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PRICING ASIAN OPTIONS IN FINANCIAL MARKETS USING MELLIN TRANSFORMS

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Abstract. We derived an expression for the floating strike put arithmetic asian options in financial market when the asset is driven by the generalized Barndorff-Nielsen and Shephard model with stochastic volatility. A solution procedure for the resulting partial differential equation is provided using the technique of Mellin transforms.

1. Introduction

Asian options are fully path-dependent exotic options that have payoffs which depend on the history of the random walk of the asset price via some sort of average. These options were first successfully priced in 1987 by David Spaughton and Mark Standish of Bankers Trust. Their payoff is typically based on arithmetic or geometric average of underlying asset prices at monitoring dates before maturity. Pricing Asian options of arithmetic type is difficult even for the simplest asset price model, as the arithmetic average of a set of lognormal random variables is not lognormally distributed. For simple asset price model when the price is driven by a Brownian motion, different methods are implemented to obtain pricing formula for arithmetic Asian options (see [6, 11, 15]). For Asian options payoff depends on the average value of the underlying asset and hence volatility in the average value tends to be smoother and lower than that of the plain vanilla options. The average is less exposed to sudden crashes or rallies in stock price and over time is harder to manipulate than a single stock price. Thus the Asian options are less expensive than comparable plain vanilla options.

For arithmetic Asian options the prices are usually approximated numerically. In [11] the computation of the price of an Asian option is obtained in two different ways. Firstly, exploiting a scaling property the problem is reduced to the problem of solving a parabolic partial differential equation (PDE) in two variables. Secondly, a reasonable lower bound is provided which is an approximation of the true price. In [7] using simple probabilistic methods the moments of all orders of an Asian option is obtained. Formulas obtained in that paper has an interesting resemblance with the Black-Scholes formula, even though the comparison cannot be carried too far. In [15] it is shown that for arithmetic Asian options, the governing PDE can not

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be transformed into a heat equation with constant coefficients, therefore does not have a closed-form solution of Black-Scholes type, i.e., in terms of cumulative normal distribution function. An analytical solution in obtained in a series form. Numerical results show that the series converges very fast and gives a good approximate value. For pricing Asian options Monte Carlo methods are applied in [8], and advanced pricing methods based on a recursive integration procedure are used in [2, 16]. An efficient partial differential equation (PDE) technique for Asian option is used in [14] where it is observed that the Asian option is a special case of the option on a traded account. The price of the Asian option is characterized by a simple one-dimensional PDE which could be applied to both continuous and discrete average Asian option.

In modern asset price models, stochastic volatility plays a crucial role in order to explain a number of stylized facts of returns. Stochastic volatility significantly increases the complexity of the problem. However, models with stochastic volatility are not well studied or understood for Asian options. In this paper we incorporate stochastic volatility for the option pricing of Asian options. We consider a generalized Barndorff-Nielsen and Shephard (BN-S) asset modeling which admits Ornstein-Uhlenbeck type stochastic volatility modeling. The objective of the present paper is to use such generalized BN-S model for the option pricing for arithmetic Asian options. Then we derive a partial differential equation that represents the arbitrage-free price of floating strike put arithmetic Asian option.

In Section 2 we present the set up of the generalized BN-S model. Main result is presented in Section 3. A very brief conclusion is provided in the last section.

2. Generalized BN-S Model

The pricing of arithmetic Asian options has been the subject of extensive research in last couple of decades. In this paper, we consider a frictionless financial market where a riskless asset with constant return rate r and a stock are traded up to a fixed horizon date T. Barndorff-Nielsen and Shephard (see [1]) assumed that the price process of the stock $S = (S_t)_{t\geq 0}$ is defined on some filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{0\leq t\leq T}, P)$ and is given by:

$$S_t = S_0 \exp(X_t), \tag{2.1}$$

$$dX_t = (\mu + bV_t)\,dt + \sqrt{V_t}\,dW_t + \rho\,dZ_{\lambda t}, \tag{2.2}$$

$$dV_t = -\lambda V_t\,dt + dZ_{\lambda t}, \quad V_0 > 0, \tag{2.3}$$

where V_t is the square of the volatility at time t, the parameters $\mu, b, \rho, \lambda \in \mathbb{R}$ with $\lambda > 0$ and $\rho \leq 0$. $W = (W_t)$ is Brownian motion and the process $Z = (Z_{\lambda t})$ is a subordinator. Barndorff-Nielsen and Shephard refer to Z as the *background driving Lévy process* (BDLP). Also W and Z are assumed to be independent and $(\mathcal{F}_t)$ is assumed to be the usual augmentation of the filtration generated by the pair (W, Z). This model is known in literature as Barndorff-Nielsen and Shephard model (BN-S model).

