Electronic Journal of Differential Equations, Vol. 2025 (2025), No. 01, pp. 1–17.
ISSN: 1072-6691. URL: https://ejde.math.txstate.edu, https://ejde.math.unt.edu
DOI: 10.58997/ejde.2025.01

EXISTENCE AND BOUNDEDNESS OF SOLUTIONS FOR A PARABOLIC-PARABOLIC PREDATOR-PREY MODEL

FENGXIANG ZHAO, HAOTIAN TANG, JIASHAN ZHENG, KAIQIANG LI

Abstract. This article concerns the fully parabolic pursuit-prey chemotaxis system

$$\begin{aligned} u_t &= \Delta u - \chi\nabla\cdot(u\nabla w) + u\left(\lambda_1 - \mu_1 u^{r_1-1} + av\right), \quad x\in\Omega,\ t>0,\\ v_t &= \Delta v + \xi\nabla\cdot(v\nabla z) + v\left(\lambda_2 - \mu_2 v^{r_2-1} - bu\right), \quad x\in\Omega,\ t>0,\\ w_t &= \Delta w - w + v, \quad x\in\Omega,\ t>0,\\ z_t &= \Delta z - z + u, \quad x\in\Omega,\ t>0, \end{aligned}$$

in a bounded domain $\Omega\subset\mathbb{R}^N$ $(N\geq 1)$ with homogeneous Neumann boundary conditions, where χ, ξ, λ_i, μ_i, a, b are positive constants and $r_i>1$ $(i=1,2)$. We show that if $(r_1-1)(r_2-1)\geq 1$, the above system exists a unique global and bounded classical solution for all appropriately regular nonnegative initial data, which extends the previous global existence result in Qi and Ke [13]).

1. Introduction

In this article, we investigate the indirect pursuit-evasion model (parabolic-parabolic system)

$$\begin{gathered} u_t = \Delta u - \chi\nabla\cdot(u\nabla w) + u\left(\lambda_1 - \mu_1 u^{r_1-1} + av\right), \quad x\in\Omega,\ t>0,\\ v_t = \Delta v + \xi\nabla\cdot(v\nabla z) + v\left(\lambda_2 - \mu_2 v^{r_2-1} - bu\right), \quad x\in\Omega,\ t>0,\\ w_t = \Delta w - w + v, \quad x\in\Omega,\ t>0,\\ z_t = \Delta z - z + u, \quad x\in\Omega,\ t>0,\\ \frac{\partial u}{\partial\nu} = \frac{\partial v}{\partial\nu} = \frac{\partial w}{\partial\nu} = \frac{\partial z}{\partial\nu} = 0, \quad x\in\partial\Omega,\ t>0,\\ u(x,0)=u_0(x), \quad v(x,0)=v_0(x), \quad w(x,0)=w_0(x),\\ z(x,0)=z_0(x), \quad x\in\Omega, \end{gathered} \tag{1.1}$$

where $\Omega\subset\mathbb{R}^N$ $(N\geq 1)$ is a bounded domain with smooth boundary, λ_1, λ_2, μ_1, μ_2, r_1, r_2, χ, ξ, a, b are positive constants. The initial data (u_0,v_0,w_0,z_0) satisfies

2020 *Mathematics Subject Classification.* 35K20, 35K55, 92C17.
Key words and phrases. Pursuit-evasion; parabolic-parabolic; boundedness; classical solution.

Submitted April 11, 2024. Published January 4, 2025.

$$
\begin{gathered}
u_0 \in C^0(\bar{\Omega}), \quad u_0 \geq 0 \quad \text{in } \bar{\Omega}, \\
v_0 \in C^0(\bar{\Omega}), \quad v_0 \geq 0 \quad \text{in } \bar{\Omega}, \\
w_0 \in W^{1,\vartheta}(\Omega), \quad w_0 \geq 0 \quad \text{in } \bar{\Omega} \text{ and some } \vartheta > N, \\
z_0 \in W^{1,\vartheta}(\Omega), \quad z_0 \geq 0 \quad \text{in } \bar{\Omega} \text{ and some } \vartheta > N.
\end{gathered} \tag{1.2}
$$

In this system, $u(x,t)$, $v(x,t)$ are the densities of the predators and the prey, respectively, meanwhile $w(x,t)$ and $z(x,t)$ represent concentrations of chemical signals emitted by $v(x,t)$ and $u(x,t)$. χ, ξ measure the strength of attractive and repulsive directed migration, respectively.

