Electronic Journal of Differential Equations, Vol. 2015 (2015), No. 130, pp. 1–9.
ISSN: 1072-6691. URL: http://ejde.math.txstate.edu or http://ejde.math.unt.edu
ftp ejde.math.txstate.edu

NON-EXISTENCE OF POSITIVE RADIAL SOLUTION FOR SEMIPOSITONE WEIGHTED P-LAPLACIAN PROBLEMS

SIGIFREDO HERRÓN, EMER LOPERA

Abstract. We prove the non-existence of positive radial solution to a semipositone weighted p-Laplacian problem whenever the weight is sufficiently large. Our main tools are a Pohozaev type identity and a comparison principle.

1. Introduction

We consider the non-existence of positive radial solutions to the problem

$$\begin{gathered} -\Delta_p u = W(\|x\|)f(u) \quad \text{in } B_1(0), \\ u = 0 \quad \text{on } \partial B_1(0), \end{gathered} \tag{1.1}$$

where $\Delta_p u = \nabla \cdot (|\nabla u|^{p-2}\nabla u)$ is the p-Laplace operator, $B_1(0)$ is the unit ball in $\mathbb{R}^N$ and $2 < p < N$.

Note that solving this problem is equivalent to solving the problem

$$[r^n \varphi_p(u')]' = -r^n W(r) f(u), \quad 0 < r < 1, \quad u'(0) = 0, \quad u(1) = 0, \tag{1.2}$$

where $r = \|x\|$, $n := N-1$ and $' = \frac{d}{dr}$. The differential equation in the last problem is equivalent to

$$(p-1)|u'|^{p-2}u'' + \frac{n}{r}|u'|^{p-2}u' + W(r)f(u) = 0, \quad 0 < r < 1. \tag{1.3}$$

We assume that the nonlinearity satisfies the following hypotheses:

- (F1) $f : [0,\infty) \to \mathbb{R}$ is a continuous function with exactly three zeros $0 < \beta_1 < \beta_2 < \beta_3$.
- (F2) $f(0) < 0$ and f is increasing from β_3 on.
- (F3) Set $F(t) := \int_0^t f(s)ds$. Then, $\beta_3 < \theta_1$ where θ_1 is the unique positive zero of F.

Let us fix $\beta_3 < \gamma < \theta_1$. We will say that a function W is an admissible weight if it satisfies the following conditions:

- (W1) $W : [0,1] \to (0,\infty)$ is continuous and differentiable in $(0,1)$.
- (W2) $\widetilde{W}(r) := N + r\frac{W'(r)}{W(r)}$ is defined a.e. in $[0,1]$.

2010 *Mathematics Subject Classification.* 35J92, 35J60, 35J62.
Key words and phrases. Semipositone; quasilinear weighted elliptic equation;
positive radial solution; non-existence result; Pohozaev identity.

Submitted January 12, 2015. Published May 7, 2015.

(W3) If we define the following real numbers associated with W:
$$\overline{\eta} := \sup \widetilde{W}, \quad \underline{\eta} := \inf \widetilde{W}, \quad \overline{\lambda} := \max W, \quad \underline{\lambda} := \min W, \quad \widetilde{C}_\lambda := \frac{\overline{\lambda}}{\underline{\lambda}},$$
then
$$\frac{\underline{\eta}}{N} F(s) - \frac{1}{p*} f(s)s \le \widetilde{C}_\lambda F(\gamma), \quad \text{for } \gamma \le s < \theta_1, \tag{1.4}$$
$$\frac{\overline{\eta}}{N} F(s) - \frac{1}{p*} f(s)s \le \widetilde{C}_\lambda F(\gamma), \quad \text{for } s \ge \theta_1, \tag{1.5}$$
where $p* = \frac{Np}{N-p}$ is the critical exponent.

For every fixed $T > 0$ we consider the class of weights
$$\mathcal{C}(T) := \{W : W \text{ is an admissible weight, } \underline{\eta} > 0 \text{ and } \widetilde{C}_\lambda \le T\}.$$
We note this class contains admissible weights so that $\underline{\eta} > 0$ and $\overline{\lambda}/\underline{\lambda}$ is bounded.