A major disadvantage of the classical BN-S model is the inclusion of single BDLP for both the log-return and volatility. In this classical model they become completely dependent. Such dependence significantly reduces the flexibility to appropriately model the volatility. Moreover, such absolute correlation is not supported by empirical observations of the implied volatility. One possible alternative is to drive the log-return and volatility by correlated (not absolutely correlated) Lévy processes. We show in this paper that this generalized model has the liberty to fit the option price and volatility in a correlated but different way, which is not possible for the case of classical BN-S model.

In this section we present a generalized version of the Barndorff-Nielsen and Shephard model. Let $Z_{\lambda t}$ and $Z^*_{\lambda t}$ be two *independent* Lévy subordinators with same (finite) variance. Then

$$d\tilde{Z}_{\lambda t} = \rho' \, dZ_{\lambda t} + \sqrt{1-\rho'^2}\, dZ^*_{\lambda t}, \tag{2.4}$$

is also a Lévy subordinator provided $0 < \rho' \leq 1$. Thus Z and $\tilde{Z}$ are positively correlated (with correlation coefficient ρ') Lévy subordinators. Here the *independence* of the Lévy processes Z are Z^* understood in the sense of [3, Proposition 5.3].

Suppose the dynamics of S_t is given by (2.1), (2.2) where V_t is given by

$$dV_t = -\lambda V_t \, dt + d\tilde{Z}_{\lambda t}, \quad V_0 > 0, \tag{2.5}$$

where $\tilde{Z} = (\tilde{Z}_{\lambda t})$ is a subordinator independent of W but has a positive correlation with Z as described above. For this paper we assume that the dynamics of $S = (S_t)$ is given by (2.1), (2.2) and (2.5) and these will be referred to as generalized BN-S model. For simplicity of notation denote the probability space of $S = (S_t)$ by $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{0\leq t\leq T}, P)$, where $(\mathcal{F}_t)$ is assumed to be the usual augmentation of the filtration generated by the pair $(W, Z, \tilde{Z})$. When the parameter $\rho < 0$ a leverage effect is incorporated in the model given by (2.2) and (2.5), due to the positive correlation between Z and $\tilde{Z}$. Empirically observed fact suggests that for most equities a fall in price is associated with an increase in volatility. The proposed model is in agreement with this fact. However this model suggests a richer structure for volatility than classical BN-S model due to the presence of Z^* which is independent of Z. This will give some additional flexibility in calibration of volatility structure.

In this work we assume that Z and Z^* have no deterministic drift (so $\tilde{Z}$ has no Brownian component).

If the Lévy measures of Z and Z^* are ν and ν^* respectively, then by assumption and [3] (Theorem 4.1), the characteristic triplet of $\tilde{Z}$ is given by $(\tilde{A}, \tilde{\gamma}, \tilde{\nu})$, where $\tilde{A} = 0$, $\tilde{\nu}(B) = \nu\big(\frac{B}{\rho'}\big) + \nu^*\big(\frac{B}{\sqrt{1-\rho'^2}}\big)$, for $B \in \mathcal{B}(\mathbb{R})$ and

$$\begin{aligned} \tilde{\gamma} &= \rho'\gamma + \sqrt{1-\rho'^2}\gamma^* - \int_{\mathbb{R}} y(1_{|y|\leq 1}(y) - 1_{S_1}(y))\tilde{\nu}(dy) \\ &= \rho'\gamma + \sqrt{1-\rho'^2}\gamma^*, \end{aligned} \tag{2.6}$$

where S_1 is given by

$$S_1 = \{\rho' x_1 + \sqrt{1-\rho'^2} x_2 : x_1^2 + x_2^2 \leq 1, x_1, x_2 \in \mathbb{R}\}.$$

