\alpha_k-1)\Big(\frac{1}{1-x-y}-1\Big)^{\alpha_k-2}\frac{1}{(1-x-y)^4} \\ &\quad + \alpha_k\Big(\frac{1}{1-x-y}-1\Big)^{\alpha_k-1}\frac{2}{(1-x-y)^3}. \end{aligned}$$

Thus, $u_{xx}-u_{yy}=0$, since $x+y\neq 1$ in Ω_2. The function in (2.2) satisfies equation (1.1) for both $y>0$ and $y<0$.

2. By (2.2), for $y<0$,

$$u(x,y)=\Big(\frac{1}{1-x-y}-1\Big)^{\alpha_k}=\Big(\frac{x+y}{1-x-y}\Big)^{\alpha_k},$$

we have

$$u|_{AC}=u|_{x+y=0}=\Big(\frac{x+y}{1-x-y}\Big)^{\alpha_k}\Big|_{x+y=0}=0,$$

since $\alpha_k>0$. Thus, function in (2.2) satisfies the boundary condition (1.2) in the hyperbolic part of the domain.

The contour σ_δ has the form

$$2y\delta=(1-x)-(1-x)^2-y^2.$$

Thus, boundary condition (1.3) can be written as

$$\frac{\partial u}{\partial n}|_{(1-x)-(1-x)^2-y^2=2y\delta}=0.$$

By definition (2.1) for the number γ_δ, we have

$$\frac{\partial u}{\partial n}|_{\sigma_\delta}=\Big(\frac{\alpha_k y^{\alpha_k-1}}{2(\sin\gamma_\delta)^{\alpha_k}((1-x)^2+y^2)^{\alpha_k}}\Big)(\cos(\alpha_k\gamma_\delta)-\sin(\alpha_k\gamma_\delta)).$$

By the definition of α_k in (2.3), we obtain

$$\alpha_k=\frac{\frac{\pi}{4}+\pi k}{\gamma_\delta}\ \Rightarrow\ \cos(\alpha_k\gamma_\delta)-\sin(\alpha_k\gamma_\delta)=0.$$

Thus,

$$\frac{\partial u}{\partial n}\Big|_{\sigma_\delta}=0.$$

The function in (2.2) satisfies the boundary condition (1.3).

3. To check conditions (1.4), from the representation of (2.2), we obtain

$$\begin{gathered} u(x,-0)=u(x,+0)=\Big(\frac{1}{1-x}-1\Big)^{\alpha_k}, \\ u_y(x,-0)=u_y(x,+0)=\alpha_k\Big(\frac{1}{1-x}-1\Big)^{\alpha_k-1}\frac{1}{(1-x)^2}, \end{gathered}$$

and conditions (1.4) are satisfied.

As a result, function in (2.2) is solution of the homogeneous Problem N-T. It is easy to see that function in (2.2) belongs to the class $C^{-\alpha_k,\alpha_k}_{A,B}(\overline{\Omega})\cap C^1(\sigma_\delta)$.

Next, we prove the final statement of theorem 2.1.

$$\|u_k\|^2_{L_2(\Omega)}=\iint_\Omega |u_k(x,y)|^2\,dx\,dy<\infty. \tag{2.6}$$

Note that

$$\iint_\Omega |u_k(x,y)|^2\,dx\,dy=\iint_{\Omega_2}|u_k(x,y)|^2\,dx\,dy+\iint_{\Omega_1}|u_k(x,y)|^2\,dx\,dy,$$

$$
\begin{aligned}
\iint_{\Omega_2} |u_k(x,y)|^2\,dx\,dy &= \iint_{\Omega_2} \Big|\Big(\frac{x+y}{1-(x+y)}\Big)^{2\alpha_k}\Big|\,dx\,dy\\
&< \iint_{\Omega_2} \big(1-(x+y)\big)^{-2\alpha_k}\,dx\,dy.
\end{aligned}
$$

Using the change of variables $x+y=\xi$, $x-y=\eta$, we obtain

$$
\begin{aligned}
\iint_{\Omega_2} |u_k(x,y)|^2\,dx\,dy &< \frac{1}{2}\iint_{\Omega_2} (1-\xi)^{-2\alpha_k} d\xi d\eta = \frac{1}{2}\int_0^1 d\eta \int_0^\eta (1-\xi)^{-2\alpha_k} d\xi\\
&= \frac{1}{2(2\alpha_k-1)}\int_0^1 \big((1-\eta)^{1-2\alpha_k}-1\big)d\eta < \infty
\end{aligned}
$$

for $1-2\alpha_k > -1$, $\Leftrightarrow \alpha_k < 1$.

