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NECESSARY AND SUFFICIENT CONDITIONS FOR THE EXISTENCE OF PERIODIC SOLUTIONS IN A PREDATOR-PREY MODEL

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Abstract. Liu et al [14] found necessary and sufficient conditions for the existence of periodic solutions of the predator-prey dynamical systems with semi-ratio dependent generalized functional response. In this work, we obtain a globally attractive or globally asymptotically stable periodic solution for the time scale $\mathbb{T}$ is taken as $\mathbb{R}$, with a small change on the condition over generalized functional response on the prey.

1. Introduction

In this study, we study the important notions of global attractive and permanent solutions for predator-prey systems. In a predator-prey dynamic system, global stability is the particular interest of the wildlife managers. If it is known that a system exhibits such global stability, then ecological planning based on a fixed eventual population can be carried out. Global stability of predator-prey dynamic systems have been studied in publications such as [1, 2, 5, 8, 9, 11, 16]. Let us give some information about these studies. [9] presents existence of the solutions of non-authonomous impulsive predator-prey system with Beddington-DeAngelis type functional response on time scales. [16] shows the existence and global attractivity of the solutions of non-authonomous predator-prey system with generalized functional response. [5] presents one of the early studies about population dynamics; there a general form of a population system with single species was studied. A very general form of a predator-prey system was studied in [1] with a time delay and a a periodic environment. In [2], another general form of a predator-prey system was studied. Nevertheless, in that study, not a constant time delay, but time varying delay system was used. The Lotka-Volterra competition model was used with a time delay which includes delay kernels in the functional response part was studied in [8]. In addition to these, one of the two-stage predator-prey interaction model was studied and again with a time delay, the analysis of that system was done in [11].

Additionally, permanence of the solution is another important notion. Permanence of a predator-prey system explains whether the prey or predator goes to extinction or not. In other words, if a system is permanent then the solution of

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prey and predator does not become extinct; this also helps the wildlife managers. About the permanence of the predator-prey system, there are also many studies: [3, 4, 6, 7, 10]. Some information about these studies can be given as follows. First of all, in [10], the main idea is about obtaining the persistant solutions in infinite dimensional systems and as an application a predator-prey system was used. Three dimensional Kolmogorov system was investigated in [7]. In that study, there are two competing predators with single prey. Uniform persistent solutions of functional differential equations are investigated and by using the results of this study, some applications to different systems are possible in [6]. In [3, 4], necessary and sufficient conditions were found for the persistent solution of the predator-prey system with Beddington-DeAngelis type functional response and Holling type functional response. These studies are different from the above cited studies, since they can be able to found a necessary and sufficient condition, not only a necessary condition. Therefore, these two studies are very important for our study. Also, inspire us to investigate the necessary and sufficient condition for the w-periodic, persistant and global attractive solutions of the considered system.

In [14], necessary and sufficient condition for the existence of w-periodic solution of the following system were found,

$$
\begin{aligned}
x^{\Delta}(t) &= a(t) - b(t)\exp(x(t)) - \phi(t, \exp(x(t)), \exp(y(t)))\exp(y(t) - x(t)),\\
y^{\Delta}(t) &= d(t) - \beta(t)\exp(y(t) - x(t)),\\
\Delta x(t) &= \ln(1 + c_{1k}), t = t_k, k \in \mathbb{N},\\
\Delta y(t) &= \ln(1 + c_{2k}), t = t_k, k \in \mathbb{N}.
\end{aligned}
$$

Additionally, assume that a is the logistic growth rate of the prey and in the absence of predator, a/b is the carrying capacity. $\phi(t, x, y)$ is the functional response which shows the effect of predator on prey. The predator also grow logistically with growth rate d and carrying capacity x/β proportional to the population size of prey (or prey abundance). The parameter β is a measure of the food quality that the prey provides for conversion into predator birth.

This investigation has remarkable importance for further developments of dynamical systems in predator-prey problems in theory of time scale calculus. In the study [14], it should be emphasized that this result only guaranties to find at least one w-periodic solution under the condition that is given in Theorem 2.6.

In the present study, when the time scale $\mathbb{T}$ is taken as $\mathbb{R}$, then one can find globally asymptotic w-periodic solution of System [14, system (1.1)] which can be found by performing a small change on the generalized functional response on prey.

Necessary and sufficient conditions for the existence of a permanent solution of the system is found in the next chapter. Then, in section 4, by applying Theorem 4.1, we give necessary and sufficient condition for the global attractivity of w-periodic solutions. In chapter 4, theoretical statements are supported by the results of two numerical examples as an application. The executions in examples are carried out by Mathlab 9.01 and obtained by a PC pentium (R) 2CPV, 2.00 6 Hz, 2.87 GB of RUN. As a last, conclusion is given as a final section.

2. Preliminaries

Definition 2.1 ([18]). Solutions of an w-periodic system generate an w-periodic semiflow $T(t) : X \to X$ (X is the initial value space) in the sense that $T(t)x$ is continuous in $(t, x) \in [0, +\infty) \times X$, $T(0) = I$ and $T(t+w) = T(t)T(w)$ for all $t > 0$.

