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PERIODIC PROBLEM FOR AN IMPULSIVE SYSTEM OF THE LOADED HYPERBOLIC EQUATIONS

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Abstract. We consider a periodic problem for a system of the loaded hyperbolic equations with impulse effects. By introducing additional functions, this problem is reduced to an equivalent problem consisting of a family of periodic boundary-value problems of loaded ordinary differential equations with impulse effects and integral relations. We obtain sufficient conditions for the existence of unique solution to the family of periodic boundary-value problems. Conditions of unique solvability of periodic problem are established in terms of initial data.

1. Introduction and statement of the problem

In this article we consider a periodic problem for the system of second-order loaded hyperbolic equations with impulse effects

$$\begin{aligned}\frac{\partial^2 u}{\partial t \partial x} &= A(t,x)\frac{\partial u}{\partial x} + B(t,x)\frac{\partial u}{\partial t} + C(t,x)u + f(t,x)\\ &\quad + \sum_{i=1}^{k} M_i(t,x)\frac{\partial u(t_i+0,x)}{\partial x} + \sum_{i=1}^{k} L_i(t,x)\frac{\partial u(t_i+0,x)}{\partial t}\\ &\quad + \sum_{i=1}^{k} K_i(t,x)u(t_i+0,x), \quad t \neq t_i,\end{aligned} \tag{1.1}$$

$$u(t,0) = \psi(t), \quad t \in [0,T], \tag{1.2}$$

$$u(0,x) = u(T,x) + \varphi_0(x), \quad x \in [0,\omega], \tag{1.3}$$

$$\frac{\partial u(t_i+0,x)}{\partial x} - \frac{\partial u(t_i-0,x)}{\partial x} = \varphi_i(x), \quad i = \overline{1,k}, \tag{1.4}$$

where the domain is $\Omega = [0,T] \times [0,\omega]$, $u = \operatorname{col}(u_1, u_2, \dots, u_n)$, the $(n \times n)$ matrices $A(t,x)$, $B(t,x)$, $C(t,x)$, and the n-vector function $f(t,x)$ are continuous on Ω, the $(n \times n)$ matrices $M_i(t,x)$, $L_i(t,x)$, $K_i(t,x)$, $i = \overline{1,k}$ are continuous on Ω, the n-vector function $\psi(t)$ is continuously differentiable on $[0,T]$, the n-vector function $\varphi_0(x)$ is continuously differentiable on $[0,\omega]$, the n-vector functions $\varphi_j(x)$, $j = \overline{1,k}$ are continuous on $[0,\omega]$, $0 < t_1 < t_2 < \dots < t_k < T$, $\|u(t,x)\| = \max_{i=\overline{1,n}} |u_i(t,x)|$.

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The data of this problem satisfy the compatibility condition $\psi(0) = \psi(T)+\varphi_0(0)$. We denote $t_0 = 0$, $t_{k+1} = T$, $\Omega_r = [t_{r-1}, t_r) \times [0, \omega]$, $r = \overline{1, k+1}$, i.e. $\Omega = \cup_{r=1}^{k+1} \Omega_r$.

Let $PC(\Omega, \{t_i\}_{i=1}^k, R^n)$ be the space of vector-functions $u(t, x)$ piecewise continuous on Ω with possible discontinuities on the lines $t = t_i$, $i = \overline{1, k}$, and let the norm be

$$\|u\|_1 = \max_{i=\overline{1,k+1}} \sup_{(t,x)\in\Omega_i} \|u(t, x)\|.$$

A function $u(t, x) \in PC(\Omega, \{t_i\}_{i=1}^k, R^n)$ with partial derivatives

$$\frac{\partial u(t,x)}{\partial x} \in PC(\Omega, \{t_i\}_{i=1}^k, R^n), \quad \frac{\partial u(t,x)}{\partial t} \in PC(\Omega, \{t_i\}_{i=1}^k, R^n),$$
$$\frac{\partial^2 u(t,x)}{\partial t \partial x} \in PC(\Omega, \{t_i\}_{i=1}^k, R^n)$$

is called a solution of problem (1.1)–(1.4) if it satisfies system (1.1) for all $(t, x) \in \Omega$ (except the lines $t = t_i$, $i = \overline{1, k}$), the boundary conditions (1.2) and (1.3), and the conditions of impulsive effect at fixed times (1.4).