Problems related to non-existence of positive radial solutions have been studied, most of them in the case $f(0) \ge 0$. For instance, in the semilinear case, in [2] considering $f(u) = u^p$ and suitable conditions on the derivative of W, and p; a result was showed in $\mathbb{R}^N$. See also [12] and references therein. The case $f(0) < 0$ is more complicated and in this direction, in [10], authors considered the non-existence of positive radial solutions for a semipositone problem (i.e. $f(0) < 0$) if f is not increasing entirely. The nonlinearity was superlinear and had more than one zero. There, the domain was an annulus and they exhibited a positive constant weight. Authors in [14], studied a semipositone elliptic system which involves positive parameters bounded away from zero, and the nonlinearities are smooth functions that satisfy certain linear growth conditions at infinity. They established non-existence of positive solutions when two of the parameters are large. Other works can be found in [1,3,9] but, we refer the reader to the survey paper [4] and references therein for a review about semipositone problems. In the quasilinear case, some papers are known in this direction. Chhetri et al [5] showed non-existence considering a C^1-nondecreasing non-linearity with a unique positive zero. They extend the result in [3] for $1 < p < \infty$. Also, Hai [8] obtained a non-existence result if f is locally Lipschitz continuous and $\liminf_{s\to\infty} f(s)/s^{p-1} > 0$. A very simple non-existence result for a semipositone quasilinear problem was showed in [13], where $W = 1$ and a non-linearity given by step function was considered.

The study of such quasilinear equations with semipositone structure is open in the case of general bounded regions. Moreover, questions on uniqueness remain open even for radial solutions when the domain is a ball or an annulus. We would like to mention that Castro *et al.* [4] wrote: "In general, studying positive solutions for semipositone problems is more difficult compared to that of positone problems. The difficulty is due to the fact that in the semipositone case, solutions have to live in regions where the reaction term is negative as well as positive". This makes remarkable our research and we emphasize that in this work we deal, mainly, with the case $2 < p < N$ and non-constant weight. Therefore, it is a generalization of [10, 13, 5, 8].

The main result reads as follows.

Theorem 1.1. *Assume hypotheses* (F1)–(F3)*. Then for every $T > 0$ there is a positive real number λ_0 such that if $W \in \mathcal{C}(T)$ and $\overline{\lambda} \ge \lambda_0$, then* (1.1) *has no positive radial solution.*

A second theorem is obtained when $p = 2$ and W is constant.

Theorem 1.2. *Set $\Omega = B_1(0)$. Assume that $p = 2 < N$, $W \equiv \lambda$ and f satisfies* (F2), (F3),

$$F(s) - \frac{N-2}{2N} f(s)s \le F(\gamma) \quad \forall s \ge \gamma, \text{ and some } \gamma \in (\beta_3, \theta_1), \tag{1.6}$$

and

(F1') *$f : [0, \infty) \to \mathbb{R}$ belongs to C^1 with exactly three zeros $0 < \beta_1 < \beta_2 < \beta_3$.*

Then there exists $\lambda_0 > 0$ such that (1.1) *has no positive radial solution in $C^2(\bar{\Omega})$ provided $\lambda > \lambda_0$.*

This article is organized as follows: In Section 2 we show some technical lemmas which will be useful in Section 3 for proving Theorems 1.1 and 1.2.

2. Qualitative Analysis

In this section we assume that there exists a positive radial solution to (1.2), which is denoted by $u(\cdot, W)$. We say that u is a solution if $r \mapsto r^n \varphi_p(u') \in C^1$.

Lemma 2.1. *If $u(0, W) > \beta_3$, then $u(\cdot, W)$ is decreasing in $[0, 1]$.*

Proof. We note that by (1.3), if $u'(r, W) = 0$ for some $0 < r < 1$ then $u(r, W) \in \{\beta_1, \beta_2, \beta_3\}$. Since $u(0, W) > \beta_3$ then we set $t_3(W) = t_3 := \min\{r \in [0, 1] : u(r, W) = \beta_3\}$. By the mean value Theorem, $u'(\xi, W) < 0$ for some $\xi \in (0, t_3)$. Since $u'(\cdot, W)$ cannot change sign in this interval then $u(\cdot, W)$ is decreasing there.