Definition 2.2 ([17, Defn. 4.2]). The periodic semi-flow $T(t)$ is said to be uniformly persistent with respect to $(X_0, \partial X_0)$ if there exists $\eta > 0$ such that for any $x \in X_0$, $\liminf_{t\to\infty} d(T(t)x, \partial X_0) \geq \eta$.

Definition 2.3 ([10]). Let $T : \mathbb{R}^n \to \mathbb{R}^n$. The map T is point dissipative if there exists a bounded set B such that, for each $x \in \mathbb{R}^n$, there is an integer $n_0 = n_0(x, B)$ such that $T^n(x) \in B$ for each $n \geq n_0$.

Lemma 2.4 ([17, Lemma 4.3]). *Let $S : X \to X$ be a continuous map with $S(X_0) \subset X_0$ Assume that S is point dissipative, compact and uniformly persistent with respect to $(X_0, \delta X_0)$. Then, there exists a global attractor A_0 for S in X_0 relative to strongly bounded sets in X_0, and S has coexistence state $x_0 \in A_0$.*

Lemma 2.5 ([17, Lemma 2.2]). *There is at least $l_0 \in 1, \dots, l$ such that $\beta_{l_0} > 0$, then* [17, system (2.1)] *admits a unique positive w-periodic solution if only if*

$$\int_0^w \alpha(t)dt + \sum_{i=1}^{q} ln(1 + h_1) > 0,$$

which, moreover, is globally asymptotically stable.

Theorem 2.6 ([14, Theorem 3.1]). *Assume that the following conditions hold.*

- (H1) *$a(t)$, $b(t)$, $d(t)$ and $\beta(t)$ are non-negative w-periodic rd-continuous real functions and $\hat{a} > 0$, $\hat{d} > 0$;*
- (H2) *The functional response $\phi : \mathbb{T} \times \mathbb{R} + \times \mathbb{R}_+ \to \mathbb{R}_+$ is rd-continuous and w-periodic with respect to t, $\phi(t, 0, y) = 0$ for any $t \in \mathbb{T}$, $y \geq 0$. In addition, there exist $m \in N$ and w-periodic rd-continuous functions $a_i : \mathbb{T} \to \mathbb{R}_+$, $i = 0, \dots, m$ such that*
$$\phi(t, x, y) \leq a_0(t)x^m + +_{m1}(t)x,$$
for $t \in \mathbb{T}$, $x \geq 0$, $y \geq 0$.

Then, [14, system (1.1)] *has at least one w-periodic solution if and only if*

$$\hat{a}w + \sum_{k=1}^{p} ln(1 + c_{1k}) > 0,$$
$$\hat{d}w + \sum_{k=1}^{p} ln(1 + c_{2k}) > 0.$$

Theorem 2.7 ([12, Theorem 1.6.1]). *Assume that*

- (i) *$m \in PC[\mathbb{R}_+, \mathbb{R}]$ with points of discontinuity at $t = \underline{t}_k$ and $\bar{t}_k$, $m(t)$ is left continuous at $t = \underline{t}_k$ for $k = 1, 2, \dots$.*
$$D^+m(t) \leq g(t, m(t)), \quad t \notin [\underline{t}_k, \bar{t}_k]$$
$$m(\bar{t}_k^+) \leq G_k(m(\underline{t}_k)),$$
$$m(t_0 \leq u_0,$$
where $g \in C[\mathbb{R}_+ \times \mathbb{R}, \mathbb{R}]$, $G_k \in C[\mathbb{R}, \mathbb{R}]$ and $G_k(u)$ is non-decreasing in u;

(ii) $r(t, t_0, u_0)$ *is the maximal solution of* [12, system (1.6.1)] *existing on* J *where*
$$J = [t_0, \infty) \setminus \cup_{k=1}^{\infty} (\underline{t}_k, \bar{t}_k],$$
Then $m(t) \leq r(t, t_0, u_0)$ *on* J.

Theorem 2.8 ([12, Theorem 2.2.1])**.** *Assume that* $f \in C[\mathbb{R}_0, \mathbb{R}^n]$, $g \in C[[t_0, t_0 + a] \times [0, 2b], \mathbb{R}_+]$ *and for* $(t, x), (t, y) \in \mathbb{R}_0$,
$$|f(t, x) - f(t, y)| \leq g(t, |x - y|)$$
where $R_0 = [(t, x) : t_0 \leq t \leq t_0 + a$ *and* $|x - x_0 \leq b]$. *Suppose further that for any* $t_0 \leq t^* < t_0 + a$, *the IVP*
$$u' = g(t, u), u(t^*) = 0,$$
has the unique solution $u(t) = 0$ *on* $[t^*, t_0 + a]$. *Then problem* [12, system (2.2.1)] *possesses at most one solution on* $[t_0, t_0 + a]$.