Therefore in general $\tilde{Z}$ has a *drift* component. However, if both Z and Z^* are processes of finite variation and $\gamma = \int_{|x|\leq 1} x\nu(dx)$ and $\gamma^* = \int_{|x|\leq 1} x\nu^*(dx)$, then $\tilde{\gamma} = \int_{|x|\leq 1} x\tilde{\nu}(dx)$ and hence the deterministing drift (in the sense of Corollary 3.1 in [3]) for Z, Z^* and $\tilde{Z}$ are zero.

For the rest of this article we assume $S_0 = 1$. The risk-neutral dynamics of $S_t = e^{X_t}$, where X_t is governed by (2.2) and (2.5), is given by $dS_t = S_{t-}(r\, dt + \sqrt{V_t}\, dW_t + dM_t)$, where $M = (M_t)_{t\geq 0}$ is the martingale Lévy process given by

$M_t = \sum_{0<s\le t}\left(e^{\rho \Delta Z_{\lambda_s}} - 1\right) - \lambda\kappa(\rho)t$. Thus

$$dS_t = S_{t-}\Big(r\,dt + \sqrt{V_t}\,dW_t + \int_{\mathbb{R}} (e^{\rho x} - 1)\tilde{J}_Z(dt, dx)\Big), \tag{2.7}$$

where $\tilde{J}_Z$ is the compensated random measure describing jumps of Z (or X) and the compensator is $\nu_Z(dt, dx) = \nu_Z(dx)dt = \lambda w(x)dxdt$, where $w(x)$ is the Lévy density for Z. Similarly, if $J_{\tilde{Z}}$ is the random measure describing jumps of $\tilde{Z}$ then

$$dV_t = -\lambda V_t\,dt + \int_{\mathbb{R}} \lambda y J_{\tilde{Z}}(dt, dy). \tag{2.8}$$

We will later use a compensator $\nu_{\tilde{Z}}(dt, dy) = \nu_{\tilde{Z}}(dy)dt = \lambda\tilde{w}(y)dydt$ related to the jumps in V_t, with $\tilde{w}(y)$ being the Lévy density for $\tilde{Z}$.

It is shown in [13] that with proper choice of parameters for the Lévy processes Z and $\tilde{Z}$, different error estimates for *market data calibration* and accuracy of implied volatility fitting can be improved significantly. With proper choice of parameters the generalized BN-S model can produce better calibration than other known models (even with more calibration parameters) such as CGMY-CIR, CGMY-Gamma-OU, CGMY-IG-OU, Meixner-IG-OU, NIG-IG-OU or GH-IG-OU.

3. Option pricing equation

In this section we present the main theorems related to the pricing of arithmetic Asian options. Let $A_t = \int_0^t S_u\,du$. Then A is an increasing continuous process and thus has no Brownian component.

There are four different types of arithmetic Asian options according to the payoff function. For *fixed strike* (E) call and put Asian options payoffs are given by $(\frac{1}{T}A_T - E)^+$ and $(E - \frac{1}{T}A_T)^+$ respectively. For *floating strike* call and put Asian options the payoffs are given by $(S_T - \frac{1}{T}A_T)^+$ and $(\frac{1}{T}A_T - S_T)^+$ respectively. In this section we develop a technique for pricing floating strike put Asian options. Option pricing for other Asian options can be done with very similar procedures.

Assumption 3.1. We assume the Lévy measure ν satisfies

$$\int_{y>1} e^{2y}\nu(dy) < \infty.$$

Also, assume when $V_t = 0$, there exist $\zeta \in (0, 2)$ such that

$$\liminf_{\epsilon\to 0} \epsilon^{-\zeta}\int_0^{\epsilon} x^2\nu(dx) > 0.$$

Assumption 3.2. At the final time T, there exist a constant that $\beta > 0$ that depends on the market, so that $\frac{1}{T}A_T - S_T \le \beta$ in the market. Let $P(T, S_T, V_T, A_T) = 0$, if $\frac{1}{T}A_T - S_T > \beta$.

