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NODAL PROPERTIES FOR p-LAPLACIAN SYSTEMS

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Abstract. We consider a system of differential equations involving the p-Laplacian. We prove the existence of oscillatory solutions with prescribed numbers of zeros, and show that the solutions satisfy the Dirichlet boundary conditions when the large parameters in the equations are suitable chosen. Our main tool in this work is a Prüfer-type substitution.

1. Introduction

The p-Laplacian operator $\Delta_p u = div(|\nabla u|^{p-2}\nabla u)$ attracts lots of attention and arises in various fields, such as non-Newtonian fluids and nonlinear diffusion problems. The quantity p is a characteristic of the medium. Media with $p > 2$ are called dilatant fluids and those with $p < 2$ are called pseudoplastics. If $p = 2$, they are Newtonian fluids. For the above topics, the readers can refer to [4, 5, 9, 12, 14, 16, 18] and their bibliographies.

In this note we consider the one-dimensional p-Laplacian system

$$\begin{gathered}-(|u'(x)|^{p-2}u'(x))' = (\lambda - w(x))|u(x)|^{p-2}u(x) - |v(x)|^{p-2}v(x),\\ -(|v'(x)|^{p-2}v'(x))' = \lambda|v(x)|^{p-2}v(x) + |u(x)|^{p-2}u(x),\end{gathered} \tag{1.1}$$

with the initial conditions

$$u(0) = v(0) = 0, \quad u'(0) = \lambda^{1/p}, \quad v'(0) = \mu. \tag{1.2}$$

Obviously, the first equation in (1.1) with $v \equiv 0$ can be regarded as a one-dimensional p-Laplacian eigenvalue problem. Recently some results related to p-Laplacian systems can be found, for example [8, 13, 21, 25]. As a result of that, many authors have studied the existence of positive solutions for p-Laplacian boundary value problems, by using topological degree theory, monotone iterative techniques, coincidence degree theory [7], and the Leggett-Williams fixed point theorem [10] or its variants; see [1, 2, 17, 22] and the references therein. Note that as $p = 2$, (1.1) reduces to

$$\begin{gathered}u''(x) + (\lambda - w(x))u(x) - v(x) = 0,\\ v''(x) + \lambda v(x) + u(x) = 0,\end{gathered} \tag{1.3}$$

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which is a linear coupled system. One can treat (1.3) as a steady state reaction diffusion model. Define $H(u,v) = \frac{(\lambda - w(x))}{2}u^2 - \frac{\lambda}{2}v^2 - uv$. Then

$$\frac{\partial H}{\partial u} = (\lambda - w(x))u - v, \quad -\frac{\partial H}{\partial v} = \lambda v + u.$$

Equation (1.3) can be viewed as a simple model of diffusion systems with skew-gradient structure (cf. [23, 24]).

In [21], we first considered a simple case related to a coupled p-Laplacian equations and developed the existence of oscillatory solutions with prescribed numbers of zeros. Motivated by [21], we study (1.1) and intend to extend the previous result. We will show that the solutions with prescribed numbers of zeros solve (1.1) and satisfy the Dirichlet boundary conditions when the large parameters λ are suitable chosen. That is, the Dirichlet boundary value problem is solvable. Essentially, the main method using in this work is a Prüfer-type substitution. Now for a solution $\{u, v\}$ we require u, u', v, and v' are absolutely continuous. Throughout the paper we assume the following conditions hold

- (A1) $p > 1$ and $\lambda, \mu > 0$;
- (A2) $w \in C(\mathbb{R})$.

Now, we have the following result which is concerned with some oscillation properties of solutions to (1.1)-(1.2).

Theorem 1.1. *Assume the conditions* (A1), (A2) *hold. Then there exists a sequence of positive parameters* $\{\lambda_k\}_{k=m}^{\infty}$ *for the one-dimensional coupled system* (1.1)-(1.2)*, where m is some positive integer, such that the corresponding solution $u(x;\lambda_k)$ has exactly $k-1$ zeros in* $(0,1)$ *for $k \geq m$. For sufficiently large λ_k, $u(x;\lambda_k)$ and $v(x;\lambda_k)$ have the same number of zeros in* $(0,1)$ *and satisfy the right-endpoint conditions $u(1;\lambda_k) = v(1;\lambda_k) = 0$ under a suitable choice of the initial parameter μ. That is, $\{u(x,\lambda_k), v(x,\lambda_k)\}$ solves* (1.1) *and satisfies the Dirichlet boundary conditions as $\mu = \lambda_k^{1/p} + o(1)$.*

2. Preliminaries and Proofs

Before to give the proof of Theorem 1.1, we first represent some elementary results for the solutions of the initial value problem (1.1)-(1.2). Here we need the following lemma to discuss the uniqueness of the local solution.