Periodic boundary-value problems for a system of differential equations hyperbolic type have been researched by many authors, see for example [5, 18, 23, 29, 31]. Some classes of boundary-value problems for ordinary and partial differential equations with impulse effects were studied in [1, 2, 11, 12, 15, 17, 25, 30]. For the prevoius decades the theory of loaded equations has been developed intensively in works of many mathematicians, see [20, 21, 22, 24, 26, 27, 28]. A review of results on boundary-value problems for the loaded differential equations of various classes can be found in [22, 28]. This periodic problem for the system of loaded hyperbolic equations second-order with impulse effects is investigated first time here.

For investigating and solving the problem, we use the method of introducing functional parameters [3]–[16]. This method is a generalization of parametrization method [19] for partial differential equations. By introducing new unknown functions the periodic problem (1.1)–(1.4) is reduced to an equivalent problem consisting a family of periodic boundary-value problems for system of loaded ordinary differential equations with impulse effects and integral relations. Then, we establish the relationship between the unique solvability of periodic problem and the unique solvability of the family of periodic boundary-value problems. Sufficient conditions for the existence of a unique solution to the family of periodic boundary value problems are obtained by the method of introduction of functional parameters. Also we present an algorithms for finding the approximate solutions of these problems. The results obtained are applied to a periodic problem for the system of loaded hyperbolic equations with impulse effects.

2. Reduction of problem (1.1)–(1.4) to an equivalent problem, and algorithm

In this section we introduce the unknown functions

$$v(t, x) = \frac{\partial u(t,x)}{\partial x}, \quad w(t, x) = \frac{\partial u(t,x)}{\partial t}$$

to reduce problem (1.1)–(1.4) to the equivalent problem

$$\begin{aligned}\frac{\partial v}{\partial t} = A(t,x)v + \sum_{i=1}^{k} M_i(t,x)v(t_i+0,x)\\ + F(t,x,u(t,x),w(t,x)), \quad t \neq t_i, \quad x \in [0,\omega],\end{aligned} \tag{2.1}$$

$$v(0,x) = v(T,x) + \varphi_0'(x), \quad x \in [0,\omega], \tag{2.2}$$

$$v(t_i+0,x) - v(t_i-0,x) = \varphi_i(x), \quad i = \overline{1,k}, \tag{2.3}$$

$$u(t,x) = \psi(t) + \int_0^x v(t,\xi)d\xi, \quad w(t,x) = \dot{\psi}(t) + \int_0^x \frac{\partial v(t,\xi)}{\partial t}d\xi, \tag{2.4}$$

where

$$\begin{aligned}F(t,x,u(t,x),w(t,x)) = f(t,x) + B(t,x)w(t,x) + \sum_{i=1}^{k} L_i(t,x)w(t_i+0,x)\\ + C(t,x)u(t,x) + \sum_{i=1}^{k} K_i(t,x)u(t_i+0,x).\end{aligned}$$

In problem (2.1)–(2.4), the condition $u(t,0) = \psi(t)$ is taken into account in relations (2.4). Condition (2.2) is equivalent to condition (1.3) and the compatibility condition.

A triple of functions $\{v(t,x), u(t,x), w(t,x)\}$ piecewise continuous on Ω is called a solution of problem (2.1)–(2.4) if the function has a piecewise continuous derivative with respect to t on Ω and satisfies a one-parameter family of periodic problems for loaded ordinary differential equations with impulse effects (2.1)–(2.3), where the functions $u(t,x)$ and $w(t,x)$ are connected with $v(t,x)$ and $\frac{\partial v(t,x)}{\partial t}$ by the integral relations (2.4).

Problem (2.1)–(2.3) at fixed $u(t,x)$ and $w(t,x)$ represents a family of periodic problems for a system of loaded ordinary differential equations with impulse effects. The relations (2.4) allows to determine unknown functions $u(t,x)$ and $w(t,x)$ by $v(t,x)$ and its derivative $\frac{\partial v(t,x)}{\partial t}$. The solution of problem (2.1)–(2.4) is unknown functions $v(t,x)$, $u(t,x)$ and $w(t,x)$ which will be find by iterative processes based on the following algorithm.