To understand the model (1.1) better, we need to introduce some results about the classical Keller-Segel model [9], which presents the aggregate and collective behavior of cells due to chemotaxis by means of a coupled system of two equations

$$
\begin{gathered}
u_t = \Delta u - \nabla \cdot (u \nabla v), \\
v_t = \Delta v - v + u.
\end{gathered} \tag{1.3}
$$

Many works are dedicated to this model and its variants, the most important results are about the existence and boundedness of classical global solutions, the occurrence of the finite-time blow-up for the solutions and large time behavior under some appropriate assumptions. See for example [4, 6, 8, 12, 20, 21, 22, 23, 24, 26] and the references therein.

Supposing predators and prey to exert species-characteristic substances, such as pheromones or scent marks, the following pursuit-evasion model was proposed

$$
\begin{gathered}
u_t = \Delta u - \chi \nabla \cdot (u \nabla w) + f(u,v), \quad x \in \Omega, t > 0, \\
v_t = \Delta v + \xi \nabla \cdot (v \nabla z) + g(u,v), \quad x \in \Omega, t > 0, \\
\tau_1 w_t = \Delta w - w + v, \quad x \in \Omega, t > 0, \\
\tau_2 z_t = \Delta z - z + u, \quad x \in \Omega, t > 0,
\end{gathered} \tag{1.4}
$$

where $\tau_i \in \{0,1\}$ $(i = 1,2)$. When $\tau_1 = \tau_2 = 0$, it has been proved that the above system possesses a unique non-negative bounded weak solution in two-dimensional space for the case $f = g = 0$ [5]. The weak solution to (1.4) with $f = uv - u$ and $g = v(1 - v - u)$ was also considered in [1]. Turning to the case $f = u(\lambda + av - u)$ and $g = v(\mu - v - bu)$, Li, Tao and Winkler [10] proved that the parabolic-elliptic system ($\tau_1 = \tau_2 = 0$) admits a global and bounded solution for any given suitably regular initial data when $N \leq 3$. Then Liu and Zheng [11] establish the existence of classical global solutions, and the asymptotic behavior for this system in N-dimensional domains. Recently, Zhang and Zheng [27] considered the above system with $r_1 > 1$, $r_2 > 1$, $f = u(\lambda_1 - \mu_1 u^{r_1 - 1} + av)$ and $g = v(\lambda_2 - \mu_2 v^{r_2 - 1} - bu)$, and proved that if $(r_1 - 1)(r_2 - 1) > \frac{(N-2)_+}{N}$, the system exists a unique global and bounded classical solution for all appropriately regular nonnegative initial data.

When $\tau_1 = \tau_2 = 1$, since accounting for two taxis mechanisms, system (1.4) can not be regarded as a triangular cross-diffusion model. Due to the complexity of this problem, there are few related works. Recently, Qi and Ke [13] showed that the system with $f = u(\mu - u + av)$ and $g = v(\lambda - v - bu)$ possesses a global bounded classical solution with a positive constant $C_{N/2+1}$ if $a < 2$ and

$$
\frac{N(2-a)}{2(C_{N/2+1})^{\frac{1}{N/2+1}}(N-2)_+} > \max\{\chi, \xi\}.
$$

Additional relevant results can be found in [2, 14, 15, 16, 19, 28] and the references therein.

Inspired by the works mentioned above, we are interested in the model (1.1), and try to establish the existence and boundedness of classical global solution under the Neumann boundary conditions.

1.1. **Statements of main results.** In this paper, we will prove that if $r_1 > 1$, $r_2 > 1$, and $(r_2 - 1)(r_1 - 1) \geq 1$, then the solution (u, v, w, z) of (1.1) is global in time and bounded for any $N \geq 1$.

