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NONEXISTENCE OF GLOBAL SOLUTIONS TO THE SYSTEM OF SEMILINEAR PARABOLIC EQUATIONS WITH BIHARMONIC OPERATOR AND SINGULAR POTENTIAL

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Abstract. In the domain $Q'_R = \{x : |x| > R\} \times (0, +\infty)$ we consider the problem

$$\frac{\partial u_1}{\partial t} + \Delta^2 u_1 - \frac{C_1}{|x|^4} u_1 = |x|^{\sigma_1} |u_2|^{q_1}, \quad u_1|_{t=0} = u_{10}(x) \geq 0,$$
$$\frac{\partial u_2}{\partial t} + \Delta^2 u_2 - \frac{C_2}{|x|^4} u_2 = |x|^{\sigma_2} |u_1|^{q_2}, \quad u_2|_{t=0} = u_{20}(x) \geq 0,$$
$$\int_0^\infty \int_{\partial B_R} u_i \, ds \, dt \geq 0, \quad \int_0^\infty \int_{\partial B_R} \Delta u_i \, ds \, dt \leq 0,$$

where $\sigma_i \in \mathbb{R}$, $q_i > 1$, $0 \leq C_i < (\frac{n(n-4)}{4})^2$, $i = 1, 2$. Sufficient condition for the nonexistence of global solutions is obtained.The proof is based on the method of test functions.

1. Introduction

Let us introduce the following notation: $x = (x_1, \dots, x_n) \in \mathbb{R}^n$, $n > 4$, $r = |x| = \sqrt{x_1^2 + \dots + x_n^2}$, $B_R = \{x; |x| < R\}$, $B'_R = \{x; |x| > R\}$, $B_{R_1,R_2} = \{x; R_1 < |x| < R_2\}$, $Q_R = B_R \times (0; +\infty)$, $Q'_R = B'_R \times (0; +\infty)$, $\partial B_R = \{x; |x| = R\}$, $\nabla u = (\frac{\partial u}{\partial x_1}, \dots, \frac{\partial u}{\partial x_n})$, $C^{4,1}_{x,t}(Q'_R)$ is the set of functions that are four times continuously differentiable with respect to x and continuously differentiable with respect to t in Q'_R.

In the domain Q'_R we consider the system of equations

$$\begin{gathered} \frac{\partial u_1}{\partial t} + \Delta^2 u_1 - \frac{C_1}{|x|^4} u_1 = |x|^{\sigma_1} |u_2|^{q_1} \\ \frac{\partial u_2}{\partial t} + \Delta^2 u_2 - \frac{C_2}{|x|^4} u_2 = |x|^{\sigma_2} |u_1|^{q_2}, \end{gathered} \tag{1.1}$$

with the initial condition

$$u_i|_{t=0} = u_{i0}(x) \geq 0, \tag{1.2}$$

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and the conditions

$$\int_0^\infty \int_{\partial B_R} u_i \,dx\,dt \geq 0, \quad \int_0^\infty \int_{\partial B_R} \Delta u_i \,dx\,dt \leq 0, \tag{1.3}$$

where $n > 4$, $q_i > 1$, $\sigma_i \in \mathbb{R}$, $0 \leq C_i < (\frac{n(n-4)}{4})^2$, $u_{i0}(x) \in C(B'_R)$, $\Delta^2 u = \Delta(\Delta u)$, $\Delta u = \sum_{k=1}^{n} \frac{\partial^2 u}{\partial x_k^2}$, $i = 1, 2$.

We will study the nonexistence of a global solution of problem (1.1)-(1.3). By a global solution of problem (1.1)-(1.3) we understand a pair of functions (u_1, u_2) such that $u_1(x,t), u_2(x,t) \in C^{4,1}_{x,t}(Q'_R) \cap C^{3,0}_{x,t}(\overline{B'_R} \times (0;+\infty)) \cap C(B'_R \times [0;+\infty))$ and satisfy the system (1.1) at every point of Q'_R, the initial condition (1.2) and conditions (1.3).

The problems of nonexistence of global solutions for differential equations and inequalities play a key role in theory and applications. Therefore, they have a constant attention of mathematicians, and a great number of works were devoted to them [1, 2, 3, 4, 9, 12, 13, 16, 21, 22]. A survey of such results can be found in the monograph [17].

In the classical paper [7], Fujita considered the following initial value problem

$$\begin{gathered} \frac{\partial u}{\partial t} = \Delta u + u^q, \quad (x,t) \in \mathbb{R}^n \times (0, +\infty), \\ u|_{t=0} = u_0(x), \quad x \in \mathbb{R}^n, \end{gathered} \tag{1.4}$$

and proved that positive global solutions of problem (1.4) do not exist for $1 < q < q^* = 1 + \frac{2}{n}$. If $q > q^*$, then there are positive global solutions for small $u_0(x)$. The case $q = q^*$ was investigated in [10, 11] and it was proved that in this case there are no positive global solutions. Pinsky [19] showed the existence and nonexistence of global solutions in $\mathbb{R}^n \times (0, +\infty)$ to the equation $u_t - \Delta u = a(x)u^q$, where $q > 1$ and $a(x)$ behaves like $|x|^\sigma$ with $\sigma > -2$ for large $|x|$. The results of Fujita's work [7] aroused great interest in the problem of the nonexistence of global solutions, and they were expanded in several directions. For example, various bounded and unbounded domains were considered instead of R^n, as well as more general operators than the Laplace operator including different type nonlinear operators were considered (for more comprehensive treatment of such problems, see [14, 17, 20] and references there in).

Another may of extending of Fujita's result is to investigate a system of Fujita-type reaction-diffusion equations, and this is what we do here. For example, many authors have investigated the existence and nonexistence of global and local solutions to the initial value problem

$$\begin{gathered} \frac{\partial u}{\partial t} = \alpha_1 \Delta u + t^{k_1} |x|^{\sigma_1} v^{q_1}, \quad u|_{t=0} = u_0(x) \geq 0 \\ \frac{\partial v}{\partial t} = \alpha_2 \Delta v + t^{k_2} |x|^{\sigma_2} u^{q_2}, \quad v|_{t=0} = v_0(x) \geq 0\,. \end{gathered} \tag{1.5}$$