Step 0. Solving the family of periodic problems with impulse effects (2.1)–(2.3) under the assumptions that $u(t,x) = \psi(t)$, $w(t,x) = \dot{\psi}(t)$ on the right-hand side of (2.1), we find the function $v^{(0)}(t,x) \in PC(\Omega, \{t_i\}_{i=1}^k, R^n)$. From integral conditions (2.4) at $v(t,x) = v^{(0)}(t,x)$, $\frac{\partial v(t,x)}{\partial t} = \frac{\partial v^{(0)}(t,x)}{\partial t}$ we determine

$$u^{(0)}(t,x) \in PC(\Omega, \{t_i\}_{i=1}^k, R^n), \quad w^{(0)}(t,x) \in PC(\Omega, \{t_i\}_{i=1}^k, R^n).$$

Step 1. Solving the family of periodic problems with impulse effects (2.1)–(2.3) under the assumptions that $u(t,x) = u^{(0)}(t,x)$, $w(t,x) = w^{(0)}(t,x)$ on the right-hand side of (2.1), we find the function $v^{(1)}(t,x) \in PC(\Omega, \{t_i\}_{i=1}^k, R^n)$. From integral conditions (2.4) at $v(t,x) = v^{(1)}(t,x)$, $\frac{\partial v(t,x)}{\partial t} = \frac{\partial v^{(1)}(t,x)}{\partial t}$ we determine

$$u^{(1)}(t,x) \in PC(\Omega, \{t_i\}_{i=1}^k, R^n), \quad w^{(1)}(t,x) \in PC(\Omega, \{t_i\}_{i=1}^k, R^n);$$

and so on.

Step m**.** Solving the family of periodic problems with impulse effects (2.1)–(2.3) under the assumptions that $u(t,x) = u^{(m-1)}(t,x)$, $w(t,x) = w^{(m-1)}(t,x)$ on the right-hand side of (2.1), we find the function $v^{(m)}(t,x) \in PC(\Omega, \{t_i\}_{i=1}^k, R^n)$. From integral conditions (2.4) at $v(t,x) = v^{(m)}(t,x)$, $\frac{\partial v(t,x)}{\partial t} = \frac{\partial v^{(m)}(t,x)}{\partial t}$ we determine

$$u^{(m)}(t,x) \in PC(\Omega, \{t_i\}_{i=1}^k, R^n), \quad w^{(m)}(t,x) \in PC(\Omega, \{t_i\}_{i=1}^k, R^n), \quad m = 1, 2, \dots.$$

The highlight of the proposed algorithm is the solvability of the family of periodic problems for the system of loaded ordinary differential equations with impulse effects (2.1)–(2.3) for fixed $u(t,x)$, $w(t,x)$. This question will be investigated in the next section. Conditions for the convergence of the algorithm will be given in Section 4.

3. Family of periodic problems for loaded ordinary differential equations with impulse effects

For fixed $u(t,x)$ and $w(t,x)$, problem (2.1)–(2.3) requires special investigations. Therefore, we consider a family of periodic problems for the following system of loaded differential equations with impulse effects

$$\frac{\partial v}{\partial t} = A(t,x)v + \sum_{i=1}^{k} M_i(t,x)v(t_i+0,x) + F(t,x), \quad t \neq t_i, \ x \in [0,\omega], \tag{3.1}$$

$$v(0,x) = v(T,x) + \varphi_0'(x), \quad x \in [0,\omega], \tag{3.2}$$

$$v(t_i+0,x) - v(t_i-0,x) = \varphi_i(x), \quad i = \overline{1,k}, \tag{3.3}$$

where $v = \mathrm{col}(v_1, v_2, \dots, v_n)$, the n-vector function $F(t,x)$ is continuous on Ω, $t_0 < t_1 < t_2 < \dots < t_k < t_{k+1}$.

A function $v(t,x) \in PC(\Omega, \{t_i\}_{i=1}^k, R^n)$ with the derivative

$$\frac{\partial v(t,x)}{\partial t} \in PC(\Omega, \{t_i\}_{i=1}^k, R^n)$$

is called a solution to the family of periodic problems with impulse effects (3.1)–(3.3) if it satisfies system of loaded ordinary differential equations (3.1) except the lines $t = t_i$, $i = \overline{1,k}$, the periodic condition (3.2), and the conditions of impulse effects (3.3) for all $x \in [0,\omega]$.

For fixed $x \in [0,\omega]$, problem (3.1)–(3.3) is a periodic problem for a system of loaded ordinary differential equations with impulse effects. Changing the variable x over $[0,\omega]$, we get a family of periodic problems for loaded ordinary differential equations with impulse effects. The boundary value problems for ordinary differential equations with impulse effects were studied by numerous authors. Conditions for the existence of solutions of the analyzed problems were obtained and expressed in different ways by using various methods and approaches [1, 2, 17, 25, 30]. The different problems for loaded differential equations were studied by many authors. For a survey, bibliography, and detailed analysis, see [22, 28].

