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SOLVABILITY OF NONLOCAL INVERSE BOUNDARY-VALUE PROBLEM FOR A SECOND-ORDER PARABOLIC EQUATION WITH INTEGRAL CONDITIONS

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Abstract. This article studies a nonlocal inverse boundary-value problem for a second-order parabolic equation on the rectangular domain. First, we introduce a definition of a classical solution, and then the original problem is reduced to an equivalent problem. Existence and uniqueness of the solution of the equivalent problem is proved using a contraction mapping. Finally, using the equivalency, the existence and uniqueness of classical solution is obtained.

1. Introduction

Practical requirements often lead to the problem of determining the coefficients or the right hand side of the differential equations for some known data about its solutions. Such problems are called inverse problems in mathematical physics. Inverse problems arise in various fields of human activity, such as seismology, mineral exploration, biology, medical visualization, computed tomography, Earth remote sensing, spectral analysis, nondestructive control, etc. Fundamentals of the theory and practice of research of inverse problems were established and developed in the pioneering works by Tikhonov [19], Lavrent'ev [11], [12], Ivanov [7], Romanov [18], Denisov [3]. Subsequently, the methods developed by them were applied to investigate a wide scale of inverse problems by their pupils and followers. Recently, there have been many studies of inverse problems for parabolic and other types of equations. A more detailed bibliography and a classification of problems are found in [1, 5, 6, 7, 8, 14, 15, 16].

Recently, problems with nonlocal conditions for partial differential equations have been of great interest. We note that most of the publications about problems with spatially nonlocal conditions and integral conditions for partial differential equations are found in[9, 4, 17]. In [10], a problem of time nonlocal integral conditions for hyperbolic conditions is investigated.

In this article we study an inverse boundary-value problem for second-order parabolic equations with nonlocal conditions. A distinctive feature of this article

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is the consideration of a parabolic equation with both spatial and time non-local conditions.

2. Formulation of the problem

Let $T > 0$ be a fixed number and $D_T := \{(x,t) : 0 \le x \le 1, 0 \le t \le T\}$. We consider the equation

$$c(t)u_t(x,t) = u_{xx}(x,t) + a(t)u(x,t) + b(t)g(x,t) + f(x,t) \tag{2.1}$$

in the rectangular domain D_T. The inverse problem has nonlocal initial condition

$$u(x,0) + \delta u(x,T) + \int_0^T p(t)u(x,t)dt = \varphi(x) \quad (0 \le x \le 1), \tag{2.2}$$

periodic boundary condition

$$u(0,t) = u(1,t) \quad (0 \le t \le T), \tag{2.3}$$

nonlocal integral condition

$$\int_0^1 u(x,t)dx = 0 \quad (0 \le t \le T), \tag{2.4}$$

and the additional conditions

$$u(x_i,t) = h_i(t) \quad (i = 1,2;\ 0 \le t \le T), \tag{2.5}$$

where $\delta \ge 0$, $x_i \in (0,1)$ $(i = 1,2;\ x_1 \ne x_2)$ are fixed numbers, $0 < c(t)$, $g(x,t)$, $f(x,t)$, $0 \le p(t)$, $\varphi(x)$, $h_i(t)$ $(i = 1,2)$ are given functions, $u(x,t), a(t), b(t)$ are the sought functions.

Definition 2.1. The triplete $\{u(x,t), a(t), b(t)\}$ is said to be a classical solution of problem (2.1)–(2.5), if the functions $u(x,t), a(t)$ and $b(t)$ satisfy the following conditions:

1. The function $u(x,t)$ and its derivatives $u_t(x,t), u_x(x,t), u_{xx}(x,t)$ are continuous in the domain D_T;
2. the functions $a(t)$ and $b(t)$ are continuous on the interval $[0,T]$;
3. equation (2.1) and conditions (2.2)–(2.5) are satisfied in the classical (usual) sense.

Lemma 2.2. *Suppose that $\delta \ge 0$, $0 < c(t) \in C[0,T]$, $a(t) \in C[0,T]$ and $0 \le p(t) \in C[0,T]$ hold. Then the problem*

$$c(t)y'(t) = a(t)y(t) \quad (0 \le t \le T), \tag{2.6}$$

$$y(0) + \delta y(T) + \int_0^T p(t)y(t)dt = 0 \tag{2.7}$$

has a unique trivial solution.

Proof. Obviously, the general solution of equation (2.6) has the form:

$$y(t) = ce^{\int_0^t \frac{a(\tau)}{c(\tau)}d\tau}. \tag{2.8}$$

Using (2.7) we obtain

$$c\Big(1 + \delta e^{\int_0^T \frac{a(t)}{c(t)}dt} + \int_0^T p(t)e^{\int_0^t \frac{a(\tau)}{c(\tau)}d\tau}dt\Big) = 0.$$

By $\delta \geq 0$, and $p(t) \geq 0$, from the latter relation it is clear that $c = 0$. Substituting the value of $c = 0$ in (2.8), we obtain that problem (2.6), (2.7) has only the trivial solution. The proof is complete. $\square$

Theorem 2.3. *Assume the following conditions are satisfied: $\delta \geq 0$, $0 < c(t) \in C[0,T]$, $0 \leq p(t) \in C[0,T]$, $f(x,t) \in C(D_T)$, $\varphi(x) \in C[0,1]$, $\int_0^1 f(x,t)dx = 0$ $(0 \leq t \leq T)$, $g(x,t) \in C(D_T)$,*

$$\int_0^1 g(x,t)dx = 0 \quad (0 \leq t \leq T),$$

$h_i(t) \in C^1[0,T]$ $(i = 1,2)$, $h(t) \equiv h_1(t)g(x_2,t) - h_2(t)g(x_1,t) \neq 0$ $(0 \leq t \leq T)$, and the compatibility conditions

$$\int_0^1 \varphi(x)dx = 0, \tag{2.9}$$

$$h_i(0) + \delta h_i(T) + \int_0^T p(t)h_i(t)dt = \varphi(x_i) \quad (i = 1,2)\,. \tag{2.10}$$

Then the problem of finding a classical solution of (2.1)–(2.5) *is equivalent to the problem of determining functions $u(x,t) \in C^{2,1}(D_T)$, $a(t) \in C[0,T]$, and $b(t) \in C[0,T]$, satisfying equation* (2.1)*, conditions* (2.2) *and* (2.3)*, and the conditions*

$$u_x(0,t) = u_x(1,t) \quad (0 \leq t \leq T), \tag{2.11}$$

$$c(t)h_i'(t) = u_{xx}(x_i,t) + a(t)h_i(t) + b(t)g(x_i,t) + f(x_i,t) \tag{2.12}$$

for $i = 1,2$; $0 \leq t \leq T$.