Escobedo and Herrero [5] considered problem (1.5) on $\mathbb{R}^n \times (0, +\infty)$ with $\alpha_i = 1, k_i = 0, \sigma_i = 0$, $q_i > 0, q_1 q_2 > 1, i = 1, 2$ and proved that if $\max(\frac{q_1+1}{q_1 q_2 - 1}, \frac{q_2+1}{q_1 q_2 - 1}) \geq \frac{n}{2}$, then for any nontrivial initial functions there are no nonnegative global solutions. Fila, Levine and Uda [6] considered problem(1.5) on $\mathbb{R}^n \times (0, +\infty)$ with $0 \leq \alpha_1 \leq 1$, $\alpha_2 = 1$, $k_i = 0$, $\sigma_i = 0$, $q_i \geq 0$, $q_1 q_2 > 1$, $i = 1, 2$ and studied the existence of nonnegative global and non-global solutions. In the case $\alpha_i = 1$, $k_i = 0$, $i = 1, 2$, Mochizuki and Huang [18] proved the existence and nonexistence theorems for global solutions and studied asymptotic behavior of the global solution of problem (1.5) on $\mathbb{R}^n \times (0,+\infty)$. Caristi [8] considered problem (1.5) for $k_i, \sigma_i \in \mathbb{R}$, $q_1, q_2 > 1$ on $\mathbb{R}^n \times (0,+\infty)$, and nonexistence of global solution is studied. Levine [15] studied nonnegative solutions of the initial boundary value problem for the system of equations in (1.5) for $\alpha_i = 1$, $k_i = 0$, $\sigma_i = 0$, $i = 1,2$ in domain $D \times (0,+\infty)$, where D is a cone or the exterior of a bounded domain. In the present paper we consider a system of semilinear parabolic equations with biharmonic operator and singular potential in the exterior domain Q'_R. Using the technique of test functions worked out by Mitidieri and Pohozaev in [16],[17], we find a sufficient condition for nonexistence of global nontrivial solution.

2. Main result and its proof

The avoid complications, we introduce the following denotation:

$$
\begin{gathered}
D_i = \sqrt{(n-2)^2 + C_i}, \quad \lambda_i^{\pm} = \sqrt{\Big(\frac{n-2}{2}\Big)^2 + 1 \pm D_i}, \\
\mu_i = \frac{1}{2}\Big(1 + \frac{D_i - \lambda_i^+}{\lambda_i^-}\Big), \quad \overline{\mu}_i = \frac{1}{2}\Big(1 - \frac{D_i - \lambda_i^+}{\lambda_i^-}\Big), \\
\alpha_1 = \frac{\lambda_1^- + \sigma_1 + \frac{n+4}{2}}{\lambda_2^- + \frac{n+4}{2}}, \quad \alpha_2 = \frac{\lambda_2^- + \sigma_2 + \frac{n+4}{2}}{\lambda_1^- + \frac{n+4}{2}}, \\
\beta_1 = \frac{\lambda_1^- + \sigma_1 + 4 + \frac{n+4}{2}}{\lambda_2^- + \frac{n-4}{2}}, \quad \beta_2 = \frac{\lambda_2^- + \sigma_2 + 4 + \frac{n+4}{2}}{\lambda_1^- + \frac{n-4}{2}}, \\
\theta_1 = \frac{\sigma_1 + 4 + q_1(\sigma_2 + 4)}{q_1 q_2 - 1} - \lambda_1^- - \frac{n+4}{2}, \\
\theta_2 = \frac{\sigma_2 + 4 + q_2(\sigma_1 + 4)}{q_1 q_2 - 1} - \lambda_2^- - \frac{n+4}{2}, \quad i = 1,2.
\end{gathered}
$$

Let us consider the functions

$$
\xi_i(x) = \mu_i |x|^{-\frac{n-4}{2} + \lambda_i^-} + \overline{\mu}_i |x|^{-\frac{n-4}{2} - \lambda_i^-} - |x|^{-\frac{n-4}{2} - \lambda_i^+}, \quad i = 1,2.
$$

It is easy to verify that $\xi_i(x)$ are the solution of the equation

$$
\Delta^2 u - \frac{C_i}{|x|^4} u = 0 \tag{2.1}
$$

in $R^n \backslash \{0\}$ and for $|x| = 1$,

$$
\xi_i = 0, \quad \frac{\partial \xi_i}{\partial r} = D_i \ge 0, \quad \Delta \xi_i = 0, \quad \frac{\partial(\Delta \xi_i)}{\partial r} \le 0. \tag{2.2}
$$

The main result of this paper reads as follows.

Theorem 2.1. *Assume that $n > 4$, $\beta_i > 1$, $0 \le C_i < (\frac{n(n-4)}{4})^2$ and $1 < q_i \le \beta_i$, $\max(\theta_1, \theta_2) \ge 0$, $(q_1, q_2) \ne (\alpha_1, \beta_2)$ in case $\alpha_1 > 1$, $(q_1, q_2) \ne (\beta_1, \alpha_2)$ in case $\alpha_2 > 1$, $i = 1,2$. Then there is no nontrivial global solution of* (1.1)-(1.3).

Proof. For simplicity we take $R = 1$. Assume that $(u_1(x,t), u_2(x,t))$ is a nontrivial solution of (1.1)-(1.3). Let us consider the following two functions:

$$\varphi(x) = \begin{cases} 1, & \text{for } 1 \le |x| \le \rho, \\ (2 - \frac{|x|}{\rho})^\kappa, & \text{for } \rho \le |x| \le 2\rho \\ 0, & \text{for } |x| \ge 2\rho, \end{cases}$$

$$T_\rho(t) = \begin{cases} 1, & \text{for } 0 \le t \le \rho^4, \\ (2 - \rho^{-4} t)^\gamma, & \text{for } \rho^4 \le t \le 2\rho^4 \\ 0, & \text{for } t \ge 2\rho^4, \end{cases}$$

where κ, γ are large positive, and κ is such number that for $|x| = 2\rho$,

$$\varphi = \frac{\partial \varphi}{\partial r} = \frac{\partial^2 \varphi}{\partial r^2} = \frac{\partial^3 \varphi}{\partial r^3} = 0. \tag{2.3}$$