By using the change of variables $x = \frac{r^2+r\cos\varphi}{1+2r\cos\varphi+r^2}$, $y = \frac{r\sin\varphi}{1+2r\cos\varphi+r^2}$ in Ω_1, we obtain

$$
\begin{aligned}
\iint_{\Omega_1} |u_k(x,y)|^2\,dx\,dy &= \iint_{\Omega_1} \frac{r^{2\alpha_k}(1+\sin 2\alpha_k\varphi)r}{(1+2r\cos\varphi+r^2)^2}\,dr\,d\varphi\\
&= \int_0^{\gamma_\delta} (1+\sin 2\alpha_k\varphi)d\varphi \int_0^\infty \frac{r^{2\alpha_k+1}dr}{(1+2r\cos\varphi+r^2)^2} < \infty
\end{aligned}
$$

for $2\alpha_k+1+1-4<0 \ \Leftrightarrow\ \alpha_k<1$.

Thus, ratio (2.6) is satisfied for $\alpha_k < 1$. Taking into account the definition of α_k in (2.3), it is easy to see that ratio (2.6) is satisfied only for $k=0$, and

$$
\frac{\pi}{4\gamma_\delta} < 1 \ \Leftrightarrow\ \gamma_\delta > \frac{\pi}{4}.
$$

□

Theorem 2.2 can be proved in a similar way; so we omit its proof.

Let us proceed to the analysis of the nonhomogeneous problem N-T and N-T*. An *n-regular solution* of Problem N-T (N-T*) is defined as a solution,

$$
\begin{gathered}
u(x,y) \in C^2(\Omega_1)\cap C^2(\Omega_2)\cap C^1(\Omega)\cap C^1(\sigma_\delta)\cap C^{-n,0}_{A,B}(\overline{\Omega})\\
(\upsilon(x,y) \in C^2(\Omega_1)\cap C^2(\Omega_2)\cap C^1(\Omega)\cap C^1(\sigma_\delta)\cap C^{0,-n}_{A,B}(\overline{\Omega})).
\end{gathered}
$$

The following theorems hold for the nonhomogeneous Problems N-T and N-T*.

Theorem 2.3. *The solution of Problem N-T is n-regular for any right-hand side $f(x,y)\in C(\overline{\Omega})$ if and only if*

$$
\gamma_\delta < \pi/(4n), \quad n=1,2,\dots \tag{2.7}
$$

The solution is n-regular for arbitrary approach angles γ_δ only if the right-hand side of (1.1) *satisfies the conditions*

$$
\iint_\Omega \upsilon_k(x,y)f(x,y)\,dx\,dy = 0, \quad k=0,\dots,j_0, \tag{2.8}
$$

where υ_k are the functions given by (2.5), *$j_0 = [n\gamma_\delta/\pi - 1/4]$, and $[z]$ is the integer part of z. In this case, the number of conditions* (2.8) *depends on the angle γ_δ, and their maximum number is equal to n (as $\gamma_\delta \to \pi$).*

Theorem 2.4. *Condition* (2.7) *is necessary and sufficient for the n-regularity of the solution of Problem N-T* for any right-hand side* $g(x,y) \in C(\overline{\Omega})$*; for arbitrary approach angles* γ_δ*, the right-hand side of* (1.5) *satisfies the relations*

$$\iint_\Omega u_k(x,y) g(x,y)\, dx\, dy = 0, \quad k = 0, \dots, j_0, \tag{2.9}$$

where the u_k *are the functions given by* (2.2) *and* $j_0 = [n\gamma_\delta/\pi - 1/4]$*. In this case, the number of conditions* (2.9) *depends on the angle* γ_δ*, and their maximum number is equal to n (as* $\gamma_\delta \to \pi$*).*

Remark 2.5. Conditions (2.8) and (2.9) with $k \geq 1$ are not orthogonality conditions in $L_2(\Omega)$, and for $k \geq 0$ they are orthogonality conditions only if inequality (2.4) holds. This immediately follows from theorems 2.1 and 2.2.