Proof. Suppose that $\{u(x,t), a(t), b(t)\}$ is a classical solution of (2.1)–(2.5). Integrating both sides of (2.1) with respect to x from 0 to 1 gives

$$\begin{aligned} c(t)\frac{d}{dt}\int_0^1 u(x,t)dx &= u_x(1,t) - u_x(0,t) + a(t)\int_0^1 u(x,t)dx \\ &\quad + b(t)\int_0^1 g(x,t)dx + \int_0^1 f(x,t)dx \quad (0 \leq t \leq T). \end{aligned} \tag{2.13}$$

Under the assumptions $\int_0^1 f(x,t)dx = 0$ and $\int_0^1 g(x,t)dx = 0$ $(0 \leq t \leq T)$, we obtain (2.11).

Setting $x = x_i$ in (2.1) we obtain

$$\begin{aligned} c(t)u_t(x_i,t) &= u(x_i,t) + a(t)u(x_i,t) + b(t)u(x_i,t) \\ &\quad + f(x_i,t) \quad (i = 1,2;\ 0 \leq t \leq T). \end{aligned} \tag{2.14}$$

Further, assuming $h_i(t) \in C^1[0,T]$ $(i = 1,2)$ and differentiating (2.5), we have

$$u_t(x_i,t) = h_i(t) \quad (i = 1,2). \tag{2.15}$$

From (2.14), by (2.5) and (2.15), we conclude that the relation (2.12) is fulfilled.

Now, assume that $\{u(x,t), a(t), b(t)\}$ is the solution of (2.1)–(2.3), (2.11), (2.12). Then from (2.13), taking into account (2.11), we find

$$c(t)\frac{d}{dt}\int_0^1 u(x,t)dx = a(t)\int_0^1 u(x,t)dx \quad (0 \leq t \leq T). \tag{2.16}$$

By (2.2) and (2.9), it is easy to see that

$$\begin{aligned} &\int_0^1 u(x,0)dx + \delta\int_0^1 u(x,T)dx + \int_0^T p(t)\int_0^1 u(x,t)dxdt \\ &= \int_0^1 \Big(u(x,0) + \delta u(x,T) + \int_0^T p(t)u(x,t)dt\Big)dx \\ &= \int_0^1 \varphi(x)dx = 0. \end{aligned} \tag{2.17}$$

Since, by Lemma 2.2, problem (2.16), (2.17) has only a trivial solution, it follows that

$$\int_0^1 u(x,t)dx = 0 \quad (0 \le t \le T),$$

i.e. the condition (2.4) holds.

Moreover, from (2.12) and (2.14) we find

$$c(t)\frac{d}{dt}(u(x_i,t) - h_i(t)) = a(t)(u(x_i,t) - h_i(t)) \quad (i = 1,2;\ 0 \le t \le T). \tag{2.18}$$

Using (2.2) and the compatibility conditions (2.10) we have

$$\begin{aligned} &u(x_i,0) - h_i(0) + \delta(u(x_i,T) - h_i(0)) + \int_0^T p(t)(u(x_i,t) - h_i(t))dt \\ &= \Big(u(x_i,0) + \delta u(x_i,T) + \int_0^T p(t)u(x_i,t)dt\Big) - \Big(h_i(0) + \delta h_i(0) + \int_0^T p(t)h_i(t)dt\Big) \\ &= \varphi(x_i) - \Big(h_i(0) + \delta h_i(0) + \int_0^T p(t)h_i(t)dt\Big) = 0 \quad (i = 1,2). \end{aligned}$$

From this equality and (2.18), by Lemma 2.2, we conclude that conditions (2.5) are satisfied. The proof is complete. □

3. Solvability of inverse boundary-value problem

In [2], it is known that the system

$$1, \cos\lambda_1 x, \sin\lambda_1 x, \dots, \cos\lambda_k x, \sin\lambda_k x, \dots, \tag{3.1}$$

where $\lambda_k = 2k\pi$ $(k = 0,1,\dots)$, is a basis for $L_2(0,1)$.

Since the system (3.1) form a basis in $L_2(0,1)$, we shall seek the first component $u(x,t)$ of classical solution $\{u(x,t), a(t), b(t)\}$ of the problem (2.1)–(2.3), (2.11), (2.12) in the form

$$u(x,t) = \sum_{k=0}^{\infty} u_{1k}(t)\cos\lambda_k x + \sum_{k=1}^{\infty} u_{2k}(t)\sin\lambda_k x \quad (\lambda = 2k\pi), \tag{3.2}$$

where

$$\begin{gathered} u_{10}(t) = \int_0^1 u(x,t)dx, \\ u_{1k}(t) = 2\int_0^1 u(x,t)\cos\lambda_k x\, dx \quad (k = 1,2,\dots), \\ u_{2k}(t) = 2\int_0^1 u(x,t)\sin\lambda_k x\, dx \quad (k = 1,2,\dots). \end{gathered}$$