We multiply the first equation by $\psi_1(x,t) = T_\rho(t)\xi_1(x)\varphi(x)$, the second by $\psi_2(x,t) = T_\rho(t)\xi_2(x)\varphi(x)$ and integrate over Q_1'. After integration by parts, we obtain the following relations

$$\begin{aligned} &\iint_{Q_1'} |x|^{\sigma_i} |u_j|^{q_i} T_\rho(t) \xi_i(x) \varphi(x)\, dx\, dt \\ &= -\iint_{Q_1'} u_i \xi_i \varphi \frac{dT_\rho}{dt}\, dx\, dt + \iint_{Q_1'} u_i T_\rho \Delta^2(\xi_i \varphi)\, dx\, dt \\ &\quad - \iint_{Q_1'} \frac{C_i}{|x|^4} u_i T_\rho \xi_i \varphi\, dx\, dt - \int_{B_1'} u_{i0}(x) \xi_i(x) \varphi(x) dx \\ &\quad + \int_0^\infty T_\rho(t) dt \Big[\int_{\partial B_{1,2\rho}} \frac{\partial(\Delta u_i)}{\partial \nu} \xi_i \varphi ds - \int_{\partial B_{1,2\rho}} \Delta u_i \frac{\partial(\xi_i \varphi)}{\partial \nu} ds \\ &\quad + \int_{\partial B_{1,2\rho}} \frac{\partial u_i}{\partial \nu} \Delta(\xi_i \varphi) ds - \int_{\partial B_{1,2\rho}} u_i \frac{\partial}{\partial \nu} \Delta(\xi_i \varphi) ds \Big], \end{aligned} \tag{2.4}$$

where ν is a unit vector of external normal to $\partial B_{1,2}\rho$, $i, j = 1, 2$, $i \ne j$.

In order not to be repeated, in what follows, we will take into account that $i, j = 1, 2$, $i \ne j$ and in all expressions will write the same constant C, but in fact, in each expression C indicates different constants.

Using (2.2), (2.3), we estimate the integrals in square brackets in (2.4).

$$\int_{\partial B_{1,2\rho}} \frac{\partial(\Delta u_i)}{\partial \nu} \xi_i \varphi ds = 0,$$

$$\begin{aligned} -\int_{\partial B_{1,2\rho}} \Delta u_i \frac{\partial(\xi_i \varphi)}{\partial \nu} ds &= -\int_{|x|=1} \Delta u_i \frac{\partial(\xi_i \varphi)}{\partial \nu} ds \\ &= \int_{|x|=1} \Delta u_i (\frac{\partial \xi_i}{\partial r} \varphi + \xi_i \frac{\partial \varphi}{\partial r}) ds \\ &= \int_{|x|=1} \Delta u_i \frac{\partial \xi_i}{\partial r} ds \le 0, \end{aligned}$$

$$
\begin{aligned}
\int_{\partial B_{1,2\rho}} \frac{\partial u_i}{\partial \nu}\Delta(\xi_i\varphi)ds &= \int\limits_{\partial B_{1,2\rho}} \frac{\partial u_i}{\partial \nu}(\Delta\xi_i\varphi + 2(\nabla\xi_i,\nabla\varphi) + \xi_i\Delta\varphi)ds \\
&= -\int_{|x|=1} \frac{\partial u_i}{\partial r}\Delta\xi_i ds = 0,
\end{aligned}
$$

$$
\begin{aligned}
-\int_{\partial B_{1,2\rho}} u_i\frac{\partial}{\partial \nu}(\Delta(\xi_i\varphi))ds &= -\int_{|x|=1} u_i\frac{\partial}{\partial \nu}(\Delta\xi_i\varphi)ds \\
&= \int_{|x|=1} u_i\frac{\partial(\Delta\xi_i)}{\partial r}ds \le 0.
\end{aligned}
$$

Since

$$
\int_{B_1'} u_{i0}(x)\xi_i(x)\varphi(x)dx \ge 0, \quad \text{and} \quad \int_0^\infty T_\rho(t)dt \ge 0,
$$

taking into account that ξ_i is the solution of n (2.1) and using the above estimates, from (2.4) we obtain

$$
\begin{aligned}
&\iint_{Q_1'} |x|^{\sigma_i}|u_j|^{q_i}T_\rho(t)\xi_i(x)\varphi(x)\,dx\,dt \\
&\le -\iint_{Q_1'} u_i\xi_i\varphi\frac{dT\rho}{dt}\,dx\,dt + \iint_{Q_1'} u_iT_\rho\Delta^2(\xi_i\varphi)\,dx\,dt - \iint_{Q_1'}\frac{C_i}{|x|^4}u_iT_\rho\xi_i\varphi\,dx\,dt \\
&= -\iint_{Q_1'} u_i\xi_i\varphi\frac{dT\rho}{dt}\,dx\,dt + \iint_{Q_1'} u_iT_\rho\varphi(\Delta^2\xi_i - \frac{C_i}{|x|^4}\xi_i)\,dx\,dt \\
&\quad + \iint_{Q_1'} u_iT_\rho\Big[4(\nabla(\Delta\xi_i),\nabla\varphi) + 4(\nabla\xi_i,\nabla(\Delta\varphi)) + 2\Delta\xi_i\Delta\varphi \\
&\quad + 4\sum_{k,m=1}^n \frac{\partial^2\xi_i}{\partial x_k\partial x_m}\frac{\partial^2\varphi}{\partial x_k\partial x_m}\Big]\,dx\,dt \\
&\le -\int_{\rho^4}^{2\rho^4}\int_{B_1'} u_i\xi_i\varphi\frac{dT_\rho}{dt}\,dx\,dt + \int_0^{2\rho^4}\int_{B_{\rho,2\rho}} u_iT_\rho H_i(\xi_i,\varphi)\,dx\,dt,
\end{aligned} \tag{2.5}
$$

where $H_i(\xi_i,\varphi)$ denotes the expression in the square brackets, i.e.

$$
\begin{aligned}
H_i(\xi_i,\varphi) &= 4(\nabla(\Delta\xi_i),\nabla\varphi) + 4(\nabla\xi_i,\nabla(\Delta\varphi)) + 2\Delta\xi_i\Delta\varphi \\
&\quad + 4\sum_{k,m=1}^n \frac{\partial^2\xi_i}{\partial x_k\partial x_m}\frac{\partial^2\varphi}{\partial x_k\partial x_m}.
\end{aligned} \tag{2.6}
$$

Using the Holder's inequality, we estimate the right-hand side of (2.5). We can write:

$$
\begin{aligned}
&\iint_{Q_1'} |x|^{\sigma_i}|u_j|^{q_i}T_\rho\xi_i\varphi\,dx\,dt \\
&\le \Big(\int_{\rho^4}^{2\rho^4}\int_{B_1'} |x|^{\sigma_j}|u_i|^{q_j}T_\rho\xi_j\varphi\,dx\,dt\Big)^{1/q_j} \\
&\quad \times \Big(\int_{\rho^4}^{2\rho^4}\int_{B_1'} \frac{|\frac{dT_\rho}{dt}|^{q_j'}\xi_i^{q_j'}\varphi}{T_\rho^{q_j'-1}|x|^{\sigma_j(q_j'-1)}\xi_j^{q_j'-1}}\,dx\,dt\Big)^{1/q_j'}
\end{aligned}
$$

$$
\begin{aligned}
&+\Big(\int_0^{2\rho^4}\int_{B_{\rho,2\rho}}|x|^{\sigma_j}|u_i|^{q_j}T_\rho\xi_j\varphi\,dx\,dt\Big)^{1/q_j}\\
&\times\Big(\int_0^{2\rho^4}\int_{B_{\rho,2\rho}}\frac{|H_i(\xi_i,\varphi)|^{q_j'}T_\rho}{|x|^{\sigma_j(q_j'-1)}\xi_j^{q_j'-1}\varphi^{q_j'-1}}\,dx\,dt\Big)^{1/q_j'},
\end{aligned}
$$

where $\frac{1}{q_j}+\frac{1}{q_j'}=1$.