Now, for all $r \ge t_3$, we claim $u(r, W) \le \beta_3$. Indeed, if $u(r_1, W) > \beta_3$ for some $r_1 \in (t_3, 1)$ then there would be an interval $[r_2, r_3] \subseteq [t_3, 1)$ such that $u(r, W) > \beta_3$ for all $r \in (r_2, r_3)$ and $u(r_2, W) = u(r_3, W) = \beta_3$. Again, the mean value Theorem implies $u'(\xi_0, W) = 0$ for some $\xi_0 \in (r_2, r_3)$. Thus $u(\xi_0, W) \in \{\beta_1, \beta_2, \beta_3\}$, which is a contradiction and the claim is proved.

Let $t_2 := t_2(W) := \min\{r \in (t_3, 1] : u(r, W) = \beta_2\}$. Then the same argument from above shows that $u(\cdot, W)$ is decreasing in the interval $[t_3, t_2]$. Afterwards we see that for all $r \in [t_2, 1)$, $u(r, W) \le \beta_2$. Assume on the contrary that there exists $r_1 \in (t_2, 1)$ such that $u(r_1, W) > \beta_2$. Then we can find an interval $[r_2, r_3] \subseteq [t_2, 1)$ such that $u(r, W) > \beta_2$ for all $r \in (r_2, r_3)$ and $u(r_2, W) = u(r_3, W) = \beta_2$. The function $v(r, W) := u(r, W) - \beta_2$ satisfies

$$-\Delta_p(v) = W(r)f(u), \text{ in } (r_2, r_3); \quad v(r_2) = v(r_3) = 0.$$

Since $u(\cdot, W) \in [\beta_2, \beta_3]$ in (r_2, r_3), it follows that $W(r)f(u(r)) \le 0$. Therefore, a comparison principle (see [6, Proposition 6.5.2]) lead us to $v(r, W) \le 0$ for all $r \in (r_2, r_3)$. But this means $u(r, W) \le \beta_2$ for r in this interval. This is an absurd.

Now let $t_1 := t_1(W) := \min\{r \in (t_2, 1] : u(r, W) = \beta_1\}$. Then, as at the beginning of this proof, $u(\cdot, W)$ is decreasing in $[t_2, t_1]$. We claim that for all $r \ge t_1$, $u(r, W) \le \beta_1$. Arguing by contradiction, if there exists $r_1 \in (t_1, 1)$ such that $u(r_1, W) > \beta_1$ then this forces $u(r, W) \ge \beta_1$ for all $r \in (t_1, r_1)$, otherwise there exists $r_0 \in (t_1, r_1)$ with $u(r_0, W) < \beta_1$ and $u'(r_0, W) = 0$. Thus $u(r_0, W) \in \{\beta_1, \beta_2, \beta_3\}$, which cannot be. Hence there would exists $\gamma_0 \in (\beta_1, \beta_2)$ such that $\beta_1 \le u(r) < \gamma_0$ in some neighborhood (r_2, r_3) of t_1 and $u(r_2) = u(r_3) = \gamma_0$. Then $w(r, W) := u(r, W) - \gamma_0$ satisfies

$$-\Delta_p(w) = W(r)f(u(r)), \text{ in } (r_2, r_3); \quad w(r_2) = w(r_3) = 0.$$

Because of behaviour of $u(\cdot,W)$ in (r_2,r_3), $f(u(r))\geq 0$. Using again the same comparison principle we deduce that $w(r,W)\geq 0$, that is $u(r,W)\geq\gamma_0$ for all $r\in(r_2,r_3)$. This contradiction shows the claim.

Finally, using the same arguments as before it follows that $u(\cdot,W)$ is decreasing in $[t_1,1]$. Thus, the lemma is proved. □

Remark 2.2. If $p=2$ our previous demonstration does not work. However, it is well known that when $p=2$, $W\equiv\lambda$ (a constant) and $f\in C^1$, a regular positive solution turns out to be radially symmetric and decreasing (see [7]).