Then applying the formal scheme of the Fourier method, for determining of unknown coefficients $u_{1k}(t)$ $(k = 0, 1, \dots)$ and $u_{2k}(t)$ $(k = 1, 2, \dots)$ of function $u(x, t)$ from (2.1) and (2.2) we have

$$c(t)u'_{10}(t) = F_{10}(t; u, a, b) \quad (0 \le t \le T), \tag{3.3}$$

$$c(t)u'_{ik}(t) + \lambda_{ik}^2 u_{ik}(t) = F_{ik}(t; u, a, b) \quad (i = 1, 2; k = 1, 2 \dots;\ 0 \le t \le T), \tag{3.4}$$

$$u_{ik}(0) + \delta u_{ik}(T) + \int_0^T p(t)u_{ik}(t)dt = \varphi_{ik} \quad (i = 1, 2;\ k = 0, 1, 2 \dots), \tag{3.5}$$

where

$$F_{ik}(t; u, a, b) = f_{ik}(t) + b(t)g_{ik}(t) + a(t)u_{ik}(t) \quad (i = 1, 2;\ k = 0, 1, 2 \dots),$$
$$f_{10}(t) = \int_0^1 f(x, t)dx, \quad g_{10}(t) = \int_0^1 g(x, t)dx,$$
$$f_{1k}(t) = 2\int_0^1 f(x, t)\cos\lambda_k x\,dx, \quad f_{2k}(t) = 2\int_0^1 f(x, t)\sin\lambda_k x\,dx,$$
$$g_{1k}(t) = 2\int_0^1 g(x, t)\cos\lambda_k x\,dx, \quad g_{2k}(t) = 2\int_0^1 g(x, t)\sin\lambda_k x\,dx,$$
$$\varphi_{10} = \int_0^1 \varphi(x)dx, \quad \varphi_{1k}(t) = 2\int_0^1 \varphi(x)\cos\lambda_k x\,dx,$$
$$\varphi_{2k}(t) = 2\int_0^1 \varphi(x)\sin\lambda_k x\,dx$$

for $k = 1, 2, \dots$.

Solving problem (3.3)–(3.5) we obtain

$$\begin{aligned} u_{10}(t) &= (1+\delta)^{-1}\Big(\varphi_{10} - \int_0^T p(t)u_{10}(t)dt - \delta\int_0^T \frac{1}{c(t)}F_{10}(t; u, a, b)dt\Big) \\ &\quad + \int_0^t \frac{1}{c(\tau)}F_{10}(\tau; u, a, b)d\tau, \end{aligned} \tag{3.6}$$

$$\begin{aligned} u_{ik}(t) &= \frac{e^{-\int_0^t \frac{\lambda_k^2}{c(s)}ds}}{1 + \delta e^{-\int_0^T \frac{\lambda_k^2}{c(s)}ds}}\Big(\varphi_{ik} - \int_0^T p(t)u_{ik}(t)dt\Big) \\ &\quad - \frac{\delta e^{-\int_0^T \frac{\lambda_k^2}{c(s)}ds}}{1 + \delta e^{-\int_0^T \frac{\lambda_k^2}{c(s)}ds}}\int_0^T \frac{1}{c(\tau)}F_{ik}(\tau; u, a, b)e^{-\int_\tau^t \frac{\lambda_k^2}{c(s)}ds}d\tau \\ &\quad + \int_0^t \frac{1}{c(\tau)}F_{ik}(\tau; u, a, b)e^{-\int_\tau^t \frac{\lambda_k^2}{c(s)}ds}d\tau \quad (i = 1, 2;\ k = 1, 2, \dots). \end{aligned} \tag{3.7}$$

After substituting expressions $u_{1k}(t)$ $(k=0,1,\dots)$ and $u_{2k}(t)$ $(k=1,2,\dots)$ in (3.2), we obtain

$$\begin{aligned}
u(x,t) &= (1+\delta)^{-1}\Big(\varphi_{10} - \int_0^T p(t)u_{10}(t)dt - \delta\int_0^T \frac{1}{c(t)}F_{10}(t;u,a,b)dt\Big)\\
&\quad + \int_0^t \frac{1}{c(\tau)}F_{10}(\tau;u,a,b)d\tau\\
&\quad + \sum_{k=1}^{\infty}\Big\{\frac{e^{-\int_0^t \frac{\lambda_k^2}{c(s)}ds}}{1+\delta e^{-\int_0^T \frac{\lambda_k^2}{c(s)}ds}}\Big(\varphi_{1k} - \int_0^T p(t)u_{1k}(t)dt\Big)\\
&\quad - \frac{\delta e^{-\int_0^T \frac{\lambda_k^2}{c(s)}ds}}{1+\delta e^{-\int_0^T \frac{\lambda_k^2}{c(s)}ds}}\int_0^T \frac{1}{c(\tau)}F_{1k}(\tau;u,a,b)e^{-\int_\tau^t \frac{\lambda_k^2}{c(s)}ds}d\tau\\
&\quad + \int_0^t \frac{1}{c(\tau)}F_{1k}(\tau;u,a,b)e^{-\int_\tau^t \frac{\lambda_k^2}{c(s)}ds}d\tau\Big\}\cos\lambda_k x\\
&\quad + \sum_{k=1}^{\infty}\Big\{\frac{e^{-\int_0^t \frac{\lambda_k^2}{c(s)}ds}}{1+\delta e^{-\int_0^T \frac{\lambda_k^2}{c(s)}ds}}\Big(\varphi_{2k} - \int_0^T p(t)u_{2k}(t)dt\Big)\\
&\quad - \frac{\delta e^{-\int_0^T \frac{\lambda_k^2}{c(s)}ds}}{1+\delta e^{-\int_0^T \frac{\lambda_k^2}{c(s)}ds}}\int_0^T \frac{1}{c(\tau)}F_{2k}(\tau;u,a,b)e^{-\int_\tau^t \frac{\lambda_k^2}{c(s)}ds}d\tau\\
&\quad + \int_0^t \frac{1}{c(\tau)}F_{2k}(\tau;u,a,b)e^{-\int_\tau^t \frac{\lambda_k^2}{c(s)}ds}d\tau\Big\}\sin\lambda_k x.
\end{aligned} \tag{3.8}$$