Let us denote the second integral in the first addend above by I_i, and the second integral in the second addend by J_i. If we write separately, then from (2.6) we obtain the following:

$$
\begin{aligned}
&\int_{Q_1'}\int|x|^{\sigma_1}|u_2|^{q_1}T_\rho\xi_1\varphi\,dx\,dt\\
&\le\Big(\int_{Q_1'}\int|x|^{\sigma_2}|u_1|^{q_2}T_\rho\xi_2\varphi\,dx\,dt\Big)^{1/q_2}\big[I_1^{1/q_2'}+J_1^{1/q_2'}\big],
\end{aligned}
\tag{2.7}
$$

$$
\begin{aligned}
&\int_{Q_1'}\int|x|^{\sigma_2}|u_1|^{q_2}T_\rho\xi_2\varphi\,dx\,dt\\
&\le\Big(\int_{Q_1'}\int|x|^{\sigma_1}|u_2|^{q_1}{}_\rho\xi_1\varphi\,dx\,dt\Big)^{1/q_1}\big[I_2^{1/q_1'}+J_2^{\frac{1}{q_1'}}\big].
\end{aligned}
\tag{2.8}
$$

Using (2.6), from these inequalities we obtain

$$
\begin{aligned}
&\iint_{Q_1'}|x|^{\sigma_1}|u_2|^{q_1}T_\rho\xi_1\varphi\,dx\,dt\\
&\le\Big[\Big(\int_{\rho^4}^{2\rho^4}\int_{B_1'}|x|^{\sigma_1}|u_2|^{q_1}T_\rho\xi_1\varphi\,dx\,dt\Big)^{1/q_1}I_2^{1/q_1'}\\
&\quad+\Big(\int_0^{2\rho^4}\int_{B_{\rho,2\rho}}|x|^{\sigma_1}|u_2|^{q_1}T_\rho\xi_1\varphi\,dx\,dt\Big)^{1/q_1}J_2^{1/q_1'}\Big]^{1/q_2}\big[I_1^{1/q_2'}+J_1^{1/q_2'}\big],
\end{aligned}
\tag{2.9}
$$

$$
\begin{aligned}
&\iint_{Q_1'}|x|^{\sigma_2}|u_1|^{q_2}T_\rho\xi_2\varphi\,dx\,dt\\
&\le\Big[\Big(\int_{\rho^4}^{2\rho^4}\int_{B_1'}|x|^{\sigma_2}|u_1|^{q_2}T_\rho\xi_2\varphi\,dx\,dt\Big)^{1/q_2}I_1^{1/q_2'}\\
&\quad+\Big(\int_0^{2\rho^4}\int_{B_{\rho,2\rho}}|x|^{\sigma_2}|u_1|^{q_2}T_\rho\xi_2\varphi\,dx\,dt)^{1/q_2}J_1^{1q_2'}\Big]^{1/q_1}\big[I_2^{1/q_1'}+J_2^{1/q_1'}\big].
\end{aligned}
\tag{2.10}
$$

Substituting (2.8) in (2.7) and (2.7) in (2.8), we obtain

$$
\begin{aligned}
&\int_{Q_1'}\int|x|^{\sigma_1}|u_2|^{q_1}T_\rho\xi_1\varphi\,dx\,dt\\
&\le\Big(\int_{Q_1'}\int|x|^{\sigma_1}|u_2|^{q_1}T_\rho\xi_1\varphi\,dx\,dt\Big)^{\frac{1}{q_1q_2}}\big[I_1^{1/q_2'}+J_1^{1/q_2'}\big]\big[I_2^{1/q_1'}+J_2^{1/q_1'}\big]^{1/q_2},\\
&\int_{Q_1'}\int|x|^{\sigma_2}|u_1|^{q_2}T_\rho\xi_2\varphi\,dx\,dt\\
&\le\Big(\int_{Q_1'}\int|x|^{\sigma_2}|u_1|^{q_2}T_\rho\xi_2\varphi\,dx\,dt\Big)^{\frac{1}{q_1q_2}}\big[I_2^{1/q_1'}+J_2^{1/q_1'}\big]\big[I_1^{1/q_2'}+J_1^{1/q_2'}\big]^{1/q_1}.
\end{aligned}
$$

Hence

$$\begin{aligned} &\int_{Q_1'}\int |x|^{\sigma_1}|u_2|^{q_1}T_\rho\xi_1\varphi\,dx\,dt \\ &\le \big[I_1^{1/q_2'}+J_1^{1/q_2'}\big]^{\frac{q_1q_2}{q_1q_2-1}}\big[I_2^{\frac{1}{q_1'}}+J_2^{1/q_1'}\big]^{\frac{q_1}{q_1q_2-1}}, \end{aligned} \tag{2.11}$$

$$\begin{aligned} &\int_{Q_1'}\int |x|^{\sigma_2}|u_1|^{q_2}T_\rho\xi_2\varphi\,dx\,dt \\ &\le \big[I_2^{1/q_1'}+J_2^{1/q_1'}\big]^{\frac{q_1q_2}{q_1q_2-1}}\big[I_1^{1/q_2'}+J_1^{1/q_2'}\big]^{\frac{q_2}{q_1q_2-1}}. \end{aligned} \tag{2.12}$$

Making the substitutions

$$\begin{gathered} t=\rho^4\tau,\quad r=\rho s,\quad x=\rho y,\quad \widetilde{T}(\tau)=T_\rho(\rho^4\tau),\\ \widetilde{\xi}_i(y)=\xi_i(\rho y),\quad \widetilde{\varphi}(y)=\varphi(\rho y), \end{gathered} \tag{2.13}$$

we estimate the right-hand sides of (2.11) and (2.12).