We note that Assumption 3.1 implies that X_t in has a smooth C^2 density with derivatives vanishing at infinity (see [12, Proposition 28.3]). Based on these two assumptions we state the following option pricing equation. The solution of this equation gives the price of Asian floating put options.

Theorem 3.3. *Consider the generalized BN-S model given by* (2.1), (2.2) *and* (2.5). *Then for* $0 \le t < T$, *the price of Asian floating put option* $P(t, S_t, A_t, V_t)$ *is* *given by*

$$\begin{aligned}
&\frac{\partial P}{\partial t} + \frac{1}{2} V S^2 \frac{\partial^2 P}{\partial S^2} + S \frac{\partial P}{\partial A} + rS \frac{\partial P}{\partial S} - \lambda V \frac{\partial P}{\partial V} - rP \\
&+ \int_{\mathbb{R}} \Big(P(t, Se^x, V, A) - P(t, S, V, A) - S(e^{\rho x} - 1) \frac{\partial P}{\partial S} \Big) \nu_Z(dx) \\
&+ \int_{\mathbb{R}} \left(P(t, S, V + y, A) - P(t, S, V, A) \right) \nu_{\tilde{Z}}(dy) = 0,
\end{aligned} \tag{3.1}$$

with final condition

$$P(T, S_T, A_T, V_T) = \big(\frac{A_T}{T} - S_T \big)^+. \tag{3.2}$$

Proof. Suppose $\hat{P}(t, S_t, V_t, A_t) = e^{r(T-t)} P(t, S_t, V_t, A_t)$. Then under the equivalent martingale measure

$$\hat{P}_t = E[(\frac{A_T}{T} - S_T)^+ | \mathcal{F}_t].$$

Clearly this is a martingale. Denote the continuous part of the stochastic processes S, V and A by S^c, V^c and A^c respectively. Applying the Itô formula to $\hat{P}_t$ and observing the quadratic variations

$$d[S^c, S^c] = S^2 V \, dt$$

and

$$d[V^c, V^c] = d[A^c, A^c] = d[V^c, A^c] = d[S^c, A^c] = d[V^c, A^c] = 0,$$

we obtain

$$d\hat{P}_t = a(t) \, dt + dR_t,$$

where

$$\begin{aligned}
a(t) = e^{r(T-t)} [&\frac{\partial P}{\partial t} + \frac{1}{2} V S^2 \frac{\partial^2 P}{\partial S^2} + S \frac{\partial P}{\partial A} + rS \frac{\partial P}{\partial S} - \lambda V \frac{\partial P}{\partial V} - rP \\
&+ \int_{\mathbb{R}} \Big(P(t, Se^x, V, A) - P(t, S, V, A) - S(e^{\rho x} - 1) \frac{\partial P}{\partial S} \Big) \nu_Z(dx) \\
&+ \int_{\mathbb{R}} \left(P(t, S, V + y, A) - P(t, S, V, A) \right) \nu_{\tilde{Z}}(dy)],
\end{aligned}$$

and

$$\begin{aligned}
dR_t = e^{r(T-t)} [S\sqrt{V} \, dW_t &+ \int_{\mathbb{R}} \left(P(t, Se^x, V, A) - P(t, S, V, A) \right) \tilde{J}_Z(dt, dx) \\
&+ \int_{\mathbb{R}} \left(P(t, S, V + y, A) - P(t, S, V, A) \right) \tilde{J}_{\tilde{Z}}(dy)].
\end{aligned}$$

With Assumption 3.1 and procedures in [4] it can shown that R_t is a martingale. Therefore $\hat{P}_t - R_t$ is a (square integrable) martingale. But $\hat{P}_t - R_t = \int_0^t a(u) \, du$ is a continuous process with finite variation. Hence $a(t) = 0$ almost surely with respect to the equivalent martingale measure. This gives the required result. □

We now find a solution of (3.1) with final condition (3.2). We show that the application of Mellin transform and a proper form of solution reduce the complexity of this problem. For the rest of the paper we assume for simplicity $\rho' = 1$. In other words, we consider the classical BN-S model for which $Z = \tilde{Z}$.