Let us define the Energy associated to the problem (1.2) by

$$E(t,W):=\frac{|u'(t,W)|^p}{p'W(t)}+F(u(t,W)).$$

Also, we define

$$H(t,W):=tW(t)E(t,W)+\frac{N-p}{p}\varphi_p(u'(t,W))u(t,W).$$

Suppose that $u(\cdot,W)$ is a solution of (1.2). Then a Pohozaev type identity takes place,

$$\begin{aligned} &t^nH(t,W)-s^nH(s,W)\\ &=\int_s^t r^nW(r)\big[\big(N+r\frac{W'(r)}{W(r)}\big)F(u)-\frac{N-p}{p}f(u)u\big]dr, \end{aligned}\tag{2.1}$$

whenever $0\leq s\leq t\leq 1$ (see [11, formula (2.2)]). Now, from the definition of the energy we have

$$E'(r,W)=-\frac{|u'(r,W)|^p}{p'W(r)r}[\frac{np}{p-1}+\frac{rW'(r)}{W(r)}]\leq 0,$$

since $\underline{\eta}>0$ and $p<N$. Thus the energy is a decreasing function of r. Hence $E(r,W)\geq E(1,W)=\frac{|u'(1,W)|^p}{p'W(1)}\geq 0$ for all $r\in[0,1]$. In particular $E(0,W)=F(u(0,W))\geq 0$. Thus $u(0,W)\geq\theta_1$. So, in view of (F3) and the previous lemma, we conclude the following result.

Lemma 2.3. *Under assumption* (F3)*, every positive radially symmetric solution of* (1.1) *is radially decreasing.*

Lemma 2.4. *Let $u(\cdot,W)$ be a positive solution of problem* (1.2) *and set $t_\gamma=t_\gamma(W)$ the unique number in* $(0,1)$ *such that $u(t_\gamma,W)=\gamma$. Then there exists a constant C independent on W such that $|u'(t_\gamma,W)|t_\gamma\leq C$.*

Proof. Using the Pohozaev identity (see (2.1)) with $s=0$ and $t=t_\gamma$ we have

$$\begin{aligned} &t_\gamma^n|u'(t_\gamma,W)|^{p-1}\big[\frac{t_\gamma}{p'}|u'(t_\gamma,W)|-\frac{N-p}{p}\gamma\big]\\ &=\int_0^{t_\gamma} r^nW(r)\big[\widetilde{W}F(u)-\frac{N-p}{p}f(u)u\big]dr-t_\gamma^NW(t_\gamma)F(\gamma). \end{aligned}\tag{2.2}$$

Now, from (1.4) and (1.5) we have that for all $r\in[0,t_\gamma]$,

$$\widetilde{W}(r)F(u)-\frac{N-p}{p}f(u)u\leq N\widetilde{C}_\lambda F(\gamma).$$

Taking into account $0 < W(t_\gamma) \leq \widetilde{C}_\lambda W(r)$ for all $r \in [0, t_\gamma]$ (indeed it is true for all $r \in [0,1]$), $F(\gamma) < 0$ and the above inequality, we obtain

$$r^n W(r)\big(\widetilde{W}(r)F(u(r)) - \frac{N-p}{p} f(u(r))u(r)\big) \leq r^n W(t_\gamma) N F(\gamma).$$

Integrating over the interval $[0, t_\gamma]$,

$$\int_0^{t_\gamma} r^n W(r)\big(\widetilde{W}(r)F(u(r)) - \frac{N-p}{p} f(u(r))u(r)\big) dr \leq F(\gamma) W(t_\gamma) t_\gamma^N.$$

Therefore, from (2.2),

$$t_\gamma^n |u'(t_\gamma, W)|^{p-1} \big[\frac{t_\gamma}{p'} |u'(t_\gamma, W)| - \frac{N-p}{p}\gamma\big] \leq 0;$$

that is,

$$\frac{t_\gamma}{p'} |u'(t_\gamma, W)| \leq \frac{N-p}{p}\gamma.$$

The lemma is proved $\square$

Lemma 2.5. *Let $\alpha \in (0, \beta_1)$ be fixed and set $b = b(W, \alpha)$ the unique number in $(0,1)$ such that $u(b, W) = \alpha$. Then $b(W, \alpha) \to 1$ as $\overline{\lambda} \to \infty$.*