First, we estimate the integrals I_i, $i=1,2$.

$$\begin{aligned} I_i&=\int_{\rho^4}^{2\rho^4}\int_{B_1'}\frac{|\frac{dT\rho}{dt}|^{q_j'}\xi_i^{q_j'}\varphi}{T_\rho^{q_j'-1}|x|^{\sigma_j(q_j'-1)}\xi_j^{q_j'-1}}\,dx\,dt\\ &\le\int_{\rho^4}^{2\rho^4}\frac{|\frac{dT\rho}{dt}|^{q_j'}}{T_\rho^{q_j'-1}}dt\int_{B_1'}\frac{\xi_i^{q_j'}}{|x|^{\sigma_j(q_j'-1)}\xi_j^{q_j'-1}}dx\\ &\le C\rho^{-4(q_j'-1)}\int_1^2\frac{|\frac{d\widetilde{T}}{d\tau}|^{q_j'}}{\widetilde{T}^{q_j'-1}}d\tau\int_{B_1'}\frac{\xi_i^{q_j'}}{|x|^{\sigma_j(q_j'-1)}\xi_j^{q_j'-1}}dx\\ &\le C\rho^{-4q_j'/q_j}\widetilde{I}_j(\widetilde{T})\int_{B_1'}\frac{\xi_i^{q_j'}}{|x|^{\sigma_j(q_j'-1)}\xi_j^{q_j'-1}}dx, \end{aligned} \tag{2.14}$$

where

$$\widetilde{I}_j(\widetilde{T})=\int_1^2\frac{|\frac{d\widetilde{T}}{d\tau}|^{q_j'}}{\widetilde{T}^{q_j'-1}}d\tau.$$

Since for $|x|=1$ in the last integral (2.14) there is a singularity, then we estimate it separately.

$$\begin{aligned} &\int_{B_1'}\frac{\xi_i^{q_j'}}{|x|^{\sigma_j(q_j'-1)}\xi_j^{q_j'-1}}dx\\ &=\int_1^{2\rho}\frac{(\mu_ir^{-\frac{n-4}{2}+\lambda_i^-}+\overline{\mu}_ir^{-\frac{n-4}{2}-\lambda_i^-}-r^{-\frac{n-4}{2}-\lambda_i^+})^{q_j'}r^{n-1}}{r^{\sigma_j(q_j'-1)}(\mu_jr^{-\frac{n-4}{2}+\lambda_j^-}+\overline{\mu}_jr^{-\frac{n-4}{2}-\lambda_j^-}-r^{-\frac{n-4}{2}-\lambda_j^+})^{q_j'-1}}dr\\ &=\int_1^{2\rho}r^{\lambda_i^-q_j-\lambda_j^-(q_j'-1)-\sigma_j(q_j'-1)-\frac{n-4}{2}+n-1}\\ &\quad\times\frac{(\mu_i+\overline{\mu}_ir^{-2\lambda_i^-}-r^{-\lambda_i^+-\lambda_i^-})^{q_j'}}{(\mu_j+\overline{\mu}_jr^{-2\lambda_j^-}-r^{-\lambda_j^+-\lambda_j^-})^{q_j'-1}}dr. \end{aligned} \tag{2.15}$$

Using the L'Hopital's rule, we obtain

$$\begin{aligned}
&\lim_{r\to 1}\frac{\mu_i+\overline{\mu}_i r^{-2\lambda_i^-}-r^{-\lambda_i^+-\lambda_i^-}}{\mu_j+\overline{\mu}_j r^{-2\lambda_j^-}-r^{-\lambda_j^+-\lambda_j^-}}\\
&=\lim_{r\to 1}\frac{-2\lambda_i^-\overline{\mu}_i r^{-2\lambda_i^- -1}+(\lambda_i^+ +\lambda_i^-)r^{-\lambda_i^+-\lambda_i^- -1}}{-2\lambda_j^-\overline{\mu}_j r^{-2\lambda_j^- -1}+(\lambda_j^+ +\lambda_j^-)r^{-\lambda_j^+-\lambda_j^- -1}}\\
&=\frac{-\lambda_i^- +D_i-\lambda_i^+ +\lambda_i^+ +\lambda_i^-}{-\lambda_j^- +D_j-\lambda_j^+ +\lambda_j^+ +\lambda_j^-}=\frac{D_i}{D_j}.
\end{aligned}$$

Then there exists $r_0>1$ such that for $r<r_0$,

$$\frac{D_i}{D_j}-1<\frac{\mu_i+\mu_i^- r^{-2\lambda_i^-}-r^{-\lambda_i^+-\lambda_i^-}}{\mu_j+\mu_j^- r^{-2\lambda_j^-}-r^{-\lambda_j^+-\lambda_j^-}}<\frac{D_i}{D_j}+1.$$

So, for $r<r_0$,

$$\mu_i+\mu_i^- r^{-2\lambda_i^-}-r^{-\lambda_i^+-\lambda_i^-}<\big(\frac{D_i}{D_j}+1\big)\big(\mu_j+\mu_j^- r^{-2\lambda_j^-}-r^{-\lambda_i^+-\lambda_j^-}\big).$$

On the other hand, for $r\ge r_0$,

$$\frac{\mu_j+\overline{\mu}_j r^{-\lambda_j^-}-r^{-\lambda_j^+-\lambda_j^-}}{\mu_j+\overline{\mu}_j r^{-\lambda_j^-}-r^{-\lambda_j^+-\lambda_j^-}}\le C(r_0).$$

Taking into account the above two relations, from (2.15) we obtain

$$\begin{aligned}
\int_{B_1'}\frac{\xi_i^{q_j'}}{|x|^{\sigma_j(q_j'-1)}\xi_j^{q_j'-1}}dx&\le C\int_1^{2\rho}r^{\lambda_i^- q_j'-\lambda_j^-(q_j'-1)-\sigma_j(q_j'-1)+\frac{n+4}{2}-1}dr\\
&=C\int_1^{2\rho}r^{\frac{q_j'}{q_j}(\lambda_i^- q_j-\lambda_j^- -\sigma_j+\frac{n+4}{2}(q_j-1))-1}dr\\
&\le C\begin{cases}\rho^{\frac{q_j'}{q_j}\eta_i}, & \text{for } \eta_i>0\\ \ln(2\rho), & \text{for } \eta_i=0\\ 1, & \text{for} \eta_i<0,\end{cases}
\end{aligned}\tag{2.16}$$

where

$$\eta_i=\lambda_i^- q_j-\lambda_j^- -\sigma_j+\frac{n+4}{2}(q_j-1).$$

Using (2.16), from (2.14) we obtain

$$I_i\le C\begin{cases}\widetilde{I}_j(\widetilde{T})\rho^{\frac{q_j'}{q_j}(\eta_i-4)}, & \text{for } \eta_i>0\\ \ln(2\rho)\rho^{-4q_j'/q_j}, & \text{for } \eta_i=0\\ \rho^{-4q_j'/q_j}, & \text{for } \eta_i<0.\end{cases}\tag{2.17}$$