Theorem 1.1. *Let $\Omega \subset \mathbb{R}^N$ $(N \geq 1)$ be bounded domain with smooth boundary. Suppose that $\chi, \xi, \lambda_i, \mu_i$ are positive constants, $r_i > 1$ $(i = 1, 2)$ and the initial data (u_0, v_0, w_0, z_0) fulfills* (1.2). *If $(r_2 - 1)(r_1 - 1) > 1$, for all μ_i $(i = 1, 2) > 0$, or $(r_2 - 1)(r_1 - 1) = 1$, μ_1 and μ_2 are appropriately large, then system* (1.1)-(1.2) *possesses a unique global classical solution*

$$
\begin{aligned}
u &\in C^0(\overline{\Omega} \times [0, \infty) \cap C^{2,1}(\overline{\Omega} \times (0, \infty)), \\
v &\in C^0(\overline{\Omega} \times (0, \infty) \cap C^{2,1}(\overline{\Omega} \times (0, \infty)), \\
w &\in C^0(\overline{\Omega} \times [0, \infty) \cap C^{2,0}(\overline{\Omega} \times (0, \infty)), \\
z &\in C^0(\overline{\Omega} \times (0, \infty) \cap C^{2,0}(\overline{\Omega} \times (0, \infty)).
\end{aligned}
$$

Moreover, there exists a constant $C > 0$ such that

$$
\|u(\cdot, t)\|_{L^\infty(\Omega)} + \|v(\cdot, t)\|_{L^\infty(\Omega)} + \|w(\cdot, t)\|_{W^{1,\infty}(\Omega)} + \|z(\cdot, t)\|_{W^{1,\infty}(\Omega)} \leq C
$$

for all $t > 0$.

Remark 1.2. For the case of $r_1 = r_2 = 2$, our condition turns into $(r_1-1)(r_2-1) = 1$, for which the global and bounded solution has been obtained by Qi and Ke [13].

In this article, we will use C and C_i $(i = 1, 2, \cdots)$ to represent different positive constants which may vary in the context. Besides, we write $u(x, t)$ as u and $\int_\Omega u dx$ as $\int_\Omega u$ for convenience.

The rest of this article is arranged as follows. In Section 2, we give some preliminary works, such as the local existence of solutions and some classical inequalities. In Section 3, the L^p estimates of u and v are given by using elementary energy method. Finally, the proof of our main result is given in Section 4 by using Neumann heat semigroup theory.

2. Preliminaries

In this section, we give the following lemmas which will be used in the later proofs. Firstly, we recall the well-known result about the existence of local solutions to model (1.1). Readers can refer to [18] for a detailed proof.

Lemma 2.1. *Let $\Omega \subset \mathbb{R}^N$ $(N \geq 1)$ be a bounded domain with smooth boundary. Assume that the initial data satisfies* (1.2). *Then system* (1.1)-(1.2) *has a unique nonnegative classical solution*

$$
\begin{aligned}
u &\in C^0(\bar{\Omega} \times [0, T_{\max})) \cap C^{2,1}(\bar{\Omega} \times (0, T_{\max})), \\
v &\in C^0(\bar{\Omega} \times [0, T_{\max})) \cap C^{2,1}(\bar{\Omega} \times (0, T_{\max})), \\
w &\in C^0(\bar{\Omega} \times [0, T_{\max})) \cap C^{2,1}(\bar{\Omega} \times (0, T_{\max})) \cap L^\infty_{loc}((0, T_{\max}); W^{1,\vartheta}(\Omega)), \\
z &\in C^0(\bar{\Omega} \times [0, T_{\max})) \cap C^{2,1}(\bar{\Omega} \times (0, T_{\max})) \cap L^\infty_{loc}((0, T_{\max}); W^{1,\vartheta}(\Omega)),
\end{aligned}
$$

where $\vartheta > N$ and $T_{\max} \in (0,+\infty]$ denotes the maximal existence time. Moreover, if $T_{\max} < \infty$, then as $t \nearrow T_{\max}$, we have

$$\|u(\cdot,t)\|_{L^\infty(\Omega)} + \|v(\cdot,t)\|_{L^\infty(\Omega)} + \|w(\cdot,t)\|_{W^{1,\infty}(\Omega)} + \|z(\cdot,t)\|_{W^{1,\infty}(\Omega)} \to \infty\,.$$

In view of Lemma 2.1, we know that for any $s_0 \in (0, T_{\max})$, $s_0 \le 1$ and $p > 1$, for all $\tau \in [0, s_0)$, there exists $K > 0$ satisfying

$$\|u(\cdot,\tau)\|_{L^\infty(\Omega)} + \|v(\cdot,\tau)\|_{L^\infty(\Omega)} + \|w(\cdot,\tau)\|_{W^{2,p}(\Omega)} + \|z(\cdot,\tau)\|_{W^{2,p}(\Omega)} \le K. \tag{2.1}$$

To prove the main theorem, we apply the following lemmas.