Now, using (3.2) and (2.12) we have

$$\begin{aligned}
a(t) &= [h(t)]^{-1}\Big\{(c(t)h_1'(t) - f(x_1,t))g(x_2,t) - (c(t)h_2'(t) - f(x_2,t))g(x_1,t)\\
&\quad + \sum_{k=1}^{\infty}\lambda_k^2 u_{1k}(t)(g(x_2,t)\cos\lambda_k x_1 - g(x_1,t)\cos\lambda_k x_2)\\
&\quad + \sum_{k=1}^{\infty}\lambda_k^2 u_{2k}(t)(g(x_2,t)\sin\lambda_k x_1 - g(x_1,t)\sin\lambda_k x_2)\Big\},
\end{aligned} \tag{3.9}$$

$$\begin{aligned}
b(t) &= [h(t)]^{-1}\Big\{h_1(t)(c(t)h_2'(t) - f(x_2,t)) - h_2(t)(c(t)h_1'(t) - f(x_1,t))\\
&\quad + \sum_{k=1}^{\infty}\lambda_k^2 u_{1k}(t)(h_1(t)\cos\lambda_k x_2 - h_2(t)\cos\lambda_k x_2)\\
&\quad + \sum_{k=1}^{\infty}\lambda_k^2 u_{2k}(t)(h_1(t)\sin\lambda_k x_2 - h_2(t)\sin\lambda_k x_1)\Big\},
\end{aligned} \tag{3.10}$$

where

$$h(t) \equiv h_1(t)g(x_2,t) - h_2(t)g(x_1,t) \neq 0 \quad (0 \le t \le T). \tag{3.11}$$

Taking into account (3.7), from (3.9) and (3.10) we obtain

$$
\begin{aligned}
a(t) = {} & [h(t)]^{-1}\{(c(t)h_1'(t) - f(x_1,t))g(x_2,t) - (c(t)h_2'(t) - f(x_2,t))g(x_2,t) \\
& + \sum_{k=1}^{\infty} \lambda_k^2 \Big[\frac{e^{-\int_0^t \frac{\lambda_k^2}{c(s)}ds}}{1+\delta e^{-\int_0^T \frac{\lambda_k^2}{c(s)}ds}} \Big(\varphi_{1k} - \int_0^T p(t)u_{1k}(t)dt \Big) - \\
& - \frac{\delta e^{-\int_0^T \frac{\lambda_k^2}{c(s)}ds}}{1+\delta e^{-\int_0^T \frac{\lambda_k^2}{c(s)}ds}} \int_0^T \frac{1}{c(\tau)} F_{1k}(\tau;u,a,b) e^{-\int_\tau^t \frac{\lambda_k^2}{c(s)}ds} d\tau \\
& + \int_0^t \frac{1}{c(\tau)} F_{1k}(\tau;u,a,b) e^{-\int_\tau^t \frac{\lambda_k^2}{c(s)}ds} d\tau \Big] (g(x_2,t)\cos\lambda_k x_1 \\
& - g(x_1,t)\cos\lambda_k x_2) \\
& + \sum_{k=1}^{\infty} \lambda_k^2 \Big[\frac{e^{-\int_0^t \frac{\lambda_k^2}{c(s)}ds}}{1+\delta e^{-\int_0^T \frac{\lambda_k^2}{c(s)}ds}} \Big(\varphi_{2k} - \int_0^T p(t)u_{2k}(t)dt \Big) \\
& - \frac{\delta e^{-\int_0^T \frac{\lambda_k^2}{c(s)}ds}}{1+\delta e^{-\int_0^T \frac{\lambda_k^2}{c(s)}ds}} \int_0^T \frac{1}{c(\tau)} F_{2k}(\tau;u,a,b) e^{-\int_\tau^t \frac{\lambda_k^2}{c(s)}ds} d\tau \\
& + \int_0^t \frac{1}{c(\tau)} F_{2k}(\tau;u,a,b) e^{-\int_\tau^t \frac{\lambda_k^2}{c(s)}ds} d\tau \Big] (g(x_2,t)\sin\lambda_k x_1 \\
& - g(x_1,t)\sin\lambda_k x_2) \Big\},
\end{aligned} \tag{3.12}
$$

$$
\begin{aligned}
b(t) = {} & [h(t)]^{-1}\Big\{ h_1(t)(c(t)h_2'(t) - f(x_2,t)) - h_2(t)(c(t)h_1'(t) - f(x_1,t)) \\
& + \sum_{k=1}^{\infty} \lambda_k^2 \Big[\frac{e^{-\int_0^t \frac{\lambda_k^2}{c(s)}ds}}{1+\delta e^{-\int_0^T \frac{\lambda_k^2}{c(s)}ds}} \Big(\varphi_{1k} - \int_0^T p(t)u_{1k}(t)dt \Big) \\
& - \frac{\delta e^{-\int_0^T \frac{\lambda_k^2}{c(s)}ds}}{1+\delta e^{-\int_0^T \frac{\lambda_k^2}{c(s)}ds}} \int_0^T \frac{1}{c(\tau)} F_{1k}(\tau;u,a,b) e^{-\int_\tau^t \frac{\lambda_k^2}{c(s)}ds} d\tau \\
& + \int_0^t \frac{1}{c(\tau)} F_{1k}(\tau;u,a,b) e^{-\int_\tau^t \frac{\lambda_k^2}{c(s)}ds} d\tau \Big] (h_1(t)\cos\lambda_k x_2 \\
& - h_2(t)\cos\lambda_k x_1) \\
& + \sum_{k=1}^{\infty} \lambda_k^2 \Big[\frac{e^{-\int_0^t \frac{\lambda_k^2}{c(s)}ds}}{1+\delta e^{-\int_0^T \frac{\lambda_k^2}{c(s)}ds}} \Big(\varphi_{2k} - \int_0^T p(t)u_{2k}(t)dt \Big) \\
& - \frac{\delta e^{-\int_0^T \frac{\lambda_k^2}{c(s)}ds}}{1+\delta e^{-\int_0^T \frac{\lambda_k^2}{c(s)}ds}} \int_0^T \frac{1}{c(\tau)} F_{2k}(\tau;u,a,b) e^{-\int_\tau^t \frac{\lambda_k^2}{c(s)}ds} d\tau \\
& + \int_0^t \frac{1}{c(\tau)} F_{2k}(\tau;u,a,b) e^{-\int_\tau^t \frac{\lambda_k^2}{c(s)}ds} d\tau \Big] (h_1(t)\sin\lambda_k x_2 \\
& - h_2(t)\sin\lambda_k x_1) \Big\}.
\end{aligned} \tag{3.13}
$$

Analogously, the following lemma was proved in [13].