3. Necessary and sufficient condition for the permanence of the semi-ratio dependent predator-prey dynamic system

We consider [14, system (1.1)] in the case $\mathbb{T} = \mathbb{R}$. The following system [14, system (1.3)] is obtained under the same conditions in (3.1) except the condition over ϕ where ϕ is the generalized functional response on the prey:
$$\begin{gathered}
x'(t) = a(t)x(t) - b(t)x^2(t) - \phi(t, x(t), y(t))y(t), \\
y'(t) = y(t)[d(t) - \frac{\beta(t)y(t)}{x(t)}], \\
\Delta x(t) = c_{1k}x(t), t = t_k, k \in \mathbb{N}, \\
\Delta y(t) = c_{2k}y(t), t = t_k, k \in \mathbb{N}.
\end{gathered} \tag{3.1}$$

In [14], $\phi(t, x, y) \leq \alpha_0(t)x^m + \dots + \alpha_{m-2}(t)x^2 + \alpha_{m-1}(t)x$ in the present study it is taken as follows:
$$\phi(t, x, y) \leq \alpha_0(t)x^m + \dots + \alpha_{m-2}(t)x^2. \tag{3.2}$$
In system (3.1), all of the coefficient $a(t)$, $b(t)$, $d(t)$, $\beta(t)$ and $\alpha_0(t)$, $\dots$, $\alpha_{m-2}(t)$ are positive, continuous and w-periodic.

Theorem 3.1. *System* (3.1) *has permanent solution if and only if*
$$\int_0^w a(t)dt + \sum_{k_1}^{p} ln(1 + c_{1k}) > 0, \tag{3.3}$$
$$\int_0^w d(t)dt + \sum_{k_1}^{p} ln(1 + c_{2k}) > 0. \tag{3.4}$$

Proof. First of all, it is obvious that
$$\begin{gathered}
x'(t) \leq a(t)x(t) - b(t)x^2(t), \\
\Delta x(t) = c_{1k}x(t), t = t_k, k \in \mathbb{N}.
\end{gathered}$$
The following equalities are considered:
$$\begin{gathered}
u'(t) = a(t)u(t) - b(t)u^2(t), \\
\Delta x(t) = c_{1k}u(t), \quad t = t_k, \ k \in \mathbb{N}.
\end{gathered} \tag{3.5}$$

If Theorem 2.7 which is the comparison theorem for impulsive differential equations from [12] is used, then, $x(t) \le u(t)$ is found. System (3.5) has unique globally asymptotically stable w-periodic solution by Lemma 2.5. Therefore, for sufficiently large t,

$$x(t) \le u(t) \le \bar{u}(t) + 1,$$

where $\bar{u}(t)$ is the unique globally attractive or globally asymptotically stable w-periodic solution of System (3.5). This shows that the solution of the prey is bounded from above. Therefore, let us say that $x(t) \le M_1$.

Since $x(t)$ is bounded from above, we can obtain the following:

$$\begin{gathered} y'(t) \le y(t)\big[d(t) - \frac{\beta(t)y(t)}{M_1}\big], \\ \Delta y(t) = c_{2k}y(t), \quad t = t_k,\ k \in \mathbb{N}. \end{gathered}$$

Then, we consider the system

$$\begin{gathered} v'(t) = v(t)\big[d(t) - \frac{\beta(t)v(t)}{M_1}\big], \\ \Delta v(t) = c_{2k}v(t), \quad t = t_k,\ k \in \mathbb{N}. \end{gathered} \tag{3.6}$$

From Theorem 2.7, $y(t) \le v(t)$ is obtained. Again from Lemma 2.5, system (3.6) has a unique globally attractive or globally asymptotically stable w-periodic solution. Let us denote this solution as $\bar{v}(t)$. Therefore, for sufficiently large t, $y(t) \le v(t) \le \bar{v}(t) + 1$. Hence, the solution of the predator is also bounded from above and let us take $y(t) \le M_2$.

By using the condition on the functional response ϕ and boundedness of $y(t)$ from below, we obtain

$$\begin{gathered} x'(t) \ge a(t)x(t) - b(t)x^2(t) - \big[\alpha_0(t)x^m(t) + \dots + \alpha_{m-2}(t)x^2(t)\big]M_2, \\ \Delta x(t) = c_{1k}x(t), \quad t = t_k,\ k \in \mathbb{N}. \end{gathered}$$

We can write the above system as

$$\begin{gathered} x'(t) \ge x(t)\big[a(t) - \sum_{i=1}^{m-1} b_i(t)x^i(t)\big], \\ \Delta x(t) = c_{1k}x(t), \quad t = t_k,\ k \in \mathbb{N}. \end{gathered}$$

Here $b_1(t) = M_2\alpha_{m-2}(t) + b(t)$, $b_2(t) = M_2\alpha_{m-1}(t)$, $\dots$, $b_{m-1}(t) = M_2\alpha_0(t)$. Then, we consider

$$\begin{gathered} {u_1}'(t) = u_1[a(t) - \sum_{i=1}^{m-1} b_i(t){u_1}^i(t)], \\ \Delta u_1(t) = c_{1k}u_1(t), \quad t = t_k,\ k \in \mathbb{N}. \end{gathered} \tag{3.7}$$

where $b_1(t) = M_2\alpha_{m-2}(t) + b(t)$, $b_2(t) = M_2\alpha_{m-1}(t)$, $\dots$, $b_{m-1}(t) = M_2\alpha_0(t)$.