Lemma 2.1 ([19, p.180])**.** *Let $W \in C^1(I)$, $x_0 \in I$ and $W(x_0) = 0$, where I is a compact interval containing x_0. Denote by $\|W\|_x$ the maximum of W in the interval from x_0 to x. Then $|W'(x)| \leq K\|W\|_x$ in I implies*

$$W = 0 \quad \text{for } |x - x_0| \leq \frac{1}{K},\ x \in I. \tag{2.1}$$

In [21, Proposition 2.2], the proof of the uniqueness gave some inconsistencies. Here we refine the proof.

Proposition 2.2. *For any fixed $\lambda, \mu \in \mathbb{R}^+$, problem* (1.1)-(1.2) *has a unique pair of solutions which exist on an open interval I containing zero.*

Proof. System (1.1) can be written as

$$\begin{gathered} u' = |U|^{p^*-2}U, \\ U' = |v|^{p-2}v - (\lambda - w(x))|u|^{p-2}u, \\ v' = |V|^{p^*-2}V, \\ V' = -|u|^{p-2}u - \lambda|v|^{p-2}v, \end{gathered} \tag{2.2}$$

with $u(0) = v(0) = 0$, $U(0) = \lambda^{1/p^*}$ and $V(0) = \mu^{1/p^*}$, where $p^* = \frac{p}{p-1}$ is the conjugate exponent of p. Then the local existence of a solution is valid by the Cauchy-Peano theorem. For the uniqueness, we define $M \equiv \max\{\lambda^{1/p}, \mu\}$. By (1.2), we may find an interval I containing zero such that

$$\frac{M}{2}|x-0| < |u(x)|, \quad |v(x)| < 2M|x-0| \quad \text{for } x \in I. \tag{2.3}$$

Suppose that $\{u_1(x), v_1(x)\}$ and $\{u_2(x), v_2(x)\}$ are two distinct local solutions of (1.1)-(1.2). For $x \in I$ we can assume that

$$\|v_1 - v_2\|_x \le c_1\|u_1 - u_2\|_x \tag{2.4}$$

for some constant c_1, where the notation $\|\cdot\|_x$ is defined in the statement of Lemma 2.1 with $x_0 = 0$.

Then, for $x \in I$ one has

$$\begin{aligned} &|u_1'(x)|^{p-2}u_1'(x) - |u_2'(x)|^{p-2}u_2'(x) \\ &= \int_0^x [|v_1(t)|^{p-2}v_1(t) - |v_2(t)|^{p-2}v_2(t)]dt \\ &- \int_0^x (\lambda - w(t))[|u_1(t)|^{p-2}u_1(t) - |u_2(t)|^{p-2}u_2(t)]dt. \end{aligned}$$

The following is a version of the mean value theorem, that for a_1 and a_2 of the same sign

$$|a_1|^{p-2}a_1 - |a_2|^{p-2}a_2 = (p-1)(a_1 - a_2)|\bar{a}|^{p-2},$$

where $\bar{a}$ lies between a_1, a_2. So, for $x \in I$ one can obtain

$$\begin{aligned} &[u_1'(x) - u_2'(x)]|\bar{u'}|^{p-2} \\ &= \int_0^x [v_1(t) - v_2(t)]|\bar{v}|^{p-2}dt - \int_0^x (\lambda - w(t))[u_1(t) - u_2(t)]|\bar{u}|^{p-2}dt, \end{aligned}$$

for some $\bar{u'}$, $\bar{v}$ and $\bar{u}$. By (2.3)-(2.4), for $x \in I$ one can get

$$c_2|u_1'(x) - u_2'(x)| \le \Big((c_1 + \lambda + \|w\|_x)\|u_1 - u_2\|_x \int_0^x (2M)^{p-2}t^{p-2}dt\Big),$$

where c_2 is some constant. Now, set $W(x) = u_1(x) - u_2(x)$. By Lemma 2.1, one can obtain that $W(x) = 0$ near $x = 0$. Applying the similar arguments on $v_1' - v_2'$, one can prove the uniqueness for $v(x)$. $\square$