Lemma 3.1. *If* $\{u(x,t), a(t), b(t)\}$ *is a solution of* (2.1)–(2.3), (2.11), (2.12), *then the functions*

$$u_{10}(t) = \int_0^1 u(x,t)dx,$$
$$u_{1k}(t) = 2\int_0^1 u(x,t)\cos\lambda_k x\,dx \quad (k = 1,2,\dots),$$
$$u_{2k}(t) = 2\int_0^1 u(x,t)\sin\lambda_k x\,dx \quad (k = 1,2,\dots)$$

satisfy system (3.6), (3.7) *on the interval* $[0,T]$.

From Lemma 3.1 it follows that to prove the uniqueness of the solution of problem (2.1)–(2.3), (2.11), (2.12), it suffices to prove the uniqueness of the solution of (3.8), (3.12), (3.13).

Now, consider the space $B^3_{2,T}$ [13] consisting of functions of the form

$$u(x,t) = \sum_{k=0}^{\infty} u_{1k}(t)\cos\lambda_k x + \sum_{k=1}^{\infty} u_{2k}(t)\sin\lambda_k x \quad (\lambda = 2k\pi)$$

in domain D_T, where the functions $u_{1k}(t)$ $(k = 0,1,\dots)$, $u_{2k}(t)$ $(k = 1,2,\dots)$, are continuous on $[0,T]$ and satisfy the condition

$$\|u_{10}(t)\|_{C[0,T]} + \Big(\sum_{k=1}^{\infty}\Big(\lambda_k^3\|u_{1k}(t)\|_{C[0,T]}\Big)^2\Big)^{1/2} + \Big(\sum_{k=1}^{\infty}\Big(\lambda_k^3\|u_{2k}(t)\|_{C[0,T]}\Big)^2\Big)^{1/2} < +\infty.$$

The norm in the space $B^3_{2,T}$ is

$$\begin{aligned}\|u(x,t)\|_{B^3_{2,T}} &= \|u_{10}(t)\|_{C[0,T]} + \Big(\sum_{k=1}^{\infty}\Big(\lambda_k^3\|u_{1k}(t)\|_{C[0,T]}\Big)^2\Big)^{1/2}\\ &\quad + \Big(\sum_{k=1}^{\infty}\Big(\lambda_k^3\|u_{2k}(t)\|_{C[0,T]}\Big)^2\Big)^{1/2}.\end{aligned}$$

We denote by E^3_T, the Banach space $B^3_{2,T}\times C[0,T]\times C[0,T]$ of vector functions $z(x,t) = \{u(x,t), a(t), b(t)\}$ with norm

$$\|z(x,t)\|_{B^3_{2,T}} = \|u(x,t)\|_{B^3_{2,T}} + \|a(t)\|_{C[0,T]} + \|b(t)\|_{C[0,T]}.$$

It is known that $B^3_{2,T}$ and E^3_T are Banach spaces

Now consider the operator

$$\Phi(u,a,b) = \{\Phi_1(u,a,b), \Phi_2(u,a,b), \Phi_3(u,a,b)\}$$

in the space E^3_T, where

$$\Phi_1(u,a,b) = \tilde u(x,t) \equiv \sum_{k=0}^{\infty}\tilde u_{1k}(t)\cos\lambda_k x + \sum_{k=1}^{\infty}\tilde u_{2k}(t)\sin\lambda_k x,$$
$$\Phi_2(u,a,b) = \tilde a(t), \quad \Phi_3(u,a,b) = \tilde b(t)$$

and the functions $\tilde u_{10}(t)$, $\tilde u_{ik}(t)$ $(i = 1,2;\ k = 0,1,2,\dots)$, $\tilde a(t)$ and $\tilde b(t)$ are equal to the right-hand sides of (3.6), (3.7), (3.12), and (3.13) respectively.

Using simple transformations from (3.6), (3.7), (3.12), and (3.13) we obtain

$$
\begin{aligned}
&\|\tilde u_{10}(t)\|_{C[0,T]}\\
&\le (1+\delta)^{-1}\Big[|\varphi_{10}| + T\|p(t)\|_{C[0,T]}\|u_{10}(t)\|_{C[0,T]}\\
&\quad + \delta\|\frac{1}{c(t)}\|_{C[0,T]}\Big(\sqrt{T}\Big(\int_0^T |f_{10}(\tau)|^2 d\tau\Big)^{1/2} + T\|a(t)\|_{C[0,T]}\|u_{10}(t)\|_{C[0,T]}\\
&\quad + \sqrt{T}\|b(t)\|_{C[0,T]}\Big(\int_0^T |g_{10}(\tau)|^2 d\tau\Big)^{1/2}\Big)\Big] \qquad (3.14)\\
&\quad + \|\frac{1}{c(t)}\|_{C[0,T]}\Big[\sqrt{T}\Big(\int_0^T |f_{10}(\tau)|^2 d\tau\Big)^{1/2}\\
&\quad + T\|a(t)\|_{C[0,T]}\|u_{10}(t)\|_{C[0,T]} + \sqrt{T}\|b(t)\|_{C[0,T]}\Big(\int_0^T |g_{10}(\tau)|^2 d\tau\Big)^{1/2}\Big],
\end{aligned}
$$