Proof of Theorem 2.3. Set $u(x,y) = \tau(x)$ and $u_y(x,0) = \nu(x)$. In the hyperbolic part of the domain Ω_2, we consider the Cauchy-Goursat problem

$$-u_{xx} + u_{yy} = f(x,y), \quad u|_{AC} = 0, \quad u_y(x,0) = \nu(x).$$

The solution of this problem has the form [12, p.121]:

$$u(x,y) = \int_0^{x+y} \nu(t)dt - \frac{1}{2}\int_0^{x+y} d\xi_1 \int_{\xi_1}^{x-y} f\big(\frac{\xi_1+\eta_1}{2}, \frac{\xi_1-\eta_1}{2}\big)\, d\eta_1.$$

Then we obtain the main relation

$$\tau(x) = \int_0^x \nu(t)dt - \frac{1}{2}\int_0^x d\xi_1 \int_{\xi_1}^x f\big(\frac{\xi_1+\eta_1}{2}, \frac{\xi_1-\eta_1}{2}\big) d\eta_1, \quad 0 < x < 1.$$

It is convenient to represent it in the form

$$\tau\big(\frac{x_1}{1+x_1}\big) = x_1 \int_0^1 \frac{\nu(x_1\theta/(1+x_1\theta))}{(1+x_1\theta)^2} d\theta + F_1\big(\frac{x_1}{1+x_1}\big), \quad 0 < x_1 < \infty, \tag{2.10}$$

where

$$F_1(x) = -\frac{1}{2}\int_0^x d\xi_1 \int_{\xi_1}^x f\big(\frac{\xi_1+\eta_1}{2}, \frac{\xi_1-\eta_1}{2}\big) d\eta_1.$$

We apply the Mellin transform $F(s) = \int_0^\infty x^{s-1} f(x) dx$ to both sides of relation (2.10). By using the formula [2, p. 269]

$$\int_0^\infty x^{s-1} dx\, x \int_0^\infty u(x\theta)\upsilon(\theta) d\theta = \int_0^\infty x^s u(x) dx \int_0^\infty x^{-s-1} \upsilon(x) dx,$$

for $u(x_1) = \frac{\nu(x_1/(1+x_1))}{(1+x_1)^2}$ and

$$\upsilon(x) = \begin{cases} 1 & \text{for } 0 \leq x < 1, \\ 0 & \text{for } x > 1, \end{cases}$$

from (2.10), we obtain the relation

$$\overline{\tau}(s) = -\frac{1}{s}\overline{\nu}(s) + \overline{F_1}(s). \tag{2.11}$$

Here

$$\overline{\tau}(s) = \int_0^\infty x^{s-1} \tau\big(\frac{x}{1+x}\big) dx, \quad \overline{\nu}(s) = \int_0^\infty x^{s-1} x \frac{\nu(x/(1+x))}{(1+x)^2} dx, \tag{2.12}$$

$$\overline{F_1}(s) = \int_0^\infty x^{s-1} F_1\big(\frac{x}{1+x}\big) dx. \tag{2.13}$$

In the elliptic part Ω_1, we consider the problem

$$u_{xx} + u_{yy} = f(x,y), \quad \frac{\partial u}{\partial n}\big|_{\sigma_\delta} = 0, \quad u(x,0) = \tau(x).$$

By making the change of variables

$$x = \frac{r^2 + r\cos\varphi}{1 + 2r\cos\varphi + r^2}, \quad y = \frac{r\sin\varphi}{1 + 2r\cos\varphi + r^2}, \tag{2.14}$$

by using the Mellin transform, and by solving the resulting problem, we obtain

$$\overline{\nu}(s) = \tan(s\gamma_\delta)s\overline{\tau}(s) - \int_0^{\gamma_\delta} \overline{u_2}(s,t)\overline{f}(s,t)dt, \tag{2.15}$$

where

$$\overline{f}(s,\varphi) = \int_0^\infty r^{s-1}\frac{r^2}{(1+2r\cos\varphi+r^2)^2} f\big(\frac{r^2 + r\cos\varphi}{1 + 2r\cos\varphi + r^2}, \frac{r\sin\varphi}{1 + 2r\ cos\varphi + r^2}\big)dr, \tag{2.16}$$

$$\overline{u_2}(s,\varphi) = \cos s\varphi + \sin s\varphi \frac{\sin s\gamma_\delta}{\cos s\gamma_\delta}, \tag{2.17}$$

and the functions $\overline{\tau}(s)$ and $\overline{\nu}(s)$ are defined in (2.12).