Now we introduce a Prüfer-type substitution for the solution of (1.1)-(1.2) by using the generalized sine function $S_p(x)$. The generalized sine function S_p has been well studied in the literature (see Lindqvist [11] or [3, 6, 15] with a minor difference in setting). Here we outline some properties for the readers' convenience. The function S_p satisfies

$$|S_p'(x)|^p + \frac{|S_p(x)|^p}{p-1} = 1, \tag{2.5}$$

$$(|S_p'|^{p-2}S_p')' + |S_p|^{p-2}S_p = 0. \tag{2.6}$$

Moreover,

$$\pi_p \equiv 2\int_0^{(p-1)^{1/p}} \frac{dt}{(1-\frac{t^p}{p-1})^{1/p}} = \frac{2(p-1)^{1/p}\pi}{p\sin(\pi/p)}$$

is the first zero of S_p in the positive real axis. Similarly, one has

$$S_p(\frac{\pi_p}{2}) = \sqrt[p]{p-1},\ S_p'(0) = 1 \quad \text{and} \quad S_p'(\frac{\pi_p}{2}) = 0.$$

With the help of the generalized sine function and denoting $' = \frac{d}{dx}$, we introduce phase-plane coordinates $R,\ r > 0$ and $\theta,\ \phi$ for the solution $\{u(x;\lambda), v(x;\lambda)\}$ as follows:

$$u(x;\lambda) = R(x;\lambda)S_p(\lambda^{1/p}\theta(x;\lambda)), \quad u'(x;\lambda) = \lambda^{1/p}R(x;\lambda)S_p'(\lambda^{1/p}\theta(x;\lambda)), \tag{2.7}$$

$$v(x;\lambda) = r(x;\lambda)S_p(\lambda^{1/p}\phi(x;\lambda)), \quad v'(x;\lambda) = \lambda^{1/p}r(x;\lambda)S_p'(\lambda^{1/p}\phi(x;\lambda)). \tag{2.8}$$

with $\theta(0;\lambda) = \phi(0;\lambda) = 0$. Then

$$\lambda R(x;\lambda)^p = \frac{\lambda|u(x;\lambda)|^p}{p-1} + |u'(x;\lambda)|^p, \quad \lambda r(x;\lambda)^p = \frac{\lambda|v(x;\lambda)|^p}{p-1} + |v'(x;\lambda)|^p \tag{2.9}$$

with $R(0;\lambda) = 1$ and $r(0;\lambda) = \frac{\mu}{\lambda^{1/p}}$. Moreover,

$$\frac{|u'(x;\lambda)|^{p-2}u'(x;\lambda)}{|u(x;\lambda)|^{p-2}u(x;\lambda)} = \frac{\lambda^{\frac{p-1}{p}}|S_p'(\lambda^{1/p}\theta(x;\lambda))|^{p-2}S_p'(\lambda^{1/p}\theta(x;\lambda))}{|S_p(\lambda^{1/p}\theta(x;\lambda))|^{p-2}S_p(\lambda^{1/p}\theta(x;\lambda))},$$

$$\frac{|v'(x;\lambda)|^{p-2}v'(x;\lambda)}{|v(x;\lambda)|^{p-2}v(x;\lambda)} = \frac{\lambda^{\frac{p-1}{p}}|S_p'(\lambda^{1/p}\phi(x;\lambda))|^{p-2}S_p'(\lambda^{1/p}\phi(x;\lambda))}{|S_p(\lambda^{1/p}\phi(x;\lambda))|^{p-2}S_p(\lambda^{1/p}\phi(x;\lambda))}.$$

Differentiating both sides of the above two identities with respect to x and employing (1.1), one can obtain the following result.