By the comparison theorem for impulsive differential equations $x(t) \ge u_1(t)$ and from Lemma 2.5, this system has unique globally attractive or globally asymptotically stable w-periodic solution. Let us say this solution $\bar{u_1}(t)$. Then, for sufficiently large t, $x(t) \ge u_1(t) \ge \bar{u_1}(t) - 1$, which shows solution of the prey is bounded from below and denote it as $x(t) \ge m_1$.

By using the boundedness of the solutions of prey from below, we obtain

$$y'(t) \ge y(t)\big[d(t) - \frac{\beta(t)y(t)}{m_1}\big],$$

$$\Delta y(t) = c_{2k} y(t), \quad t = t_k,\ k \in \mathbb{N}.$$

Then, we consider the system

$$\begin{aligned} {v_1}'(t) &= v_1(t)\big[d(t) - \frac{\beta(t)v_1(t)}{m_1}\big], \\ \Delta v_1(t) &= c_{2k} v_1(t), \quad t = t_k,\ k \in \mathbb{N}. \end{aligned} \tag{3.8}$$

From Theorem 2.7, $y(t) \geq v_1(t)$ is obtained. From Lemma 2.5, System (3.6) has a unique globally asymptotically stable w-periodic solution. Let us denote this solution as $\bar{v_1}(t)$. Then, for sufficiently large t, $y(t) \geq v_1(t) \geq \bar{v_1}(t) - 1$. Thus, the solution of the predator is also bounded from below. By taking $y(t) \geq m_2$, it is shown that solution of System (3.1) is bounded from both above and below. □

4. Necessary and sufficient condition for globally attractivity of the w-periodic solution

Theorem 4.1. *Assume System* (3.1) *satisfies* (3.3) *and* (3.4)*. Then the w-periodic solution of the system is globally asymptotically stable.*

Proof. We apply Lemma 2.4. Let us consider the ordinary differential equation

$$\begin{aligned} z'(t) &= F(t, z(t)), \\ z(t_k^+) - z(t_k) &= I_k(z(t_k)), \\ z(0) &= \phi. \end{aligned} \tag{4.1}$$

Here, $F \in C([0,\infty) \times \mathbb{R}^2, \mathbb{R}^2)$, $\phi \in \mathbb{R}^2$, $F(t+w, u) = F(t, u)$, $I_k \in C(\mathbb{R}^2, \mathbb{R}^2)$ and there exists an integer q such that $I_{k+q} = I_k$, $t_{k+q} = t_k + w$. Then, the operator that solves system (4.1) can be written as

$$\hat{T}(t)z = ze^{-\lambda t} + \int_0^t e^{-\lambda(t-s)}[F(s, \hat{T}(s)z) + \lambda \hat{T}(s)z]ds + \sum_{0<t_k<t} e^{-\lambda(t-t_k)} I_k(\hat{T}(t_k)z).$$

In the above equation λ is a positive constant. It is obvious that $T(0) = I$. Also, we can verify that the equation

$$u(s) = \begin{cases} T(s)z, & 0 \leq s \leq w, \\ T(s-w)T(w)z, & w \leq s \leq t+w, \end{cases}$$

where $s \in [0, t+w]$ is the solution of System (4.1) with the initial value $u(0) = z$. By using Theorem 2.8 which is the uniqueness theorem, System (4.1) has a unique solution, therefore $T(t+w)z = u(t+w) = T(t)T(w)z$. This is true when $t \neq t_k$. For $t = t_k$,

$$\begin{aligned} T(t_k^+ + w)z &= T(t_k + w)z + I_k(T(t_k + w)z) \\ &= T(t_k)T(w)z + I_k(T(t_k)T(w)z) = T(t_k^+)T(w)z. \end{aligned}$$

To apply Lemma 2.4, let $S = T(w)$, $S^2 = SoS = T(w)oT(w) = T(2w)$. Here System (4.1) is a periodic system, therefore, we can apply Arzela-Ascoli theorem for impulsive differential equations. By this way, it is obtained that $T(t)$ is a compact operator.