$$
\begin{aligned}
&\Big(\sum_{k=1}^\infty Big(\lambda_k^3\|\tilde u_{ik}(t)\|_{C[0,T]}\Big)^2\Big)^{1/2}\\
&\le 2\sqrt{2}\Big(\sum_{k=1}^\infty\Big(\lambda_k^3|\varphi_{ik}|\Big)^2\Big)^{1/2} + 2\sqrt{2}T\|p(t)\|_{C[0,T]}\Big(\sum_{k=1}^\infty\Big(\lambda_k^3\|u_{ik}\|_{C[0,T]}\Big)^2\Big)^{1/2}\\
&\quad + 2\sqrt{2}(1+\delta)\|\frac{1}{c(t)}\|_{C[0,T]}\sqrt{T}\Big(\int_0^T\sum_{k=1}^\infty\Big(\lambda_k^3|f_{ik}(\tau)|\Big)^2 d\tau\Big)^{1/2} \qquad (3.15)\\
&\quad + 2\sqrt{2}(1+\delta)\|\frac{1}{c(t)}\|_{C[0,T]}T\|a(t)\|_{C[0,T]}\Big(\sum_{k=1}^\infty\Big(\lambda_k^3\|u_{ik}(t)\|_{C[0,T]}\Big)^2\Big)^{1/2}\\
&\quad + 2\sqrt{2T}(1+\delta)\|\frac{1}{c(t)}\|_{C[0,T]}\|b(t)\|_{C[0,T]}\Big(\int_0^T\sum_{k=1}^\infty\Big(\lambda_k^3|g_{ik}(\tau)|\Big)^2 d\tau\Big)^{1/2},
\end{aligned}
$$

$$
\begin{aligned}
&\|\tilde a(t)\|_{C[0,T]}\\
&\le \|[h(t)]^{-1}\|_{C[0,T]}\Big\{\big\|(c(t)h_1'(t) - f(x_1,t))g(x_2,t) - (c(t)h_2'(t)\\
&\quad - f(x_2,t))g(x_1,t)\big\|_{C[0,T]}\\
&\quad + \Big(\sum_{k=1}^\infty \lambda_k^{-2}\Big)^{1/2}\||g(x_2,t)| + |g(x_1,t)|\|_{C[0,T]}\Big[\sum_{i=1}^2\Big(\sum_{k=1}^\infty\Big(\lambda_k^3|\varphi_{ik}|\Big)^2\Big)^{1/2}\\
&\quad + T\|p(t)\|_{C[0,T]}\sum_{i=1}^2\Big(\sum_{k=1}^\infty\Big(\lambda_k^3\|u_{ik}(t)\|_{C[0,T]}\Big)^2\Big)^{1/2}\\
&\quad + (1+\delta)\|\frac{1}{c(t)}\|_{C[0,T]}\sqrt{T}\sum_{i=1}^2\Big(\int_0^T\sum_{k=1}^\infty\Big(\lambda_k^3|f_{ik}(\tau)|\Big)^2 d\tau\Big)^{1/2} \qquad (3.16)\\
&\quad + (1+\delta)\|\frac{1}{c(t)}\|_{C[0,T]}T\|a(t)\|_{C[0,T]}\sum_{i=1}^2\Big(\sum_{k=1}^\infty\Big(\lambda_k^3\|u_{ik}(t)\|_{C[0,T]}\Big)^2\Big)^{1/2}\\
&\quad + (1+\delta)\|\frac{1}{c(t)}\|_{C[0,T]}\sqrt{T}\|b(t)\|_{C[0,T]}\sum_{i=1}^2\Big(\int_0^T\sum_{k=1}^\infty\big(\lambda_k^3|g_{ik}(\tau)d\tau|\big)^2\Big)^{1/2}\Big]\Big\},
\end{aligned}
$$

$$
\begin{aligned}
&\|\tilde{b}(t)\|_{C[0,T]}\\
&\leq \|[h(t)]^{-1}\|_{C[0,T]}\Big\{\|(h_1(t)(c(t)h_2'(t)-f(x_2,t))-h_2(t)(c(t)h_1'(t)\\
&\quad -f(x_1,t))\|_{C[0,T]}\\
&\quad +\Big(\sum_{k=1}^{\infty}\lambda_k^{-2}\Big)^{1/2}\||h_1(t)|+|h_2(t)|\|_{C[0,T]}\Big[\sum_{i=1}^{2}\Big(\sum_{k=1}^{\infty}\Big(\lambda_k^3|\varphi_{ik}|\Big)^2\Big)^{1/2}\\
&\quad +T\|p(t)\|_{C[0,T]}\sum_{i=1}^{2}\Big(\sum_{k=1}^{\infty}\Big(\lambda_k^3\|u_{ik}(t)\|_{C[0,T]}\Big)^2\Big)^{1/2}\\
&\quad +(1+\delta)\|\frac{1}{c(t)}\|_{C[0,T]}\sqrt{T}\sum_{i=1}^{2}\Big(\int_0^T\sum_{k=1}^{\infty}\Big(\lambda_k^3|f_{ik}(\tau)|\Big)^2d\tau\Big)^{1/2}\\
&\quad +(1+\delta)\|\frac{1}{c(t)}\|_{C[0,T]}T\|a(t)\|_{C[0,T]}\sum_{i=1}^{2}\Big(\sum_{k=1}^{\infty}\Big(\lambda_k^3\|u_{ik}(t)\|_{C[0,T]}\Big)^2\Big)^{1/2}\\
&\quad +(1+\delta)\|\frac{1}{c(t)}\|_{C[0,T]}\sqrt{T}\|b(t)\|_{C[0,T]}\\
&\quad \times\sum_{i=1}^{2}\Big(\int_0^T\sum_{k=1}^{\infty}\Big(\lambda_k^3|g_{ik}(\tau)d\tau|\Big)^2\Big)^{1/2}\Big]\Big\}.
\end{aligned}
\tag{3.17}
$$

Assume that the data for problem (2.1)–(2.3), (2.11) and (2.12) satisfy the following conditions

- (A1) $\varphi(x)\in C^2[0,1],\ \varphi'''(x)\in L_2(0,1),\ \varphi(0)=\varphi(1),\ \varphi'(0)=\varphi'(1),\varphi''(0)=\varphi''(1)$;
- (A2) $f(x,t),f_x(x,t),f_{xx}(x,t)\in C^2[0,1],\ f_{xxx}(x,t)\in L_2(D_T),\ f(0,t)=f(1,t),\ f_x(0,t)=f_x(1,t),\ f_{xx}(0,t)=f_{xx}(1,t)=0\ (0\leq t\leq T)$;
- (A3) $g(x,t),g_x(x,t),g_{xx}(x,t)\in C^2[0,1],\ g_{xxx}(x,t)\in L_2(D_T),\ g(0,t)=g(1,t),\ g_x(0,t)=f_x(1,t),\ g_{xx}(0,t)=g_{xx}(1,t)=0\ (0\leq t\leq T)$;
- (A4) $\delta\geq 0,\ 0\leq p(t)\in C[0,T],\ h_i(t)\in C^1[0,T]\ (i=1,2),\ h(t)=h_1(t)g(x_2,t)-h_2(t)g(x_1,t)\neq 0\ (0\leq t\leq T)$.