The parametrization method is used for the investigation and solving of the family of periodic problems with impulse effects. The idea of this method is to introduce additional parameters as values of the required function on the lines of partition of the domain with respect to the variable t. By the change of unknown functions, the original problem is reduced to an equivalent problem with functional parameters. The properties of these parameters inherit the properties of the solutions.

In this section, the partition of the domain Ω is non-uniform and the additional parameters are introduced as values of the required function on the lines $t = t_i$, $i = \overline{0,k}$.

By using the straight lines $t = t_i$, $i = \overline{1,k}$, we split the domain Ω into the subdomains Ω_i, $i = \overline{1,k+1}$. Let $v_r(t,x)$ be the restriction of the function $v(t,x)$ to Ω_r, $r = \overline{1,k+1}$. We introduce the parameters $\lambda_r(x) = v_r(t_{r-1},x)$, $r = \overline{1,k+1}$. By the change of the unknown function

$$v_r(t,x) = \widetilde{v}_r(t,x) + \lambda_r(x), \quad (t,x) \in \Omega_r, \quad r = \overline{1,k+1},$$

we reduce problem (3.1)–(3.3) to the following equivalent periodic problem with parameters:

$$\begin{gathered}\frac{\partial \widetilde{v}_r}{\partial t} = A(t,x)\widetilde{v}_r + A(t,x)\lambda_r(x) + \sum_{i=1}^{k} M_i(t,x)\lambda_{i+1}(x) \\ + F(t,x), \quad (t,x) \in \Omega_r,\ r = \overline{1,k+1},\end{gathered} \tag{3.4}$$

$$\widetilde{v}_r(t_{r-1},x) = 0, \quad r = \overline{1,k+1}, \quad x \in [0,\omega], \tag{3.5}$$

$$\lambda_1(x) = \lim_{t\to T-0} \widetilde{v}_{k+1}(t,x) + \lambda_{k+1}(x) + \varphi_0'(x), \quad x \in [0,\omega], \tag{3.6}$$

$$\lambda_{i+1}(x) - \lim_{t\to t_i-0} \widetilde{v}_i(t,x) - \lambda_i(x) = \varphi_i(x), \quad i = \overline{1,k}. \tag{3.7}$$

The solution of problem (3.4)–(3.7) is obtained as a system of pairs $(\lambda(x), \widetilde{v}([t],x))$ with elements $\lambda(x) = (\lambda_1(x), \lambda_2(x), \dots, \lambda_{k+1}(x))'$ and

$$\widetilde{v}([t],x) = (\widetilde{v}_1(t,x), \widetilde{v}_2(t,x), \dots, \widetilde{v}_{k+1}(t,x))',$$

where the functions $\widetilde{v}_r(t,x)$ are continuous on Ω_r together with their partial derivatives $\frac{\partial \widetilde{v}_r(t,x)}{\partial t}$ on Ω_r, $r = \overline{1,k+1}$, have a finite left limit $\lim_{t\to t_r-0} \widetilde{v}_r(t,x)$, $r = \overline{1,k+1}$, and satisfy, for $\lambda_r(x) = \lambda_r^*(x)$, the system of differential equations (3.4) and conditions (3.5)–(3.7).

Unlike problem (3.1)–(3.3), here, we get the initial conditions (3.5) as values of the unknown function on the lines $t = t_{r-1}$, $r = \overline{1,k+1}$. For fixed $\lambda_r(x)$, $r = \overline{1,k+1}$, there exist the solutions of the Cauchy problem on Ω_r for system (3.4) with condition (3.5).

The Cauchy problem (3.4), (3.5) is equivalent to the Volterra integral equation of second kind

$$\begin{aligned}\widetilde{v}_r(t,x) &= \int_{t_{r-1}}^{t} A(\tau,x)\widetilde{v}_r(\tau,x)d\tau + \int_{t_{r-1}}^{t} A(\tau,x)d\tau\lambda_r(x) \\ &\quad + \sum_{i=1}^{k} \int_{t_{r-1}}^{t} M_i(\tau,x)d\tau\lambda_{i+1}(x) + \int_{t_{r-1}}^{t} F(\tau,x)d\tau.\end{aligned} \tag{3.8}$$