If we take $X_i^+ = \{z_i : z_i \in \mathbb{R}, z_i \geq 0\}$ for $i = 1, 2$ and $X_{i_0}^+ = \{z_i : z_i \in \mathbb{R}, z_i > 0\}$ for $i = 1, 2$, then $X = X_1^+ \times X_2^+$, $X = X_{1_0}^+ \times X_{2_0}^+$ and $\delta X_0 = X/X_0$. When system (3.1) satisfies inequalities (3.3) and (3.4), the system becomes permanent system from Theorem 3.1. Therefore, S satisfies the conditions of Lemma 2.4. Therefore, S admits a global attractor which means the system has globally asymptotically stable w-periodic solution. Similar proof was also made in [15]. $\square$

Corollary 4.2. *For system* (3.1)*, there exist globally attractive w-periodic solution if and only if inequalities* (3.3) *and* (3.4) *are satisfied.*

5. Application

The system in the following examples are written appropriate to [14, system (1.01)] which is equivalent to system (3.1) in that study and [14, system (1.3)].

Example 5.1. As a first example, the following system is considered for testing the permanence and periodicity of the solutions to support the theoretical statement in [14, Theorem 3.1] and Theorem 3.1 in the previous sections.

$$
\begin{gathered}
x' = (0.2\sin(2\pi t) + 0.3) - (0.2\sin(2\pi t) + 0.2)\exp(x) \\
- \frac{(0.1 + 0.1\cos(2\pi t))\exp(2x)}{(0.5\sin(2\pi t) + 0.7) + (1 + 0.5\cos(2\pi t))\exp(x) + \exp(y)} \exp(y - x), \; t \neq t_k \\
y'(t) = (0.3sin(2\pi t) + 1) - \frac{(4\cos(2\pi t) + 6.5)}{(1 + 0.5\cos(2\pi t)} \exp(y - x), \quad t \neq t_k \\
\Delta x(t_k) = ln(1 + c_{1k}) \\
\Delta y(t_k) = ln(1 + c_{2k})
\end{gathered}
$$

Impulse points: $t_1 = 2k + 1/4$, $t_2 = 2k + 3/4$ and $p = 2$.

$$
c_{11} = e^{-0.01} - 1, \quad c_{12} = e^{-0.01} - 1, \quad c_{21} = e^{0.1} - 1, \quad c_{22} = e^{0.1} - 1 .
$$

It is clear that $\phi(x, y, t) = \frac{(0.1+0.1\cos(2\pi t))x^2}{(0.5\sin(2\pi t)+0.7)+(1+0.5\cos(2\pi t))x+y}$. $\phi(x, y, t)$ should satisfy inequality (3.2) and with some simple calculations, we obtain that $\phi(x, y, t) \leq (0.1 + 0.1cos(2\pi t))x^2$. Therefore the conditions of System (3.1) is satisfied by this example.

Now, another important point is to observe that the permanent and w-periodic solution of the system in Example 5.1 numerically satisfies the theoretical results in Theorem 4.1; in other words, this solution satisfies the global attractivity property. To show that the solution of the system in Example 5.1 satisfies this condition, we should change the initial conditions. In Figure 2, although we change the initial conditions of the same system, still we obtain the same solution after sufficiently large t which means the result of Theorem 4.1 is numerically satisfied.

In the following figure, since Example 5.1 satisfies the condition of Corollary 4.2, we would like to show that the solutions of the same system with different initial conditions have the same solutions after a while and we try to show this numerically by the displacement of the Figure 1 and 2 and another plot with same system and different initial conditions in a single figure.

Example 5.2. As a second example, we consider the following one to observe that although the periodicity of the system is changed, still the results of the Theorems 2.6 and 3.1 is numerically satisfied.

$$
\begin{gathered}
x' = 0.5 - (0.3\cos(t) + 0.6)\exp(x) \\
- \frac{(0.15 + 0.5\sin(t))\exp(2x)}{0.4 + (1.3 + 0.5\sin(t))\exp(x) + 0.9\exp(y)} \exp(y - x), \quad t \neq t_k,
\end{gathered}
$$

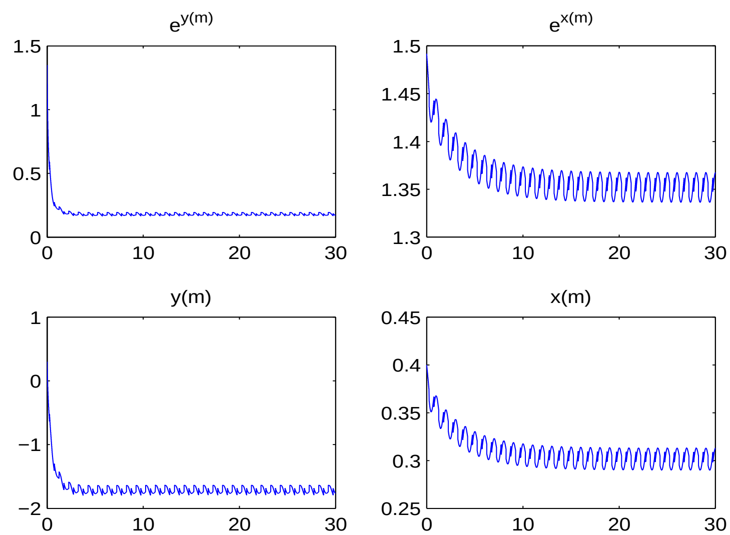

FIGURE 1. Solution when $x(0) = 0.4$ and $y(0) = 0.3$.