If f is an integrable complex valued function defined over the positive real numbers, then its Mellin transform (if exists) is defined by

$$\mathcal{M}(f)(\eta) = \int_0^\infty f(s)\eta^{s-1}\, ds, \quad \eta \in \mathbb{C}.$$

If $\mathcal{M}(f)(\eta)$ exists for $a < \mathrm{Re}(\eta) < b$, then the latter is called the *fundamental strip*. We use the following three properties of Mellin transform for the proof of the next theorem. For proofs of these properties see [5].

(1) (Scaling property) $\mathcal{M}(f(as)) = a^{-\eta}F(\eta)$, where $a > 0$.
(2) $\mathcal{M}(sf'(s)) = -\eta F(\eta)$.
(3) $\mathcal{M}(s^2 f''(s)) = \eta(\eta+1)F(\eta)$.

In the next theorem, we provide an integral representation of the floating strike put Asian option price P in Theorem 3.3. In [9] the authors derive an expression for pricing perpetual options using Mellin transform. In [10] integral equation representations for the price of European and American basket put options have been derived using Mellin transform techniques. We denote the indicator function of a set B by χ_B.

The Mellin transform defined in the following theorem exists for $\eta \in \mathbb{C}$ such that $\mathrm{Re}(\eta) < 0$ and $Re(\eta) \neq -1$. The region $-1 < \mathrm{Re}(\eta) < 0$ may be taken as the *fundamental strip*.

Theorem 3.4. *There exists a solution of* (3.1) *with final condition* (3.2) *of the form*

$$P(t, S_t, V_t, A_t) = g(t, V_t, \frac{1}{T}A_t - S_t).$$

For fixed $m > T$, on the hyper-plane $S_t = \frac{A_t}{m+t}$, $g(t, V_t, \frac{1}{T}A_t - S_t) = g(t, V_t, \kappa_t S_t)$, where $\kappa_t = (\frac{m+t}{T} - 1)$ is given by

$$g(t, V_t, \kappa_t S_t) = \frac{1}{2\pi i}\int_{c^*-i\infty}^{c^*+i\infty} (\kappa_t S_t)^{-\eta} \exp\left[q(t,\eta)V_t\right] H(t,\eta)\, d\eta, \tag{3.3}$$

where

$$q(t,\eta) = \frac{1}{2\lambda}\eta(\eta+1)[1 - e^{-\lambda(T-t)}], \tag{3.4}$$

and $H(t,\eta)$ is given by

$$H(t,\eta) = \frac{\beta^{\eta+1}}{\eta+1}\exp\Big(\int_t^T L(t,\eta)\, dt\Big), \tag{3.5}$$

where

$$\begin{aligned} L(t,\eta) = -(r - \frac{1}{T})\eta - r + \int_{\mathbb{R}} \Big(\chi_{x\in(-\infty, \ln(\frac{m+t}{T}))}{\alpha_t}^{-\eta} - 1 \\ + (e^{\rho x} - 1)\eta\Big)\nu_Z(dx) + \int_{\mathbb{R}} (e^{q(t,\eta)y} - 1)\nu_Z(dy), \end{aligned} \tag{3.6}$$

with

$$\alpha_t = \Big(\frac{\frac{m+t}{T} - e^x}{\frac{m+t}{T} - 1}\Big),$$

and β is obtained from market by Assumption 3.2.

Once $g(t, V_t, \kappa_t S_t)$ is known for the hyper-plane $S_t = \frac{A_t}{m+t}$, the solution is extended to other points in the $S > 0$, $A > 0$ region by $g(t, V_t, \frac{1}{T}A_t - S_t)$. On the other hand, if $\frac{A_t}{T} \leq S_t$, $P(t, S_t, V_t, A_t) = 0$.

The quantity c^* *in the right hand side of* (3.3) *appears due to the inverse Mellin transform. It is any real value so that the integrand in* (3.3) *is analytic in a neighborhood of* c^* *and the integrand tends to zero uniformly along* $c^* \pm i\infty$*.*

Proof. We fix a calibration parameter $m > T$. At time $t = 0$, consider the following straight line in the stock-price (S) and average price (A) hyper-space:

$$A = mS. \tag{3.7}$$

Since for a *fixed* S, $A = A_0 + St$, therefore the nature of (3.7) at time t will be given by

$$A_t = (m + t)S_t. \tag{3.8}$$

Notice that since $m > T$ therefore for any t, (3.8) always remain in the side for which $\frac{A}{T} - S > 0$.