To estimate J_i, $i=1,2$, we estimate each addend of $H_i(\xi_i,\varphi)$ separately.

$$\begin{aligned}
|(\nabla(\Delta\xi_i),\nabla\varphi)|&\le\Big|\frac{\partial^3\xi_i}{\partial r^3}+\frac{n-1}{r}\frac{\partial^2\xi_i}{\partial r^2}-\frac{n-1}{r^2}\frac{\partial\xi_i}{\partial r}\Big|\Big|\frac{\partial\varphi}{\partial r}\Big|\\
&\le Cr^{-\frac{n-4}{2}+\lambda_i^- -3}\Big|\frac{\partial\varphi}{\partial r}\Big|,
\end{aligned}$$

$$
\begin{aligned}
|\Delta\xi_i\Delta\varphi| &\le \Big|\frac{\partial^2\xi_i}{\partial r^2}+\frac{n-1}{r}\frac{\partial\xi_i}{\partial r}\Big|\Big|\frac{\partial^2\varphi}{\partial r^2}+\frac{n-1}{r}\frac{\partial\varphi}{\partial r}\Big|\\
&\le Cr^{-\frac{n-4}{2}+\lambda_i^- -2}\Big|\frac{\partial^2\varphi}{\partial r^2}+\frac{n-1}{r}\frac{\partial\varphi}{\partial r}\Big|,\\
|(\nabla\xi_i\nabla(\Delta\varphi))| &\le Cr^{-\frac{n-4}{2}+\lambda_i^- -1}\Big|\frac{\partial^3\varphi}{\partial r^3}+\frac{n-1}{r}\frac{\partial^2\varphi}{\partial r^2}+\frac{n-1}{r^2}\frac{\partial\varphi}{\partial r}\Big|,
\end{aligned}
$$

$$
\begin{aligned}
&\Big|\sum_{i,j=1}^n\frac{\partial^2\xi_i}{\partial x_i\partial x_j}\frac{\partial^2\varphi}{\partial x_i\partial x_j}\Big|\\
&\le \Big|\sum_{i,j=1}^n\frac{\partial}{\partial x_j}\Big(\frac{\partial\xi_i}{\partial r}\frac{x_i}{r}\Big)\frac{\partial}{\partial x_j}\Big(\frac{\partial\varphi}{\partial r}\frac{x_i}{r}\Big)\Big|\\
&\le \sum_{i,j=1}^n\Big|\frac{\partial^2\xi_i}{\partial r^2}\frac{x_ix_j}{r^2}+\frac{\partial\xi_i}{\partial r}\Big(\frac{\delta_{ij}}{r}-\frac{x_ix_j}{r^3}\Big)\Big|\Big|\frac{\partial^2\varphi}{\partial r^2}\frac{x_ix_j}{r^2}+\frac{\partial\varphi}{\partial r}\Big(\frac{\delta_{ij}}{r}-\frac{x_ix_j}{r^3}\Big)\Big|\\
&\le C\Big(\Big|\frac{\partial^2\xi_i}{\partial r^2}\Big|+\frac1r\Big|\frac{\partial\xi_i}{\partial r}\Big|\Big)\Big(\Big|\frac{\partial^2\varphi}{\partial r^2}\Big|+\frac1r\Big|\frac{\partial\varphi}{\partial r}\Big|\Big)\\
&\le Cr^{-\frac{n-4}{2}+\lambda_i^- -2}\Big(\Big|\frac{\partial^2\varphi}{\partial r^2}\Big|+\frac1r\Big|\frac{\partial\varphi}{\partial r}\Big|\Big).
\end{aligned}
$$

Now, taking into account these relations and (2.13), we estimate J_i, $i=1,2$:

$$
\begin{aligned}
J_i &= \int_0^{2\rho^4}\int_{B_{\rho,2\rho}}\frac{|H_i(\xi_i,\varphi)|^{q_j'}T_\rho}{|x|^{\sigma_j(q_j'-1)}\xi_j^{q_j'-1}\varphi^{q_j'-1}}\,dx\,dt\\
&\le \int_0^{2\rho^4}T_\rho dt\int_{B_{\rho,2\rho}}\frac{|H_i(\xi_i,\varphi)|^{q_j'}}{|x|^{\sigma_j(q_j'-1)}\xi_j^{q_j'-1}\varphi^{q_j'-1}}dx\\
&\le C\rho^{(-\frac{n-4}{2}+\lambda_i^- -4)q_j'-\sigma_j(q_j'-1)-(-\frac{n-4}{2}+\lambda_j^-)(q_j'-1)+n+4}\\
&\quad\times\int_1^2\frac{(|\frac{d^3\widetilde\varphi}{ds^3}|+|\frac{d^2\widetilde\varphi}{ds^2}|+|\frac{d\widetilde\varphi}{ds}|)^{q_j'}}{s^{\sigma_j(q_j-1)}\widetilde\varphi^{q_j'-1}}ds\\
&\le C\rho^{-4(q_j'-1)+\lambda_i^-q_j'-\lambda_j^-(q_j'-1)-\sigma_j(q_j'-1)+\frac{n+4}{2}}\widetilde J_j(\widetilde\varphi)\\
&= C\rho^{\frac{q_j'}{q_j}(\eta_i-4)}\widetilde J_j(\widetilde\varphi),
\end{aligned}
\tag{2.18}
$$

where $\widetilde J_j(\widetilde\varphi)$ denotes the last integral.

Using the estimates (2.17),(2.18), we estimate the right-hand sides of (2.11), (2.12). It is known that for large κ and γ, the integrals $\widetilde I_j(\widetilde T)$, $\widetilde J_j(\widetilde\varphi)$ are bounded [17].

Depending on the sign of η_i, $i=1,2$, we consider various variants.

I. $\alpha_1>1$, $\alpha_2>1$. This is equivalent to

$$
\lambda_1^- -\lambda_2^- +\sigma_1>0\quad\text{and}\quad \lambda_2^- -\lambda_1^- +\sigma_2>0. \tag{2.19}
$$

Subject to relation (2.19), we consider the following cases.