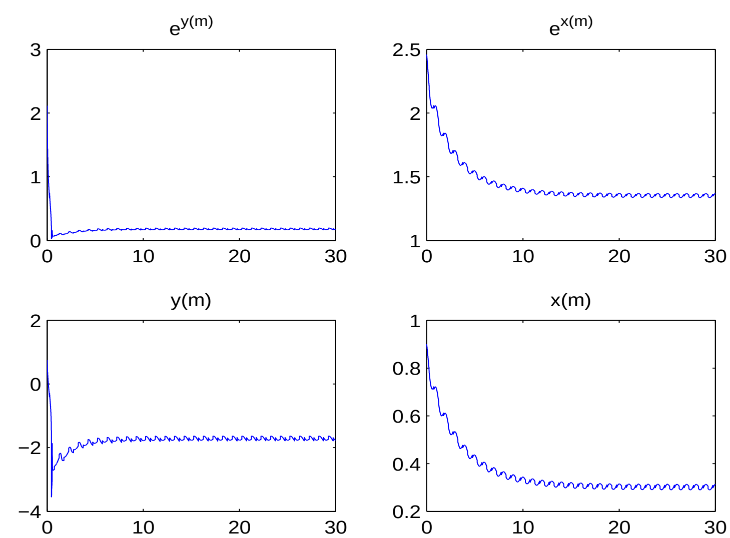

FIGURE 2. Solution when $x(0) = 0.9$ and $y(0) = 0.75$.

$$
\begin{gathered}
y'(t) = 1.1 - \frac{4.5}{(1.3 + 0.5\sin(t))}\exp(y - x), \quad t \neq t_k, \\
\Delta x(t_k) = \ln(1 + c_{1k}), \\
\Delta y(t_k) = \ln(1 + c_{2k}).
\end{gathered}
$$

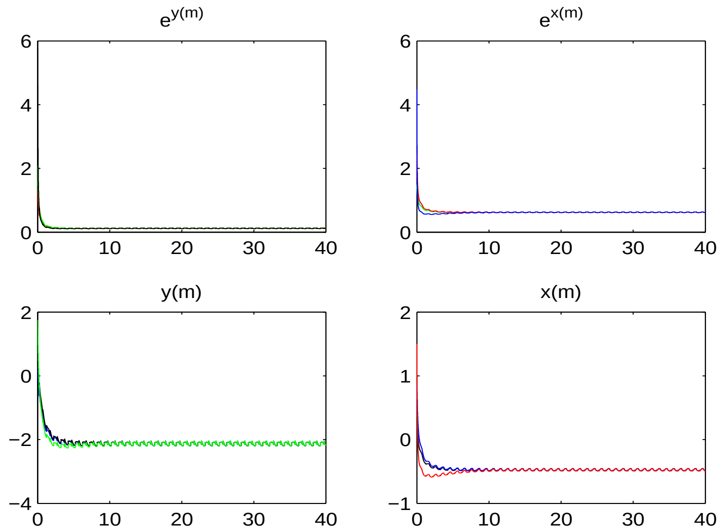

FIGURE 3. Solution when $x(0) = 0.4$, $y(0) = 0.3$; $x(0) = 0.9$, $y(0) = 0.75$; $x(0) = 1.5$, $y(0) = 1.75$. Blue, black, green lines expresses the solution $y(t)$; black, blue, red lines expresses respectively the solution $x(t)$; red, green, black lines expresses respectively the solution $e^{(y)}$; green, red, blue lines expresses respectively the solution $e^{(x)}$.

Impulse points: $t_1 = 2k + 1/4$, $t_2 = 2k + 3/4$ and $p = 2$.

$$c_{11} = e^{-0.03} - 1, \quad c_{12} = e^{-0.03} - 1, \quad c_{21} = e^{0.15} - 1, \quad c_{22} = e^{0.15} - 1\,.$$

It is clear that $\phi(x, y, t) = \frac{(0.15+0.5\sin(t))x^2}{0.4+(1.3+0.5\sin(t))x+0.9y}$. $\phi(x, y, t)$ should satisfy inequality (3.2) and with some simple calculations, we obtain that $\phi(x, y, t) \le (0.15 + 0.5\sin(t)/0.4)x^2$. Therefore the conditions of System (3.1) is satisfied by this example.

Here, we should make the observation about the global attractivity of the permanent and w-periodic solutions of the system in Example 5.2. In other words, we can obtain that the theoretical results in Theorem 4.1 is numerically satisfied by the Figure 5. To show that the solution of the system in Example 5.2 is globally attractive, we should change the initial conditions. Now, although we change the initial conditions of the same system, still we obtain the same solution after sufficiently large t which means the result of Theorem 4.1 is numerically satisfied.