We look for a solution of (3.1) the form $P(t, S_t, V_t, A_t) = g(t, V_t, \frac{1}{T}A_t - S_t)$. This solution will be referred to as *traveling wave solution* with respect to S and A variable. Then (3.1) gives

$$\begin{aligned}
&\frac{\partial g}{\partial t} + \frac{1}{2}VS^2\frac{\partial^2 g}{\partial S^2} + (r - \frac{1}{T})S\frac{\partial g}{\partial S} - \lambda V\frac{\partial g}{\partial V} - rg \\
&+ \int_{\mathbb{R}} \Big(g(t, V, \frac{1}{T}A - Se^x) - g(t, V, \frac{1}{T}A - S) - S(e^{\rho x} - 1)\frac{\partial g}{\partial S}\Big)\nu_Z(dx) \\
&+ \int_{\mathbb{R}} \Big(g(t, V + y, \frac{1}{T}A - S) - g(t, V, \frac{1}{T}A - S)\Big)\nu_Z(dy) = 0.
\end{aligned} \tag{3.9}$$

Since the solution (for a given t and V) is of nature of a traveling wave in S and A plane, it is sufficient to determine g for some line in S and A plane which is *not* parallel to $S_t - \frac{A_t}{T} = 0$, for $0 \le t \le T$. Once $g(t, V_t, \frac{1}{T}A_t - S_t)$ is known on that line the solution is extended to the $S > 0$ and $A > 0$ region.

For this end, let us consider that at time t, S and A are related by (3.8). In this case, suppose $g(t, V_t, \frac{1}{T}A_t - S_t) = g(t, V_t, \kappa_t S_t)$, where $\kappa_t = (\frac{m+t}{T} - 1)$.

Denote the Mellin transform of $g\,(t, V_t, S_t)$ with respect to S_t by $\tilde{g}(t, V_t, \eta)$. We have the following relations from the property of Mellin transformation (with respect to S):

$$\begin{gathered}
\mathcal{M}\Big(S\frac{\partial g(t, V_t, \kappa_t S_t)}{\partial S}\Big) = -\eta\kappa_t^{-\eta}\tilde{g}(t, V_t, \eta), \\
\mathcal{M}\Big(S^2\frac{\partial^2 g(t, V_t, \kappa_t S_t)}{\partial S^2}\Big) = \eta(\eta + 1)\kappa_t^{-\eta}\tilde{g}(t, V_t, \eta).
\end{gathered} \tag{3.10}$$

Observe that $g(t, V, \frac{1}{T}A - Se^x) = 0$ when $\frac{1}{T}A - Se^x \le 0$. On the hyper-plane $S = \frac{1}{m+t}A$, $\frac{1}{T}A - Se^x = (\frac{m+t}{T} - e^x)S$. Thus on this hyper-plane the first term of the first integral term in (3.9) is zero when $x > \ln(\frac{m+t}{T})$. Thus, for this case, taking Mellin transform with respect to S for (3.9) and using (3.10) we obtain (after dividing by $\kappa_t^{-\eta}$ and writing $\tilde{g}$ for $\tilde{g}(t, V_t, \eta)$)

$$\begin{aligned}
&\frac{\partial \tilde{g}}{\partial t} + \frac{1}{2}V\eta(\eta + 1)\tilde{g} - (r - \frac{1}{T})\eta\tilde{g} - \lambda V\frac{\partial \tilde{g}}{\partial V} - r\tilde{g} \\
&+ \tilde{g}\int_{\mathbb{R}} \Big(\chi_{x \in (-\infty, \ln(\frac{m+t}{T}))}\Big(\frac{\frac{m+t}{T} - e^x}{\kappa_t}\Big)^{-\eta} - 1 + (e^{\rho x} - 1)\eta\Big)\nu_Z(dx) \\
&+ \int_{\mathbb{R}} (\tilde{g}(t, V + y, \eta) - \tilde{g}(t, V, \eta))\nu_Z(dy) = 0.
\end{aligned}$$