(a) $\eta_1\le 0$, $\eta_2\le 0$ or $q_1\le\alpha_1$, $q_2\le\alpha_2$. Then, taking into account (2.17), (2.18), from (2.11), (2.12) we obtain

$$
\int_{Q_1'}\int|x|^{\sigma_i}|u_j|^{q_i}T_\rho\xi_i\varphi\,dx\,dt
$$

$$\le C\rho^{-\frac{4}{q_1q_2-1}(q_i+1)}\big[f_i^{\frac{1}{q_j'}}\widetilde{I}_j^{1/q_j'}+\widetilde{J}_j^{1/q_j'}\big]^{\frac{q_1q_2}{q_1q_2-1}}\big[f_j^{\frac{1}{q_i'}}\widetilde{I}_i^{\frac{1}{q_i'}}+\widetilde{J}_i^{\frac{1}{q_i'}}\big]^{\frac{q_i}{q_1q_2-1}},$$

where

$$f_i(\rho)=\begin{cases}1, & \text{if } \eta_i<0\\ \ln(2\rho), & \text{if } \eta_i=0.\end{cases}$$

When we pass to limit as $\rho\to+\infty$, we obtain

$$\int_{Q_1'}\int |x|^{\sigma_i}|u_j|^{q_i}T_\rho\xi_i\varphi\,dx\,dt\le 0.$$

Hence $u_1\equiv 0$, $u_2\equiv 0$.

(b) Now let $\eta_1>0$, $\eta_2>0$ or $q_1>\alpha_1, q_2>\alpha_2$. Again using (2.17), (2.18), from (2.11), (2.12) we obtain

$$\begin{aligned}&\int_{Q_1'}\int |x|^{\sigma_i}|u_j|^{q_i}T_\rho\xi_i\varphi\,dx\,dt\\ &\le C\rho^{\frac{1}{q_1q_2-1}(q_i(\eta_i-4)+\eta_j-4)}\big[I_j^{1/q_j'}(\widetilde{T})+\widetilde{J}_j^{1/q_j'}(\widetilde{T})\big]^{\frac{q_1q_2}{q_1q_2-1}}\\ &\quad\times\big[\widetilde{I}_i^{\frac{1}{q_i'}}(\widetilde{T})+\widetilde{J}_i^{\frac{1}{q_i'}}(\widetilde{T})\big]^{\frac{q_i}{q_1q_2-1}}.\end{aligned}\tag{2.20}$$

Assume that

$$\min\{q_1(\eta_1-4)+\eta_2-4,\quad q_2(\eta_2-4)+\eta_1-4\}<0.\tag{2.21}$$

Since

$$\begin{aligned}&q_i(\eta_i-4)+\eta_j-4\\ &=\lambda_i^-q_iq_j-\lambda_j^-q_i-\sigma_jq_i+\frac{n+4}{2}(q_iq_j-q_i)+\lambda_j^-q_i-\lambda_i^--\sigma_i\\ &\quad+\frac{n+4}{2}(q_i-1)-4-4q_i\\ &=-(q_iq_j-1)\theta_i,\end{aligned}$$

then we can write (2.21) as $\max(\theta_1,\theta_2)>0$.

For definiteness, we assume $q_1(\eta_1-4)+\eta_2-4<0$. Then for $i=1$, from (2.20) we obtain

$$\begin{aligned}&\int_{Q_1'}\int |x|^{\sigma_1}|u_2|^{q_1}T_\rho\xi_1\varphi\,dx\,dt\\ &\le C\rho^{\frac{1}{q_1q_2-1}(q_1(\eta_1-4)+\eta_2-4)}\big[\widetilde{I}_2^{\frac{1}{q_2'}}+\widetilde{J}_2^{1/q_2'}\big]^{\frac{q_1q_2}{q_1q_2-1}}\big[\widetilde{I}_1^{1/q_1'}+\widetilde{J}_1^{1/q_1'}\big]^{\frac{q_1}{q_1q_2-1}}.\end{aligned}$$

Passing to the limit as $\rho\to+\infty$, we obtain

$$\int_{Q_1'}\int |x|^{\sigma_1}|u_2|^{q_1}\xi_1\,dx\,dt\le 0.$$

Hence $u_2\equiv 0$. Then from the second equation of the system it follows that $u_1\equiv 0$. Similarly, for $q_2(\eta_2-4)+\eta_1-4<0$, we obtain $u_1\equiv 0$, $u_2\equiv 0$. Now let $\min\{q_1(\eta_1-4)+\eta_2-4, q_2(\eta_2-4)+\eta_1-4\}=0$ or the same $\max(\theta_1,\theta_2)=0$. For example, take $q_1(\eta_1-4)+\eta_2-4=0$. Then from (2.20) it follows

$$\int_{Q_1'}\int |x|^{\sigma_1}|u_2|^{q_1}T_\rho\xi_1\,dx\,dt\le C.$$

From the properties of the integral, it follows that

$$\int_0^\infty \int_{B_{\rho,2\rho}} |x|^{\sigma_1}|u_2|^{q_1}\xi_1\,dx\,dt \to 0, \tag{2.22}$$

$$\int_{\rho^4}^{2\rho^4} \int_{B_1'} |x|^{\sigma_1}|u_2|^{q_1}\xi_1\,dx\,dt \to 0. \tag{2.23}$$

Then from (2.9), by (2.22) and (2.23) we obtain

$$\begin{aligned}
&\int_{Q_1'}\int |x|^{\sigma_1}|u_2|^{q_1}T_\rho\xi_1\varphi\,dx\,dt\\
&\le \Big[\Big(\int_{\rho^4}^{2\rho^4}\int_{B_1'} |x|^{\sigma_1}|u_2|^{q_1}\xi_1\,dx\,dt\Big)^{1/q_1} I_2^{1/q_1'}\\
&\quad + \Big(\int_0^\infty\int_{B_1'} |x|^{\sigma_1}|u_2|^{q_1}\xi_1\,dx\,dt\Big)^{1/q_1} J_2^{1/q_1'}\Big]^{1/q_2}\big[I_1^{1/q_2'}+J_1^{1/q_2'}\big]\\
&\le C\rho^{-\frac{1}{q_1q_2}(q_1(\eta_1-4)+\eta_2-4)}\Big[\Big(\int_{\rho^4}^{2\rho^4}\int_{B_1'} |x|^{\sigma_1}|u_2|^{q_1}\xi_1\,dx\,dt\Big)^{\frac{1}{q_1'}}\widetilde{I}_1^{1/q_1'}\\
&\quad + \Big(\int_0^\infty\int_{B_{\rho,2\rho}} |x|^{\sigma_1}|u_2|^{q_1}\xi_1\,dx\,dt\Big)^{1/q_1'}\widetilde{J}_2^{1/q_1'}\Big]^{1/q_2}\big[\widetilde{I}_2^{1/q_2'}+\widetilde{J}_2^{1/q_2'}\big] \to 0.
\end{aligned}$$

So, again

$$\iint_{Q_1'} |x|^{\sigma_1}|u_2|^{q_1}\xi_1\,dx\,dt \le 0.$$

Hence $u_2 \equiv 0$ and respectively $u_1 \equiv 0$. If $q_2(\eta_2-4)+\eta_1-4=0$, then in the same way, we obtain $u_1\equiv 0$, $u_2\equiv 0$.