In the following figure, since Example 5.2 satisfies the condition of Corollary 4.2, we would like to show that the solutions of the same system with different initial conditions have the same solutions after a while and we try to show this numerically by the displacement of the Figure 5 and two other plots with same system and different initial conditions in a single figure.

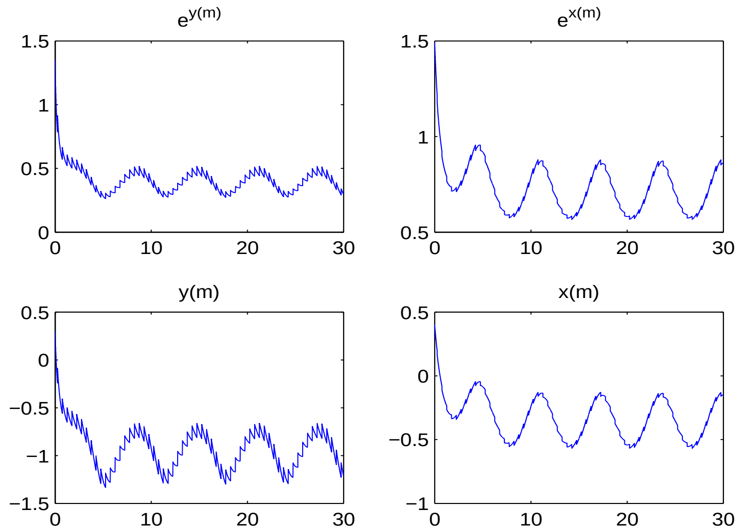

FIGURE 4. Solution when $x(0) = 0.3$ and $y(0) = 0.4$.

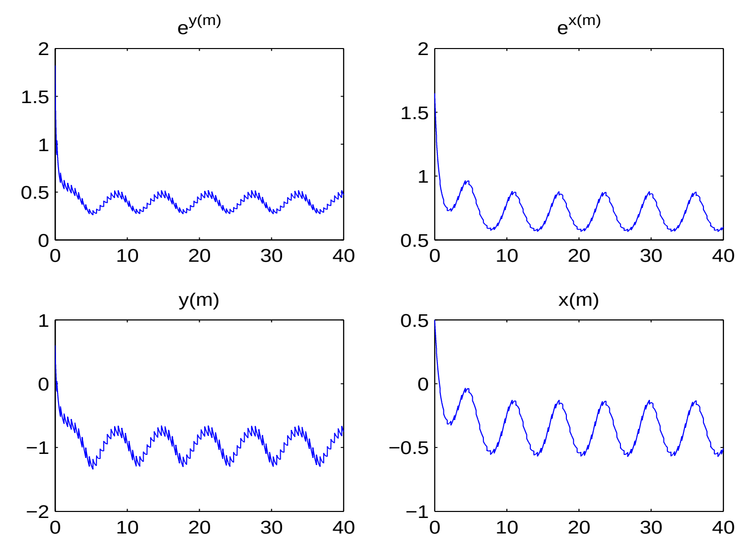

FIGURE 5. Solution when $x(0) = 0.5$ and $y(0) = 0.6$.

6. CONCLUSION

In this study, two important things are achieved. The first one is to find a necessary and sufficient condition for the permanent solution of the predator-prey system with generalized semi-ratio type functional response for the continuous case.

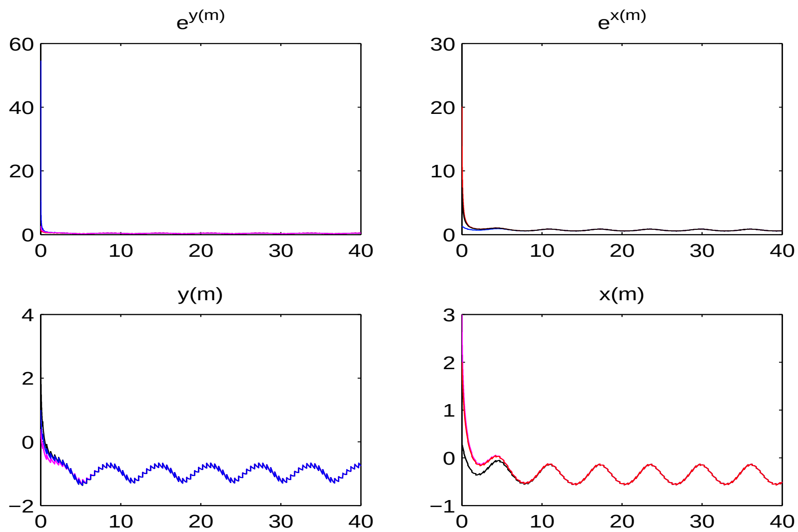

FIGURE 6. Solution when $x(0) = 0.5$, $y(0) = 0.6$; $x(0) = 1.2$, $y(0) = 0.9$; $x(0) = 5$, $y(0) = 4$. Blue, red and pink lines expresses respectively the solution $y(t)$; red, blue, green lines expresses the solution $x(t)$; pink, black, blue lines expresses the solution $e^{(y)}$ for the system; black, pink, red lines expresses the solution $e^{(x)}$ for the system.