Proof. Integrating the equation in (1.2) on $[b, 1]$ we obtain

$$\begin{aligned} \varphi_p(u'(1)) - b^n \varphi_p(u'(b)) &= -\int_b^1 r^n W(r) f(u) dr \\ &\geq K \int_b^1 r^n W(r) dr \geq \frac{K\lambda}{N}(1 - b^N), \end{aligned} \tag{2.3}$$

where $-K = -K(\alpha) := \max\{f(s) : s \in [0, \alpha]\} < 0$. Multiplying the differential equation in (1.2) by $r^m u'$, with $n+m := np'$, and integrating by parts the left-hand side of the resulting equation, we have

$$\begin{aligned} &b^{n+m}|u'|^p - 1^{n+m}|u'|^p + \int_b^1 r^n \varphi_p(u')[m r^{m-1} u'(r) + r^m u''(r)] dr \\ &= \int_b^1 r^{n+m} W(r) [F(u)]' dr. \end{aligned}$$

Now, integrating by parts the right-hand side of the above equation we can estimate it as

$$\begin{aligned} &\int_b^1 r^{n+m} W(r) [F(u)]' dr = -b^{n+m} W(b) F(u(b)) - \int_b^1 F(u) (r^{n+m} W(r))' dr \\ &\leq -b^{n+m} W(b) F(u(b)) - F(\alpha) \int_b^1 (r^{n+m} W(r))' dr. \end{aligned}$$

This estimate is due to the fact that $F(\alpha) \leq F(u(r))$ for all $r \in (b, 1)$ and the assumption that $\widetilde{W}$ is positive, which implies that $[r^{n+m} W(r)]' > 0$. In consequence,

$$\begin{aligned} &b^{n+m}|u'|^p - 1^{n+m}|u'|^p \\ &\leq -F(\alpha) W(1) - \int_b^1 r^n \varphi_p(u') [m r^{m-1} u'(r) + r^m u''(r)] dr \\ &= -F(\alpha) W(1) + \int_b^1 r^{n+m-1} |u'|^{p-1} [m u'(r) + r u''(r)] dr, \end{aligned} \tag{2.4}$$

where we have noted that by Lemma 2.3, $u'(r) \le 0$ for all $r \in [b, 1]$. On the other hand, from (1.3) and $M = M(\alpha) := \max_{s\in[0,\alpha]} |f(s)| > 0$, we have for all $r \in [b, 1]$

$$(p-1)|u'|^{p-2}u'' + \frac{n}{r}|u'|^{p-2}u' = -W(r)f(u) = W(r)|f(u)| \le \overline{\lambda}M.$$

Then

$$|u'|^{p-1}[ru'' + mu'] \le \frac{\overline{\lambda}M}{p-1} r|u'|.$$

Therefore, from (2.4),

$$\begin{aligned} b^{n+m}|u'|^p - |u'(1)|^p &\le -F(\alpha)\overline{\lambda} + \frac{\overline{\lambda}M}{p-1}\int_b^1 r^{n+m}|u'|dr \\ &\le -F(\alpha)\overline{\lambda} + \frac{\overline{\lambda}M}{p-1}\int_b^1 |u'|dr \\ &= [-F(\alpha) + \frac{\alpha M}{p-1}]\overline{\lambda} =: C_0\overline{\lambda}, \end{aligned} \tag{2.5}$$

where $C_0 = C_0(\alpha) > 0$. Now, from (2.3) and taking into account that $u' \le 0$,

$$0 < b^{np'}|u'(b)|^p - |u'(1)|^p. \tag{2.6}$$

Hence, combining (2.5) and (2.6),

$$0 < b^{np'}|u'(b)|^p - |u'(1)|^p \le C_0\overline{\lambda}.$$

Thus, using (2.3) again,

$$\begin{aligned} 0 &\le \frac{\underline{\lambda}K}{N}(1 - b^N) \\ &\le \varphi_p(u'(1)) - b^n\varphi_p(u'(b)) \\ &\le |b^n|u'(b)|^{p-1} - |u'(1)|^{p-1}| \\ &\le \left(b^{np'}|u'(b)|^{(p-1)p'} - |u'(1)|^{(p-1)p'}\right)^{1/p'} \\ &= \left(b^{np'}|u'(b)|^p - |u'(1)|^p\right)^{1/p'} \le \overline{\lambda}^{1/p'} C_0^{1/p'}, \end{aligned}$$