Lemma 2.3. *For the sake of simplicity, write $\theta(x) = \theta(x;\lambda)$, $\phi(x) = \phi(x;\lambda)$, $R(x) = R(x;\lambda)$ and $r(x) = r(x;\lambda)$. Then, for $x \in I$,*

$$\begin{aligned}\theta'(x) &= 1 - \frac{w(x)}{(p-1)\lambda}|S_p(\lambda^{1/p}\theta(x))|^p \\ &\quad - \frac{1}{(p-1)\lambda}\big[\frac{r(x)}{R(x)}\big]^{p-1}|S_p(\lambda^{1/p}\phi(x))|^{p-2}S_p(\lambda^{1/p}\phi(x))S_p(\lambda^{1/p}\theta(x)),\end{aligned} \tag{2.10}$$

$$\begin{aligned}&\phi'(x) \\ &= 1 + \frac{1}{(p-1)\lambda}\big[\frac{R(x)}{r(x)}\big]^{p-1}|S_p(\lambda^{1/p}\theta(x))|^{p-2}S_p(\lambda^{1/p}\theta(x))S_p(\lambda^{1/p}\phi(x)),\end{aligned} \tag{2.11}$$

$$\begin{aligned}&R'(x) \\ &= \frac{w(x)R(x)}{(p-1)\lambda^{1-1/p}}|S_p(\lambda^{1/p}\theta(x))|^{p-2}S_p(\lambda^{1/p}\theta(x))S_p'(\lambda^{1/p}\theta(x)) \\ &\quad + \frac{1}{(p-1)\lambda^{1-1/p}}\big[\frac{r(x)^{p-1}}{R(x)^{p-2}}\big]|S_p(\lambda^{1/p}\phi(x))|^{p-2}S_p(\lambda^{1/p}\phi(x))S_p'(\lambda^{1/p}\theta(x)),\end{aligned} \tag{2.12}$$

$$
\begin{aligned}
&r'(x)\\
&= -\frac{1}{(p-1)\lambda^{1-1/p}}\big[\frac{R(x)^{p-1}}{r(x)^{p-2}}\big]|S_p(\lambda^{1/p}\theta(x))|^{p-2}S_p(\lambda^{1/p}\theta(x))S_p'(\lambda^{1/p}\phi(x)).
\end{aligned} \tag{2.13}
$$

Applying Lemma 2.3, we find that $\{u(x;\lambda), v(x;\lambda)\}$ is a solution of (1.1)-(1.2) if and only if $\{\theta(x;\lambda), R(x;\lambda), \phi(x;\lambda), r(x;\lambda)\}$ is a solution of (2.10)-(2.13) coupled with the following conditions

$$
\theta(0;\lambda) = \phi(0;\lambda) = 0, \quad \text{and} \quad R(0;\lambda) = 1, \quad r(0;\lambda) = \frac{\mu}{\lambda^{1/p}}. \tag{2.14}
$$

Next we derive some properties for the radial functions $R(x;\lambda)$ and $r(x;\lambda)$.

Lemma 2.4. *Write $R(x) = R(x;\lambda)$ and $r(x) = r(x;\lambda)$.*

(i) *For $x \in I$, the radial functions satisfy*

$$
\begin{aligned}
\Big(1 + \frac{\mu^{p-1}}{\lambda^{\frac{p-1}{p}}}\Big)\exp[-c_1\lambda^{\frac{1-p}{p}}x] &\le R(x)^{p-1} + r(x)^{p-1}\\
&\le \Big(1 + \frac{\mu^{p-1}}{\lambda^{\frac{p-1}{p}}}\Big)\exp[c_1\lambda^{\frac{1-p}{p}}x],
\end{aligned} \tag{2.15}
$$

where c_1 is a positive constant.

(ii) *For fixed $x \in I$ and sufficiently large λ, we can choose $\mu = \lambda^{1/p}$ and obtain that*

$$
\frac{r(x)}{R(x)} = 1 + o(1). \tag{2.16}
$$

Proof. (i) By (2.12)-(2.13), there exists some positive constant c_1 such that

$$
\begin{aligned}
-c_1\lambda^{\frac{1-p}{p}}[R(x)^{p-1} + r(x)^{p-1}] &\le (p-1)\left[R(x)^{p-2}R'(x) + r(x)^{p-2}r'(x)\right]\\
&\le c_1\lambda^{\frac{1-p}{p}}[R(x)^{p-1} + r(x)^{p-1}].
\end{aligned}
$$

Solving the above differential inequality and applying the initial condition (2.14), we obtain the inequality (2.15).