(c) Let us consider the case when $\eta_i \le 0$, $\eta_j \ge 0$. At first, let $\eta_1 \le 0$, $\eta_2 \ge 0$. As in the previous cases, from (2.11), (2.12) we obtain

$$\begin{aligned}
&\iint_{Q_1'} |x|^{\sigma_1}|u_2|^{q_1}\xi_1\,dx\,dt\\
&\le C\rho^{\frac{1}{q_1q_2-1}(-4(q_1+1)+\eta_2)}\big[f_1^{1/q_2'}\widetilde{I}_2^{1/q_2'}+\widetilde{J}_2^{1/q_2'}\big]^{\frac{q_1q_2}{q_1q_2-1}}\big[\widetilde{I}_1^{1/q_1'}+\widetilde{J}_1^{1/q_1'}\big]^{\frac{q_1}{q_1q_2-1}}.
\end{aligned} \tag{2.24}$$

If $\eta_2 < 4(q_1+1)$, then passing to limit as $\rho\to+\infty$, from (2.24) we have

$$\iint_{Q_1'} |x|^{\sigma_1}|u_2|^{q_1}\xi_1\,dx\,dt \le 0.$$

Hence $u_2\equiv 0$ and from the second equation of the system it follows $u_1\equiv 0$. Note that if $\eta_1<0$, then for $\eta_2 = 4(q_1+1)$, f rom (2.24) we obtain

$$\iint_{Q_1'} |x|^{\sigma_1}|u_2|^{q_1}\xi_1\,dx\,dt < C.$$

As in the previous case, we can show again that $u_1\equiv 0$, $u_2\equiv 0$. Note that the condition $\eta_1<0$, $0\le\eta_2\le 4(q_1+1)$ is equivalent to the condition

$$1<q_2<\alpha_2, \alpha_1\le q_1\le\beta_1,$$

and the condition $\eta_1 = 0$, $0 \le \eta_2 \le 4(q_1+1)$ to the condition

$$q_2 = \alpha_2, \quad \alpha_1 \le q_1 < \beta_1.$$

Now let $\eta_1 \ge 0$, $\eta_2 \le 0$. Then similar to the previous case we obtain that for $\eta_2 < 0$, $0 \le \eta_1 \le 4(q_2+1)$ and for $\eta_2 = 0$, $\eta_1 < 4(q_2+1)$, $u_1 \equiv 0$, $u_2 \equiv 0$.

The same condition $\eta_2 < 0$, $0 \le \eta_1 \le 4(q_2+1)$ is equivalent to the condition

$$q_1 < \alpha_1, \alpha_2 \le q_2 \le \beta_2,$$

and the condition $\eta_2 = 0$, $0 \le \eta_1 < 4(q_2+1)$ to the condition

$$q_1 = \alpha_1, \alpha_2 \le q_2 < \beta_2.$$

II. $\alpha_1 \le 1$, $\alpha_2 > 1$. Herewith, the cases $\eta_1 \le 0$, $\eta_2 > 0$ and $\eta_1 > 0$, $\eta_2 > 0$ should be considered. For $\eta_1 \le 0$, $\eta_2 > 0$ as in the previous cases, we obtain $u_1 \equiv 0$, $u_2 \equiv 0$ if $\eta_1 < 0$, $\eta_2 \le 4(q_1+1)$ and $\eta_1 = 0$, $\eta_2 < 4(q_1+1)$.

From the inequality $\eta_2 \le 4(q_1+1)$ it follows that $1 < q_1 \le \beta_1$. Since

$$\beta_1 = \frac{\lambda_1^- + \sigma_1 + 4 + \frac{n+4}{2}}{\lambda_2^- + \frac{n-4}{2}},$$

this case has meaning for $\lambda_1^- + \sigma_1 + 8 > \lambda_2^-$.

Now let $\eta_1 > 0$, $\eta_2 > 0$. Then similar to case (b), we obtain that $u_1 \equiv 0$, $u_2 \equiv 0$ if

$$q_1 > \alpha_1, q_2 > \alpha_2, \quad \max\{\theta_1, \theta_2\} \ge 0.$$

III. $\alpha_1 > 0$, $\alpha_2 \le 1$. Herewith, it is necessary to consider the case when $\eta_1 > 0$, $\eta_2 \le 0$ and $\eta_1 > 0$, $\eta_2 > 0$. For $\eta_1 > 0$, $\eta_2 \le 0$, $u_1, u_2 \equiv 0$ if $q_1 < \alpha_1$, $1 < q_2 \le \beta_2$ and $q = \alpha_1$, $1 < q_2 < \beta_2$, and in the case $\eta_1 > 0$, $\eta_2 > 0$, for $q_1 > \alpha_1$, $1 < q_2 < \beta_2$, $\max\{\theta_1, \theta_2\} \ge 0$. Obviously, this case has meaning for $\beta_2 > 1$ or for $\lambda_2^- + \sigma_2 + 8 > \lambda_1^-$.

IV. $\alpha_1 \le 1$, $\alpha_2 \le 1$. Here it is necessary to consider the only case when $\eta_1 > 0$, $\eta_2 > 0$. Then $u_1 \equiv 0$, $u_2 \equiv 0$, if $1 < q_1 < \beta_1$, $1 < q_2 < \beta_2$ and $\max\{\theta_1, \theta_2\} \ge 0$. Obviously, this set is not empty if $\lambda_1^- + \sigma_1 + 8 > \lambda_2^-$, $\lambda_2^- + \sigma_2 + 8 > \lambda_1^-$. This completely proves the theorem. □

Note that remains open the cases $q_1 = \alpha_1, q_2 = \beta_2$ and $q_1 = \beta_1, q_2 = \alpha_2$.

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