Passing on the right-hand side of (3.8) to the limit as $t \to t_r - 0$, we obtain $\lim_{t\to t_r-0} \widetilde{v}_r(t,x)$, $r = \overline{1,k+1}$, $x \in [0,\omega]$. Substituting these limits in (3.6) and (3.7), we obtain the following system of $(k+1)$ functional equations for the unknown vector functions $\lambda_r(x)$, $r=\overline{1,k+1}$:

$$\begin{aligned} &\lambda_1(x)-\int_{t_k}^{T}A(\tau,x)d\tau\lambda_{k+1}(x)-\sum_{i=1}^{k}\int_{t_k}^{T}M_i(\tau,x)d\tau\lambda_{i+1}(x)-\lambda_{k+1}(x)\\ &=\varphi_0'(x)+\int_{t_k}^{T}F(\tau,x)d\tau+\int_{t_k}^{T}A(\tau,x)\widetilde{v}_{k+1}(\tau,x)d\tau, \end{aligned} \tag{3.9}$$

$$\begin{aligned} &\lambda_{i+1}(x)-\Big[I+\int_{t_{i-1}}^{t_i}A(\tau,x)d\tau\Big]\lambda_i(x)-\sum_{j=1}^{k}\int_{t_{i-1}}^{t_i}M_j(\tau,x)d\tau\lambda_{j+1}(x)\\ &=\varphi_i(x)+\int_{t_{i-1}}^{t_i}F(\tau,x)d\tau+\int_{t_{i-1}}^{t_i}A(\tau,x)\widetilde{v}_i(\tau,x)d\tau,\quad i=\overline{1,k}. \end{aligned} \tag{3.10}$$

Now we construct the algorithm for finding the approximate solution to problem (3.1)–(3.3). The parametrization method splits the process of determination of unknown functions into two stages: (i) determination of the introduced functional parameters $\lambda_r(x)$ from the system of equations (3.9), (3.10); (ii) determination of the unknown functions $\widetilde{v}_r(t,x)$ from the integral equations (3.8).

We denote by $Q(x)$ the $(n(k+1)\times n(k+1))$ matrix corresponding to the left-hand side of system equations (3.9), (3.10) consists of the coefficients at the parameters $\lambda_r(x)$, $r=\overline{1,k+1}$.

The following assertion gives the conditions of the unique solvability of problem (3.1)–(3.3) and convergence this algorithm.

Theorem 3.1. *Suppose the $n(k+1)\times n(k+1)$ matrix $Q(x)$ is invertible for all $x\in[0,\omega]$ and that the following inequalities are true:*

(a) $\|[Q(x)]^{-1}\|\le\gamma(x)$, *where $\gamma(x)$ is a positive function continuous with respect to $x\in[0,\omega]$;*

(b)
$$q(x)=\gamma(x)\Big\{1+\sum_{i=1}^{k}\max_{t\in[0,T]}\|M_i(t,x)\|h\Big\}\Big[e^{\alpha(x)h}-1-\alpha(x)h\Big]\le\chi<1,$$
where $\alpha(x)=\max_{t\in[0,T]}\|A(t,x)\|$, $h=\max_{r=\overline{1,k+1}}(t_r-t_{r-1})$, χ is a constant.

Then the family of periodic problems with impulse effects (3.1)–(3.3) *has a unique solution $v^*(t,x)$.*

The proof of Theorem 3.1 is analogous to scheme of the proof of [12, Theorem 2]; we omit it.

4. Main results

The following statement gives the conditions of the feasibility and the convergence of the proposed algorithm, simultaneously guaranteeing the existence of unique solution of the equivalent problem (2.1)–(2.4).

Theorem 4.1. *Suppose the $n(k+1)\times n(k+1)$ matrix $Q(x)$ is invertible for all $x\in[0,\omega]$ and that inequalities (a) and (b) of Theorem 3.1 hold. Then problem* (2.1)–(2.4) *is uniquely solvable.*

The proof of Theorem 4.1 is analogous to the proof of [12, Theorems 1], taking into account the features of the considered problem. From the equivalence of the problems (2.1)–(2.4) and (1.1)–(1.4), the assertion follows.

Theorem 4.2. *Suppose the $n(k+1)\times n(k+1)$ matrix $Q(x)$ is invertible for all $x\in[0,\omega]$ and that inequalities (a) and (b) of Theorem 3.1 hold. Then the periodic problem for system of loaded hyperbolic equations with impulse effects* (1.1)–(1.4) *is uniquely solvable.*

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