(ii) As in (i), there exists a positive constant c_2 such that

$$
\frac{R(x)r'(x) - r(x)R'(x)}{R(x)^2} \le c_2\lambda^{\frac{1-p}{p}}\big[\frac{R(x)^{p-2}}{r(x)^{p-2}} + \frac{r(x)}{R(x)} + \frac{r(x)^p}{R(x)^p}\big].
$$

Letting $y(x) = \frac{r(x)}{R(x)}$, we have

$$
y'(x) \le c_2\lambda^{\frac{1-p}{p}}[y(x)^{2-p} + y(x) + y(x)^p].
$$

Note that

$$
\frac{dy}{dx} \le c_2\lambda^{\frac{1-p}{p}}\big(\frac{1 + y^{p-1} + y^{2p-2}}{y^{p-2}}\big), \text{i.e.,} \quad \frac{y^{p-2}dy}{1 + y^{p-1} + y^{2p-2}} \ leqc_2\lambda^{\frac{1-p}{p}}dx.
$$

Letting $z = y^{p-1}$ and integrating the above inequality, we obtain

$$
\frac{2}{\sqrt{3}}\Big[\tan^{-1}\frac{2z+1}{\sqrt{3}}\Big]_{y(0)}^{y(x)^{p-1}} \le (p-1)c_2\lambda^{\frac{1-p}{p}}x.
$$

i.e.,

$$
\frac{2}{\sqrt{3}}\Big[\tan^{-1}\frac{2z+1}{\sqrt{3}}\Big]_{y(0)}^{y(x)^{p-1}} = o(1).
$$

Then,

$$0 < \tan^{-1}\frac{2y(x)^{p-1}+1}{\sqrt{3}} = \frac{\pi}{3} + o(1).$$

So $y(x)^{p-1} = 1 + o(1)$ as λ is sufficiently large. This completes the proof. $\square$

Remark 2.5. From the proof of Lemma 2.4 (ii), one can obtain the boundedness of $\frac{r(x)}{R(x)}$ and $\frac{R(x)}{r(x)}$. Then, the right-hand sides of (2.10)-(2.13) satisfy the generalized Lipschitz continuous in θ, ϕ, R and r respectively. Thus, the existence of the unique absolutely continuous solutions is valid. The above can be referred to [20] (pp.121-123).

Remark 2.6. For sufficiently large λ, one can choose μ closed to $\lambda^{1/p}$; that is, $\mu = \lambda^{1/p} + o(1)$, and the asymptotic estimate (2.16) is still valid.

From Proposition 2.2 and Lemma 2.4 (i), we have the following result.

Proposition 2.7. *For fixed $\lambda, \mu > 0$, the uniquely local solution $\{u(x;\lambda), v(x;\lambda)\}$ can be extended to the whole real axis.*

Now we derive some properties related to the phase functions $\theta(x;\lambda)$ and $\phi(x;\lambda)$.

Lemma 2.8. *For $\lambda > 0$, the phase functions $\theta(x;\lambda)$ and $\phi(x;\lambda)$ satisfy the following properties.*

(i) *$\theta(\cdot;\lambda)$ and $\phi(\cdot;\lambda)$ are continuous in λ and satisfy $\theta(0;\lambda) = \phi(0;\lambda) = 0$.*
(ii) *If $\lambda^{1/p}\theta(x_n;\lambda) = n\pi_p$ for some $x_n \in (0,1)$, then $\lambda^{1/p}\theta(x;\lambda) > n\pi_p$ for every $x > x_n$.*
(iii) *For sufficiently large λ,*
$$\lambda^{1/p}\theta(1;\lambda) = \lambda^{1/p} + O\Big(\frac{1}{\lambda^{1-\frac{1}{p}}}\Big). \tag{2.17}$$
Moreover, $\lambda^{1/p}\phi(1;\lambda)$ has the same estimate as (2.17).
(iv) *For sufficiently large λ, a suitable initial parameter $\mu = \lambda^{1/p} + o(1)$ can be chosen such that $\theta(1;\lambda) = \phi(1;\lambda)$.*