The second significant achievement is to be able to find a necessary and sufficient condition for the globally attractive periodic solution of the same predator-prey model.

More concretely, the contribution of our study to [14] and literature is to be able to shown that when $\phi(t,x,y) \leq \alpha_0(t)x^m + \dots + \alpha_{m-2}(t)x^2$ and $\mathbb{T} = \mathbb{R}$, then, System (3.1) has globally attractive w-periodic solution if and only if the inequalities (3.3) and (3.4) are satisfied. Also, it is shown that this significant result is supported by the numerical examples.

The suggested problem for the future works is to find the necessary and sufficient condition for the globally attractive periodic solution of the discrete predator-prey dynamic systems. To obtain the globally attractive periodic solution of the continuous system, semi-group theory has been used. For the discrete case, it should be investigated how one can obtain the result that is related with the global attractivity of the system. Necessary and sufficient condition for the permanent and periodic solution of the considered system can be found by the help of time scale calculus in the discrete case of the considered system. However, the question that is about to find a necessary and sufficient condition for the global attractivity of the system still does not have satisfactory answer.

REFERENCES

[1] Y. Cao, I. Freedman; *Global Attractivity in Time-Delayed Predator-Prey Systems,* Journal of Australian Mathematical Society. Sen B, 38 (1996), 149-162.

[2] Y. Cao, T. C. Gard; *Ultimate bounds and global asymptotic stability for differential delay equations*, Rocky Mountain Journal of Mathematics, 25 (1995), 119-131.

[3] J. Cui; *Permanence of predator-prey system with periodic coefficients.* Mathematical and Computer Modeling, 42.1 (2005), 87-98.

[4] J. Cui, Y. Takeuchi; *Permanence, extinction and periodic solution of predator-prey system with Beddington- DeAngelis functional response* J. Math. Anal. Appl., 317 (2006), 464-474.

[5] H. I. Freedman, K. Gopalsamy; *Global stability in time-delayed single species dynamics*, Bulletin in Mathematical Biology, 48 (1986), 485-192.

[6] H. I. Freedman, S. Ruan; *Uniform persistence in functional differential equations*, Differential Equations, 115 (1995), 173-192.

[7] H. I. Freedman, P. Waltman; *Persistence in models of three interacting predator-prey populations*, Mathematical Bioscience, 68 (1984), 213-231.

[8] K. Goopalsamy; *Time lags and global stability in two species competition*, Bulletin in Mathematical Biology, 42 (1980), 729-737.

[9] A. F. Güvenilir, B. Kaymakçalan, N. N. Pelen; *Impulsive Predator-Prey Dynamic Systems with Beddington-DeAngelis Type Functional Response on the Unification of Discrete and Continuous Systems*, Applied Mathematics, 6.09 (2015), 1649.

[10] J. K. Hale, P. Waltman; *Persistence in infinite dimensional systems*, SIAM Journal of Mathematical Analysis, 20 (1989), 388-395.

[11] Y. Kuang; *Delay Differential Equations with Applications in Population Dynamics*, Academic Press, Boston, 1993.

[12] V. Lakshmikantham, D. Bainov, P. Simeonov; *Theory of Impulsive Differential Equations,* vol. 6 of Series in Modern Applied Mathematics, World Scientific, Teaneck, NJ, USA, 1989.

[13] V. Lakshmikantham, D. D. Bainov, P. S. Simeonov; *Theory of Impulsive Differential Equations*, vol. 6 of Series in Modern Applied Mathematics, World Scientific, Teaneck, NJ, USA, 1989.

[14] X. Liu, X. Liu; *Necessary and Sufficient Conditions for the Existence of Periodic Solutions in a Predator-Prey Model on Time Scales*, Electronic Journal of Differential Equations, Vol. 2012 (2012), No. 199, pp. 1-13.

[15] N. N. Pelen, A. F. Güvenilir, B. Kaymakçalan; *Necessary and sufficient condition for existence of periodic solutions of predator-prey dynamic systems with Beddington-DeAngelis-type functional response.* Advances in Difference Equations, 2016.1 (2016), 1-15.

[16] N. N. Pelen; *Stability Analysis of the Periodic Solutions of Some Kinds of Predator-Prey Dynamical Systems,* Journal of Mathematics 2016 (2016).

[17] W. Wang, J. Shen, J. Nieto; *Permanence and Periodic Solution of Predator-Prey System with Holling Type Functional Response and Impulses*, Discrete Dynamics in Nature and Society Volume 2007, Article ID 81756, 15 pages. ISSN: 1072-6691.

[18] X. Q. Zhao; *Uniform persistence and periodic coexistence states in infinite-dimensional periodic semiflows with applications*, The Canadian Applied Mathematics Quarterly, vol. 3, no. 4, pp. 473-495, 1995.

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