Then from (3.14)–(3.17) we find that

$$
\begin{aligned}
\|\tilde{u}(x,t)\|_{B_{2,T}^3}&\leq A_1(T)+B_1(T)\|a(t)\|_{C[0,T]}\|u(x,t)\|_{B_{2,T}^3}\\
&\quad +C_1(T)\|u(x,t)\|_{B_{2,T}^3}+D_1(T)\|b(t)\|_{C[0,T]},
\end{aligned}
\tag{3.18}
$$

$$
\begin{aligned}
\|\tilde{a}(t)\|_{C[0,T]}&\leq A_2(T)+B_2(T)\|a(t)\|_{C[0,T]}\|u(x,t)\|_{B_{2,T}^3}\\
&\quad +C_2(T)\|u(x,t)\|_{B_{2,T}^3}+D_2(T)\|b(t)\|_{C[0,T]},
\end{aligned}
\tag{3.19}
$$

$$
\begin{aligned}
\|\tilde{b}(t)\|_{C[0,T]}&\leq A_3(T)+B_3(T)\|a(t)\|_{C[0,T]}\|u(x,t)\|_{B_{2,T}^3}\\
&\quad +C_3(T)\|u(x,t)\|_{B_{2,T}^3}+D_3(T)\|b(t)\|_{C[0,T]},
\end{aligned}
\tag{3.20}
$$

where

$$
\begin{aligned}
A_1(T) &= (1+\delta)^{-1}\Big(2\|\varphi(x)\|_{L_2(0,1)} + 2\delta\|\frac{1}{c(t)}\|_{C[0,T]}\|f(x,t)\|_{L_2(D_T)}\Big) \\
&\quad + \|\frac{1}{c(t)}\|_{C[0,T]}\|f(x,t)\|_{L_2(D_T)} + 4\sqrt{2}\|\varphi'''(x)\|_{L_2(0,1)} \\
&\quad + 4\sqrt{2T}\|\frac{1}{c(t)}\|_{C[0,T]}\|f_{xxx}(x,t)\|_{L_2(D_T)}, \\
B_1(T) &= \big(\delta(1+\delta)^{-1}+1\big)T\|\frac{1}{c(t)}\|_{C[0,T]}, \\
C_1(T) &= \Big((1+\delta)^{-1} + 2\sqrt{2}\Big)\, T\|p(t)\|_{C[0,T]}, \\
D_1(T) &= \Big(\delta(1+\delta)^{-1} + 1 + 2\sqrt{2}\Big)\sqrt{T}\|\frac{1}{c(t)}\|_{C[0,T]}\|g_{xxx}(x,t)\|_{L_2(D_T)}, \\
A_2(T) &= \|[h(t)]^{-1}\|_{C[0,T]}\Big\{\big\|(c(t)h_1'(t) - f(x_1,t))g(x_2,t) - (c(t)h_2'(t) \\
&\quad - f(x_2,t))g(x_1,t)\big\|_{C[0,T]} + \Big(\sum_{k=1}^{\infty}\lambda_k^{-2}\Big)^{1/2}\||g(x_1,t)| + |g(x_2,t)|\|_{C[0,T]} \\
&\quad \times \Big[2\|\varphi'''(x)\|_{L_2(0,1)} + 2(1+\delta)\|\frac{1}{c(t)}\|_{C[0,T]}\sqrt{T}\|f_{xxx}(x,t)\|_{L_2(D_T)}\Big]\Big\}, \\
B_2(T) &= \|[h(t)]^{-1}\|_{C[0,T]}\Big(\sum_{k=1}^{\infty}(\lambda_k^{-2})\Big)^{1/2} \\
&\quad \times \||g(x_1,t)| + |g(x_2,t)|\|_{C[0,T]}(1+\delta)T\|\frac{1}{c(t)}\|_{C[0,T]}, \\
C_2(T) &= \|[h(t)]^{-1}\|_{C[0,T]}\Big(\sum_{k=1}^{\infty}(\lambda_k^{-2})\Big)^{1/2}\||g(x_1,t)| + |g(x_2,t)|\|_{C[0,T]}\|p(t)\|_{C[0,T]}T, \\
D_2(T) &= \|[h(t)]^{-1}\|_{C[0,T]}\Big(\sum_{k=1}^{\infty}(\lambda_k^{-2})\Big)^{1/2}\||g(x_1,t)| + |g(x_2,t)|\|_{C[0,T]} \\
&\quad \times (1+\delta)\|\frac{1}{c(t)}\|_{C[0,T]}\sqrt{T}\|g_{xxx}(x,t)\|_{L_2(D_T)}, \\
A_3(T) &= \|[h(t)]^{-1}\|_{C[0,T]}\Big\{\big\|h_1(t)(c(t)h_2'(t) - f(x_2,t)) - h_2(t)(c(t)h_1'(t) \\
&\quad - f(x_1,t))\|_{C[0,T]} + \Big(\sum_{k=1}^{\infty}\lambda_k^{-2}\Big)^{1/2}\||h_1(t)| + |h_2(t)|\|_{C[0,T]} \\
&\quad \times \Big[2\|\varphi'''(x)\|_{L_2(0,1)} + 2(1+\delta)\|\frac{1}{c(t)}\|_{C[0,T]}\sqrt{T}\|f_{xxx}(x,t)\|_{L_2(D_T)}\Big]\Big\}, \\
B_3(T) &= \|[h(t)]^{-1}\|_{C[0,T]}\Big(\sum_{k=1}^{\infty}(\lambda_k^{-2})\Big)^{1/2}\||h_1(t)| + |h_2(t)|\|_{C[0,T]}(1+\delta)T\|\frac{1}{c(t)}\|_{C[0,T]}, \\
C_3(T) &= \|[h(t)]^{-1}\|_{C[0,T]}\Big(\sum_{k=1}^{\infty}(\lambda_k^{-2})\Big)^{1/2}\||h_1(t)| + |h_2(t)|\|_{C[0,T]}\|p(t)\|_{C[0,T]}T,
\end{aligned}
$$

$$
\begin{aligned}
D_3(T) = \|[h(t)]^{-1}\|_{C[0,T]}\Big(\sum_{k=1}^{\infty}(\lambda_k^{-2})\Big)^{1/2}\||h_1(t)|+|h_2(t)|\|_{C[0,T]} \\
(1+\delta)\|\frac{1}{c(t)}\|_{C[0,T]}\sqrt{T}\|g_{xxx}(x,t)\|_{L_2(D_T)}.
\end{aligned}
$$