Proof. Item (i) is valid by (2.10)-(2.11) and (2.14). For (ii), if $\lambda^{1/p}\theta(x_n;\lambda) = n\pi_p$ for some $x_n \in (0,1)$, then by (2.10) and Lemma 2.4 (ii), we have

$$\theta'(x_n;\lambda) = 1 > 0. \tag{2.18}$$

For (iii), integrating (2.10) and (2.11) over $[0,1]$ and applying (i) and Lemma 2.4 (ii), one can obtain the asymptotic estimates (2.17) as λ is sufficiently large. This proves (iii). Besides, write $\theta'(x;\lambda) = F(x;\lambda;\theta;\phi)$ and $\phi'(x;\lambda) = H(x;\lambda;\theta;\phi)$. Then, for $x \in [0,1]$,

$$\begin{aligned}
&\theta(x;\lambda) - \phi(x;\lambda)\\
&= \int_0^x (F(t;\lambda;\theta;\phi) - H(t;\lambda;\theta;\phi))dt\\
&= \int_0^x \Big[F(t;\lambda;\theta;\phi) - F(t;\lambda;\theta;\theta) + F(t;\lambda;\theta;\theta) - H(t;\lambda;\theta;\theta)\\
&\quad + H(t;\lambda;\theta;\theta) - H(t;\lambda;\theta;\phi)\Big]dt\\
&= \int_0^x \frac{\partial}{\partial\phi}F(t;\lambda;\theta;\xi)[\phi(t;\lambda) - \theta(t;\lambda)]dt - \frac{1}{(p-1)\lambda}\int_0^x w(t)|S_p(\lambda^{1/p}\theta(t;\lambda))|^p dt
\end{aligned}$$

$$
\begin{aligned}
&- \frac{1}{(p-1)\lambda} \int_0^x \Big(\big[\frac{r(x)}{R(x)}\big]^{p-1} + \big[\frac{R(x)}{r(x)}\big]^{p-1} \Big) |S_p(\lambda^{1/p}\theta(t;\lambda))|^p dt \\
&+ \int_0^x \frac{\partial}{\partial \phi} H(t;\lambda;\theta;\eta)[\phi(t;\lambda) - \theta(t;\lambda)] dt,
\end{aligned}
$$

where $\xi(t;\lambda)$ and $\eta(t;\lambda)$ are between $\phi(t;\lambda)$ and $\theta(t;\lambda)$. Note that $|\frac{\partial}{\partial\phi}F(t;\lambda;\theta;\xi)|$ and $|\frac{\partial}{\partial\phi}H(t;\lambda;\theta;\eta)|$ are uniformly bounded by some constant K for all $t \in [0,1]$. Then, by Lemma 2.4 (ii) for any $\delta > 0$ there exists sufficiently large λ such that

$$
|\theta(x;\lambda) - \phi(x;\lambda)| \le \delta + \int_0^x 2K|\phi(t;\lambda) - \theta(t;\lambda)| dt.
$$

By the Gronwall inequality, we obtain

$$
|\theta(x;\lambda) - \phi(x;\lambda)| \le \delta e^{2K}. \tag{2.19}
$$

By (2.10)-(2.11), Remark 2.6 and (2.19), one can choose suitable μ satisfying $\mu = \lambda^{1/p} + o(1)$ such that the two Prüfer phases are identical at the right end-point as λ is sufficiently large. That is, $\theta(1;\lambda) = \phi(1;\lambda)$. Now the proof of (iv) is complete. $\square$

Proof of Theorem 1.1. By Lemma 2.8 (i) and (iii), the modified phase $\lambda^{1/p}\theta(1;\lambda)$ tends to infinity as $\lambda \to \infty$. Hence, for every sufficiently large $k \in \mathbb{N}$ there exists $\lambda_k > 0$ satisfies $\lambda_k^{1/p}\theta(1;\lambda_k) = k\pi_p$. This implies that there exists $m \in \mathbb{N}$ such that $\lambda_k^{1/p}\theta(1;\lambda_k) = k\pi_p$ for every $k \ge m$. Furthermore, the remaining results are valid by Lemma 2.8 (iii) and (iv). $\square$

References

[1] R. P. Agarwal, D. Cao, H. Lu, D. O'Regan; *Existence and mulplicity of positive solutions for singular semipositone p-Laplacian equations*, Canad. J. Math. **58** (2006), 449–475.