which implies

$$0 \le \frac{K}{N}(1 - b^N) \le \frac{\overline{\lambda}^{1/p'}}{\underline{\lambda}} C_0^{1/p'} \le \frac{1}{\overline{\lambda}^{\frac{1}{p}}} T C_0^{1/p'},$$

where $T > 0$ satisfies $\overline{\lambda}/\underline{\lambda} \le T$. The statement of the lemma follows. □

3. Proof of results and examples

Proof of theorem 1.1. Let us argue by contradiction. Suppose there exist $T > 0$ and a sequence of admissible weights $\{W_m\}_{m=1}^{\infty}$ with $\overline{\lambda}_m := \|W_m\|_\infty \to +\infty$ as $m \to \infty$ and such that for all m problem (1.1) has a positive radial solution $u(\cdot, W_m)$. If we suppose that there is a positive constant L such that $t_\gamma(m) := t_\gamma(W_m) \ge L$ for all m sufficiently large then, from Lemma 2.4, $|u'(t_\gamma(m), W_m)| \le \frac{C}{L} =: C_1$, where C is independent on the weight. This would imply

$$\begin{aligned} E(t_\gamma(m), W_m) &= \frac{|u'(t_\gamma(m), W_m)|^p}{p'W(t_\gamma(m))} + F(u(t_\gamma(m), W_m)) \\ &\le \frac{C_1^p}{p'W(t_\gamma(m))} + F(\gamma) \end{aligned}$$

$$
\begin{aligned}
&\le \frac{C_1^p}{p'\underline{\lambda}_m} + F(\gamma) \\
&\le \frac{C_1^p T}{p'\overline{\lambda}_m} + F(\gamma).
\end{aligned}
$$

Hence $0 \le \limsup_{m\to\infty} E(t_\gamma(m), W_m) \le F(\gamma) < 0$, what is an absurd. In consequence there is a sub-sequence of $\{t_\gamma(m)\}_{m=1}^\infty$ (denoted in the same way) which converges to zero. Let us fix $\alpha > 0$ and we define $b_m = b(W_m)$ as in Lemma 2.5. Then by that lemma we can choose $k > 0$ such that for all $m \ge k$, $t_\gamma(m) < 1/4$ and $3/4 < b_m$. Set $\widetilde{L} := \max_{s\in[\alpha,\gamma]} F(s) < 0$. Thus, there exists $\xi_m \in (t_\gamma(m), b_m)$ such that

$$
|u'(\xi_m, W_m)| = \frac{|u(t_\gamma(m), W_m) - u(b_m, W_m)|}{|t_\gamma(m) - b_m|} \le 2(\gamma+\alpha) =: \mu > 0.
$$

Then, taking into account that $u(\cdot, W_m)$ is decreasing we have $\alpha \le u(r, W_m) \le \gamma$ for all $r \in [t_\gamma(m), b_m]$. Therefore,

$$
\begin{aligned}
E(\xi_m, W_m) &= \frac{|u'(\xi_m, W_m)|^p}{p'W(\xi_m)} + F(u(\xi_m, W_m)) \\
&\le \frac{\mu^p}{p'W(\xi_m)} + \widetilde{L} \\
&\le \frac{\mu^p}{p'\underline{\lambda}_m} + \widetilde{L} \\
&\le \frac{\mu^p T}{p'\overline{\lambda}_m} + \widetilde{L}.
\end{aligned}
$$

Hence $0 \le \limsup_{m\to\infty} E(\xi_m, W_m) \le \widetilde{L} < 0$. This is a contradiction and so the theorem is proved. $\square$

Proof of theorem 1.2. Assume on the contrary that there exist a sequence $\lambda_m \to \infty$ and positive solutions $u_m \in C^2(\bar{\Omega})$ of the problem

$$
\Delta u + \lambda f(u) = 0, \text{ in } \Omega; \quad u = 0 \text{ on } \partial\Omega.
$$

A celebrated result in [7] (see Remark 2.2) implies that u_m is radially symmetric and decreasing. Now, after a detailed reading of proofs Lemmas 2.4 and 2.5, one can see that their conclusions hold for $p = 2$. In this case we must use hypothesis (1.6). The proof follows the same argument as in proof of theorem (1.1). $\square$

Examples. Next, we exhibit a nonlinearity f and two weights W, holding all conditions. First, we consider a constant weight.