From (3.18)–(3.20) we conclude that

$$
\begin{aligned}
&\|\tilde{u}(x,t)\|_{B^3_{2,T}} + \|\tilde{a}(t)\|_{C[0,T]} + \|\tilde{b}(t)\|_{C[0,T]} \\
&\le A(T) + B(T)\|a(t)\|_{C[0,T]}\|u(x,t)\|_{B^3_{2,T}} \\
&\quad + C(T)\|u(x,t)\|_{B^3_{2,T}} + D(T)\|b(t)\|_{C[0,T]},
\end{aligned} \tag{3.21}
$$

where

$$
\begin{gathered}
A(T) = A_1(T) + A_2(T) + A_3(T), \quad B(T) = B_1(T) + B_2(T) + B_3(T), \\
C(T) = C_1(T) + C_2(T) + C_3(T), \quad D(T) = D_1(T) + D_2(T) + D_3(T).
\end{gathered}
$$

Theorem 3.2. *If conditions* (A1)–(A4) *and the condition*

$$
(B(T)(A(T)+2) + C(T) + D(T))(A(T)+2) < 1 \tag{3.22}
$$

hold, then problem (2.1)–(2.3), (2.11), (2.12) *has a unique solution in the ball* $K = K_R$ ($\|z\|_{E^3_T} \le R \le A(T)+2$) *of the space* E^3_T.

Proof. In the space E^3_T, we consider the equation

$$
z = \Phi z, \tag{3.23}
$$

where $z = \{u, a, b\}$, the components $\Phi_i(u,a,b)$ $(i = 1,2,3)$, of operator $\Phi(u,a,b)$, defined by the right side of equations (3.8), (3.12) and (3.13).

Consider the operator $\Phi(u,a,b)$, in the ball $K = K_R$ of the space E^3_T. Similarly, with the aid of (3.21) we obtain that for any $z_1, z_2, z_3 \in K_R$ the following inequalities hold

$$
\begin{aligned}
&\|\Phi z\|_{E^3_T} \\
&\le A(T) + B(T)\|a(t)\|_{C[0,T]}\|u(x,t)\|_{B^3_{2,T}} + C(T)\|u(x,t)\|_{B^3_{2,T}} + D(T)\|b(t)\|_{C[0,T]} \\
&\le A(T) + B(T)(A(T)+2)^2 + C(T)(A(T)+2) + D(T)(A(T)+2) \\
&< A(T) + 2,
\end{aligned} \tag{3.24}
$$

$$
\begin{aligned}
&\|\Phi z_1 - \Phi z_2\|_{E^3_T} \\
&\le B(T)R\Big(\|a_1(t) - a_2(t)\|_{C[0,T]} + \|u_1(x,t) - u_2(x,t)\|_{B^3_{2,T}}\Big) \\
&\quad + C(T)\|u_1(x,t) - u_2(x,t)\|_{B^3_{2,T}} + D(T)\|b_1(t) - b_2(t)\|_{C[0,T]}.
\end{aligned} \tag{3.25}
$$

Then by (3.22), from (3.24) and (3.25) it is clear that the operator Φ on the set $K = K_R$ satisfy the conditions of the contraction mapping principle. Therefore the operator Φ has a unique fixed point $\{z\} = \{u, a, b\}$, in the ball $K = K_R$, which is a solution of equation (3.23); i.e. in the sphere $K = K_R$ is the unique solution of the systems (3.8), (3.12), (3.13). Then the function $u(x,t)$, as an element of space $B^3_{2,T}$, is continuous and has continuous derivatives $u_x(x,t)$ and $u_{xx}(x,t)$ in D_T.

Next, from (3.4) it follows that $u'_{ik}(t)$ $(i=1,2;\ k=1,2\dots)$ is continuous on $[0,T]$ and consequently we have

$$\begin{aligned}
&\Big(\sum_{k=1}^{\infty}(\lambda_k\|u'_{ik}(t)\|_{C[0,T]})^2\Big)^{1/2}\\
&\le \|\frac{1}{c(t)}\|_{C[0,T]}\sqrt{2}\Big[\Big(\sum_{k=1}^{\infty}\big(\lambda_k^3\|u_{ik}(t)\|_{C[0,T]}\big)^2\Big)^{1/2}\\
&\quad+\big\|\|f_x(x,t)+a(t)u_x(x,t)+b(t)g_x(x,t)\|_{C[0,T]}\big\|_{L_2(0,1)}\Big]<+\infty \quad (i=1,2).
\end{aligned}$$

Hence we conclude that the function $u_t(x,t)$ is continuous in the domain D_T. Further, it is possible to verify that equation (2.1) and conditions (2.2), (2.3), (2.11), (2.12) are satisfied in the usual sense. Consequently, $\{u(x,t),a(t),b(t)\}$ is a solution of (2.1)–(2.3), (2.11), (2.12), and by Lemma 3.1 it is unique in the ball $K=K_R$. The proof is complete. $\square$

From Theorem 3.2 and Theorem 2.3, it follows directly the following assertion.

Theorem 3.3. *Suppose that all assumptions of Theorem 3.2, and the compatibility conditions* (2.9), (2.10) *hold. If*

$$\int_0^1 f(x,t)dx=0,\quad \int_0^1 g(x,t)dx=0 \quad (0\le t\le T)$$

then problem (2.1)–(2.5) *has a unique classical solution in the ball* $K=K_R$.

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