Letting $\tilde{g}(t, V_t, \eta) = \exp[q(t,\eta)V_t]H(t,\eta)$, where $q(t,\eta)$ is given by (3.4) and H is a function of t and η, we obtain

$$\frac{\partial H(t,\eta)}{\partial t} + L(t,\eta)H(t,\eta) = 0, \tag{3.11}$$

where $L(t,\eta)$ is given by (3.6). With the use of Assumption 3.2 and the observation that $q(T,\eta) = 0$, the final condition will be transformed to

$$\tilde{g}(T, V_T, \eta) = H(T,\eta) = \frac{\beta^{\eta+1}}{\eta+1}.$$

Note that this is in agreement with the fact that the final condition is independent of V_T (however, the option price is dependent on the volatility).

Thus (3.5) follows from (3.11). Hence (3.3) is obtained by inverse Mellin transform of $\tilde{g}(t, V_t, \eta)$. □

Conclusion. Generalized Barndorff-Nielsen and Shephard model with stochastic volatility is becoming increasingly popular model in literature and in this paper, we have derived the pricing expression for the floating strike put arithmetic Asian options in financial market driven by such model. It is worth noting that such model can be easily generalized to floating strike put Asian options. Thus the main theorem presented in this paper gives a concrete algorithmic method for solving these pricing problems. We will demonstrate the evidence of good numerical accuracy and stability of this proposed solution technique in a longer research article which will be a sequel of this paper.

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References

[1] Barndorff-Nielsen, O. E.; Shephard, N.; Non-Gaussian Ornstein-Uhlenbeck-based models and some of their uses in financial economics, *J. R. Stat. Soc. Ser. B Stat. Methodol.*, **63** (2001) , 167-241.

[2] Černý, A.; Kyriakou, I.; An improved convolution algorithm for discretely sampled Asian options, *Quant. Finance*, **11** (2011), 381-389.

[3] Cont, R.; Tankov, P.; *Financial Modelling with Jump Processes*, 2nd edn. Chapman & Hall/CRC, (2004).

[4] Cont, R.; Voltchkova, E.; Integro-differential equations for option prices in exponential Lévy models, *Finance Stochast.*, **9** (2005), 299-325.

[5] Debnath, L.; Bhatta, D.; *Integral transforms and their applications*, 2nd edn. Chapman & Hall/CRC (2006).

[6] Elshegmani, Z. A.; Ahmad, R. R.; Solving an Asian option PDE via Laplace transform, *ScienceAsia*, **39S** (2013), 67-69.

[7] Geman, H.; Yor, M.; Bessel process, Asian options and perpetuities, *Math. Finance*, **3** (1993), 349-375.

[8] Kemna, A.; Vorst, T.; A pricing method for options based on average asset values, *J. Bank. Finance*, **14** (1990), 113-129.

[9] Panini, P.; Srivastav, R. P.; Pricing perpetual options using Mellin transforms, *Applied Mathematics Letters*, **18** (2005), 471-474.

[10] Panini, P.; Srivastav, R. P.; Option pricing with Mellin transnforms, *Mathematical and Computer Modelling*, **40** (2004), 43-56.

[11] Rogers, L.; Shi, Z.; The value of an Asian option, *J Appl Probab*, **32** (1995), 1077-1088.

[12] Sato, K.-I.; *Lévy Processes and Infinitely Divisible Distributions*, Cambridge University Press (1999).

[13] SenGupta, I.; *Generalized BN-S stochastic volatility model for option pricing*, submitted.

[14] Večeř, J.; New pricing of Asian options, *J. Comput. Finance*, **4** (2001), 105-113.

[15] Zhang, J. E.; Theory of continuously-sampled Asian option pricing, *J Futures Market*, **23** (2003), 535-560.

[16] Zhang, B.; Oosterlee, C. W.; Efficient Pricing of European-Style Asian Options under Exponential Lévy Processes Based on Fourier Cosine Expansions, *SIAM J. Financial Math.*, **4** (2013), 399-426.

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