Let $g : [0,3] \to \mathbb{R}$ defined by $g(t) = (t-1)(t-2)(t-3)$. Let

$$
f(t) := \begin{cases} g(t) & \text{if } 0 \le t \le 3 \\ (t-3)^q & \text{if } t > 3, \end{cases}
$$

with $p^* < q+1$. Thus f has exactly three zeros, $f(0) < 0$ and $f(3) = 0$. Also, $F(t) := \int_0^t f(s)ds = \frac{t^4}{4} - 2t^3 + \frac{11t^2}{2} - 6t < 0$ for all $t \in [0,3]$. Then we have $\beta_3 = 3$ and $4 = \theta_1 > \beta_3$. We fix $\gamma \in (\beta_3, \theta_1)$ and note that a simple computation shows

$$
F(t) = F(\gamma) + \frac{1}{q+1}[(t-3)^{q+1} - (\gamma-3)^{q+1}], \quad \forall t \ge \gamma. \tag{3.1}
$$

We define the family of constant weights $W_\lambda \equiv \lambda > 0$, which belongs to the class of weights $\mathcal{C}(1)$. Because of $p^* < q+1$ then $\frac{(t-3)^{q+1}}{q+1} \le \frac{t(t-3)^q}{p*}$. Therefore

$$F(t) - F(\gamma) = \frac{(t-3)^{q+1} - (\gamma-3)^{q+1}}{q+1} \le \frac{t(t-3)^q}{p*} = \frac{tf(t)}{p*}, \quad \text{for all } t \ge \gamma.$$

This gives us (1.4) and (1.5). In consequence, by Theorem 1.1, there exists $\lambda_0 > 0$ such that for all $\lambda > \lambda_0$, problem (1.2) has no radial positive solution.

For the same non-linearity f we introduce another family of weights that satisfy the hypotheses of Theorem 1.1. For $\lambda \ge 1$, define $W_\lambda(r) := \lambda + r$. Then $\widetilde{W}_\lambda(r) = N + \frac{r}{\lambda+r}$, $\widetilde{C}_\lambda = \frac{\lambda+1}{\lambda}$, $\underline{\eta} = N$ and $\overline{\eta} = N + \frac{1}{\lambda+1}$. First of all, since $\lim_{\gamma\to\theta_1} f(\gamma) = f(\theta_1) > 0$ and $\lim_{\gamma\to\theta_1} F(\gamma) = 0$, we can take γ between β_3 and θ_1 such that

$$-\frac{f(\gamma)\gamma}{p*} < \frac{\lambda+1}{\lambda} F(\gamma),$$

where γ is independent on λ due to the estimate $1 < \frac{\lambda+1}{\lambda} \le 2$. Since $F(s) \le 0$ and $-\frac{f(s)s}{p*} \le -\frac{f(\gamma)\gamma}{p*}$ for all $\gamma \le s \le \theta_1$ ($t \mapsto tf(t)$ is non-decreasing for $t > 3$), then

$$F(s) - \frac{f(s)s}{p*} < \frac{\lambda+1}{\lambda} F(\gamma),$$

which gives us (1.4). On the other hand, for λ sufficiently large,

$$\begin{gathered}
\big(\frac{1}{N(\lambda+1)} - \frac{1}{\lambda}\big) F(\gamma) \le \frac{3(\theta_1-3)^q}{p*}, \\
\frac{1}{p*} - \frac{1}{q+1} - \frac{1}{N(\lambda+1)(q+1)} > 0.
\end{gathered}$$

Then, for all $s \ge \theta_1$,

$$\begin{aligned}
&\big(\frac{1}{N(\lambda+1)} - \frac{1}{\lambda}\big) F(\gamma) \\
&\le \frac{3(s-3)^q}{p*} + \big(\frac{1}{p*} - \frac{1}{q+1} - \frac{1}{N(\lambda+1)(q+1)}\big)(s-3)^{q+1} \\
&\quad + \big(\frac{1}{q+1} + \frac{1}{N(\lambda+1)}\frac{1}{q+1}\big)(\gamma-3)^{q+1}.
\end{aligned}$$

So, keeping in mind (3.1) we have (1.5).