[2] P. Amster, P. De Napoli; *Landesman-Lazer type conditions for a system of p-Laplacian like operators*, J. Math. Anal. Appl., **326** (2007), 1236–1243.

[3] P. Binding, P. Drabek; *Sturm–Liouville theory for the p-Laplacian*, Studia Scientiarum Mathematicarum Hungarica, **40** (2003), 373-396.

[4] J. I. Diaz; *Nonlinear Partial Differential Equations and Free Boundaries*, vol. I. Elliptic Equations, Res. Notes Math., vol. 106, Pitman, Boston, MA, 1985.

[5] J. I. Diaz; *Qualitative study of nonlinear parabolic equations: an introduction*, Extracta Math., **16** (2001), 303-341.

[6] A. Elbert; *A half-linear second order differential equation*, Colloqia mathematica Societatis Jónos Bolyai, **30**, Qualitiative Theory of Differential Equations, Szeged (Hungary) (1979), 153-180.

[7] R. E. Gaines, J. L. Mawhin; *Coincidence Degree and Nonlinear Differential Equations*, Springer-Verlag, Berlin, 1977.

[8] D. D. Hai, H. Wang; *Nontrivial solutions for p-Laplacian systems*, J. Math. Anal. Appl., **330** (2007), 186–194.

[9] O. A. Ladyzhenskaya; *The Mathematical Theory of Viscous Incompressible Flow*, 2nd ed., Gordon and Breach (New York), 1969.

[10] R. W. Leggett, L. R. Williams; *Multiple positive fixed points of a nonlinear operators on ordered Banach spaces*, Indiana Univ. Math. J., **28** (1979), 673–688.

[11] P. Lindqvist; *Some remarkable sine and cosine functions*, Ricerche Mat., **44** (1995), 269-290.

[12] J. L. Lions; *Quelques methodes de resolution des problemes aux limites non lineaires*, Dunod (Paris) 1969.

[13] D. Lü; *Multiple solutions for p-Laplacian systems with critical homogeneous nonlinearity*, Boundary Value Problems (2012), 2012:27.

[14] M. C. Pelissier, M. L. Reynaud; *Etude d?n modele mathematique d?coulement de glacier*, C. R. Acad. Sci. Paris Ser. I Math., **279** (1974), 531-534.

[15] W. Reichel, W. Walter; *Sturm-Liouville type problems for the p-Laplacian under asymptotic non-resonance conditions*, J. Differential Equations **156** (1999), 50-70.
[16] R. E. Showalter, N. J. Walkington; *Diffusion of fluid in a fissured medium with microstructure*, SIAM J. Math. Anal. **22** (1991), 1702-1722.
[17] H. Sun, W. Li; *Existence theory for positive solutions to one-dimensional p-Laplacian boundary value problems*, J. Differential Equations, **240** (2007), 217–248.
[18] S. Takeuchi; *Generalized Jacobian elliptic functions and their application to bifurcation problems associated with p-Laplacian*, J. Math. Anal. Appl. **385** (2012), 24-35.
[19] W. Walter; *Sturm-Liouville theory for the radial Δ_p-operator*, Math. Z. **227** (1998), 175-185.
[20] W. Walter; *Ordinary Differential Equations*, Springer-Verlag, 1998.
[21] W. C. Wang; *Prüfer substitutions on a coupled system involving p-Laplacian*, Electronic J. Differential Equations, **2013** (213) (2013), 1-9.
[22] Z. Wang, J. Zhang; *Positive solutions for one-dimensional p-Laplacian boundary value problems with dependence on the first order derivative*, J. Math. Anal. Appl., **314** (2006), 618–630.
[23] E. Yanagida; *Mini-maximizers for reaction-diffusion systems with skew-gradient structure*, J. Differential Equations **179** (2002), 311-335.
[24] E. Yanagida; *Standing pulse solutions in reaction-diffusion systems with skew-gradient structure*, J. Dynam. Differential Equations **14** (2002), 189-205.
[25] Z. Yang; *Positive solutions for a system of p-Laplacian boundary value problems*, Computers and Mathematics with Applications **62** (2011), 4429-4438.

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