Now from relations (2.11) and (2.15), we obtain

$$\overline{\nu}(s) = \Big[\tan(s\gamma_\delta)s\overline{F_1}(s) - \int_0^{\gamma_\delta} \overline{u_2}(s,t)\overline{f}(s,t)dt\Big][1 + \tan(s\gamma_\delta)]^{-1}, \tag{2.18}$$

$$\overline{\tau}(s) = \Big[s\overline{F_1}(s) + \int_0^{\gamma_\delta} \overline{u_2}(s,t)\overline{f}(s,t)dt\Big][s(1 + \tan(s\gamma_\delta))]^{-1}. \tag{2.19}$$

First, let us analyze definitions (2.12) of the functions $\overline{\tau}(s)$ and $\overline{\nu}(s)$ and their relationship with the original functions $\tau(x)$ and $\nu(x)$. By making the obvious change of variables $t = x/(1+x)$, we reduce relation (2.12) to the form

$$\overline{\tau}(s) = \int_0^1 t^{s-1}(1-t)^{-s-1}\tau(t)dt, \quad \overline{\nu}(s) = \int_0^1 t^s(1-t)^{-s}\nu(t)dt.$$

Hence, it follows that if the function $\overline{\tau}(s)$ is continuous on the interval $(-1,0)$, then the function $\tau(t)$ is continuous at the point $t = 0$ and has a zero of order ≥ 1 there.

As a result, by taking into account the definitions of the functions $\tau(x)$ and $\nu(x)$, for the n-regularity of the solution, the right-hand sides in relations (2.18) and (2.19) should be continuous for $-n < s < 0$, whence we obtain $\gamma_\delta < \pi/(4n)$. Consequently, condition (2.7) is necessary and sufficient for the n-regularity of the solution of the Neumann-Tricomi problem for any right-hand side $f(x,y) \in C(\overline{\Omega})$.

The proof of first part of Theorem 2.3 is completed. Now let us proceed to the proof of properties of solutions for arbitrary approach angles γ_δ. Suppose that condition (2.7) fails. It follows that, for $s = -\alpha_k = -\pi(1+4k)/(4\gamma_\delta) \in (-n,0)$, for $k = 0,\dots,j_0$, and for j_0, which is defined in the statement of the theorem, the denominator in relations (2.18) and (2.19) is zero. Therefore, for the n-regularity, it is necessary and sufficient that the numerator is zero at these points as well,

$$\Big(s\overline{F_1}(s) + \int_0^{\gamma_\delta} \overline{u_2}(s,t)\overline{f}(s,t)dt\Big)\Big|_{s=-\alpha_k} = 0. \tag{2.20}$$

In this equation, we take into account relation (2.13) and the following property of the Mellin transform [6, p.567]: if

$$g(s) = \int_0^\infty x^{s-1} f(x)\, dx,$$

then

$$sg(s) = -\int_0^\infty x^{s-1} x f'(x)\, dx.$$

Now, by setting $s = -\alpha_k$, by returning to the variables x and y according to formulas (2.14), and by taking into account relations (2.3), (2.5), (2.16), and (2.17), we find that condition (2.20) can be represented in the form

$$\begin{aligned}&\iint_{\Omega-} \upsilon_k(x,y) f(x,y) dxdy + \iint_{\Omega+} \upsilon_k(x,y) f(x,y) dxdy\\ &= \iint_{\Omega} \upsilon_k(x,y) f(x,y) dxdy = 0.\end{aligned}$$

Here $k = 0, \dots, j_0$; moreover, the number of such k (by the definition of j_0) cannot exceed n. $\square$

Theorem 2.4 can be proved in a similar way, we omit its proof.

Conclusion. In this article, it has been shown that the number of solutions of the homogeneous Neumann-Tricomi problem admitting a feature at the corner points of the domain, depends on the order of the singularity and depends on the order of the singularity and on the values of approach angles of an elliptic part of boundary of the domain to the line change of type. We have shown that for what angles of approach a singular solution of homogeneous Neumann-Tricomi problem belongs to the space L_2. In case of Neumann-Tricomi problem, unlike Tricomi problem in space L_2, only the value of angle at point A solves everything and the angle of approach at point B does not react [11]. Also, we have obtained conditions of existence of n-regular solutions for the nonhomogeneous Neumann-Tricomi problem. These conditions have been formulated in terms of orthogonality of the function in the right hand side of the equation to the corresponding singular solutions of its adjoint homogeneous problem.

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Makhmud A. Sadybekov
Institute of Mathematics and Mathematical Modeling, 125 Pushkin str., 050010 Almaty, Kazakhstan
E-mail address: makhmud-s@mail.ru

Nurgissa A. Yessirkegenov
Institute of Mathematics and Mathematical Modeling, 125 Pushkin str., 050010 Almaty, Kazakhstan.
Al-Farabi Kazakh national university, 71 ave. Al-Farabi, 050040 Almaty, Kazakhstan
E-mail address: nurgisa@hotmail.com