Acknowledgments. This research was partially supported by Fondo Nacional de Financiamiento para la Ciencia, la Tecnología y la Innovación Francisco José de Caldas, contrato Colciencias FP44842-087-2015.

References

[1] D. Arcoya, A. Zertiti; *Existence and non-existence of radially symmetric non-negative solutions for a class of semi-positone problems in annulus*, Rendiconti di Mathematica, serie VII, Volume 14, Roma (1994), 625-646.

[2] G. Bianchi; *Non-existence of positive solutions to semilinear elliptic equations on R^n or R^n_+ through the method of moving planes*, Commun. in Partial Differential Equations, 22(9&10), 1671-1690 (1997).

[3] K. J. Brown, A. Castro, R. Shivaji; *Non-existence of radially symmetric non-negative solutions for a class of semi-positone problems*, Diff. and Int. Equations, 2. (1989), 541-545.

[4] A. Castro, M. Chhetri, R. Shivaji; *Nonlinear eigenvalue problems with semipositone structure*, Electron. J. Diff. Eqns., Conf. 05, 2000, pp. 33-49.

[5] M. Chhetri, P. Girg; *Nonexistence of nonnegative solutions for a class of* $(p-1)$*-superhomogeneous semipositone problems*, Journal of Mathematical Analysis and Applications Volume 322, Issue 2, 15 October 2006, pp. 957-963.

[6] L. Gasiński, N. S. Papageorgiou; *Nonlinear analysis*, Volume 9, Series in mathematical analysis and applications, Chapman & Hall/CRC, 2005.

[7] B. Gidas, W. M. Ni, L. Nirenberg; *Symmetry and related properties via the maximum principle*, Commun. Math. Phys, (68), 1979, pp. 209-243.

[8] D. D. Hai; *Nonexistence of positive solutions for a class of p-Laplacian boundary value problems*, Applied Mathematics Letters, 31 (2014), pp. 12-15.

[9] S. Hakimi; *Nonexistence of radial positive solutions for a nonpositone system in an annulus*, Electron. J. Diff. Equ., Vol. 2011 (2011), No. 152, pp. 1-7.

[10] S. Hakimi, A. Zertiti; *Nonexistence of radial positive solutions for a nonpositone problem*, Electron. J. Diff. Equ., Vol. 2011 (2011), No. 26, pp. 1-7.

[11] S. Herrón, E. Lopera; *Signed Radial Solutions for a Weighted p-Superlinear Problem*, Electron. J. Differential Equations 2014, No. 24, pp. 1-13.

[12] Y. Naito; *Nonexistence results of positive solutions for semilinear elliptic equations in* $\mathbb{R}^N$, J. Math. Soc. Japan Vol. 52, No. 3, 2000, pp. 637-644.

[13] M. Rudd; *Existence and nonexistence results for quasilinear semipositone Dirichlet problems*, Proceedings of the Seventh Mississippi State-UAB Conference on Differential Equations and Computational Simulations, Electron. J. Differ. Equ. Conf., 17, 207-212, 2009.

[14] R. Shivaji, J. Ye; *Nonexistence results for classes of* 3×3 *elliptic systems*, Nonlinear Analysis: Theory, Methods & Applications, Volume 74, Issue 4, 15 February 2011, Pages 1485-1494.

Sigifredo Herrón

Universidad Nacional de Colombia Sede Medellín, Apartado Aéreo 3840, Medellín, Colombia

E-mail address: sherron@unal.edu.co

Emer Lopera

Universidad Nacional de Colombia Sede Medellín, Apartado Aéreo 3840, Medellín, Colombia

E-mail address: edlopera@unal.edu.co