Lemma 2.2 ([3, 7]). *(Suppose that $\gamma \in (1,\infty)$, $g \in L^\gamma((0,T); L^\gamma(\Omega))$, and c is a solution to the initial boundary value problem*

$$\begin{gathered} c_t = \Delta c - c + g, \\ \frac{\partial c}{\partial \nu} = 0, \\ c(x,0) = c_0(x). \end{gathered}$$

Then there exists a positive constant C_γ such that if $s_0 \in [0,T)$ and $c(\cdot,s_0) \in W^{2,\gamma}(\Omega)$ with $\frac{\partial c(\cdot,s_0)}{\partial \nu} = 0$, one has

$$\begin{aligned} &\int_{s_0}^T e^{\gamma s}\|\Delta c(\cdot,s)\|^\gamma_{L^\gamma(\Omega)}\,ds \\ &\le C_\gamma\Big(\int_{s_0}^T e^{\gamma s}\|g(\cdot,s)\|^\gamma_{L^\gamma(\Omega)}\,ds + e^{\gamma s_0}(\|c(\cdot,s_0)\|^\gamma_{L^\gamma(\Omega)} + \|\Delta c(\cdot,s_0)\|^\gamma_{L^\gamma(\Omega)})\Big). \end{aligned}$$

Lemma 2.3 ([20]). *Let $(e^{t\Delta})_t \ge 0$ be the Neumann heat semigroup in Ω, $\lambda_1 > 0$ denote the first nonzero eigenvalue of $-\Delta$ in Ω under Neumann boundary conditions. Then there exists a constant C, if $1 \le q \le p \le \infty$, then*

$$\|\nabla e^{t\Delta} w\|_{L^p(\Omega)} \le C_2(1 + t^{-\frac{n}{2}(\frac{1}{q}-\frac{1}{p})})e^{-\lambda_1 t}\|w\|_{L^q(\Omega)} \quad \forall t > 0$$

holds for each $w \in L^q(\Omega)$.

Lemma 2.4 ([25]). *Let $y(t) \ge 0$ be a solution of problem*

$$\begin{gathered} y'(t) + Ay^p \le B, \quad t > 0, \\ y(0) = y_0, \end{gathered}$$

with $A > 0$, $p > 0$ and $B \ge 0$. Then for any $t > 0$, we have

$$y(t) \le \max\{y_0, (\frac{B}{A})^{1/p}\}.$$

3. A priori estimates

In this section, according to the L^p estimates of the parabolic equations, we obtain the a priori estimates which plays a vital role in proving our main result. The readers can find a proof of the following lemma in Zheng and Zhang [27].

Lemma 3.1. *Under the condition of Lemma 2.1, then there exists a nonnegative constant C such that the solution of (1.1) satisfies*

$$\int_\Omega u + \int_\Omega v \le C \quad \forall t \in (0, T_{\max}). \tag{3.1}$$

To address the difference betweenthe parabolic equation and the elliptic equation, we use lemma 2.1 and obtain the following conclusion. To prove the main lemma, is based on the different forms of r_1 and r_2, so we can prove the theorem in two different forms.

Lemma 3.2. *Suppose that $r_1 > 1$, $r_2 > 1$ and $(r_1-1)(r_2-1) > 1$. Then for any $p > 1$ and $q > 1$, there exists a constant $C > 0$ such that*

$$\int_\Omega u^q + \int_\Omega v^p \le C \quad \forall t \in (0, T_{\max}). \tag{3.2}$$

Proof. Assume any $p > 1$. Multiplying the second equation in (1.1) by v^{p-1} and integrating by parts, we have

$$\begin{aligned}
&\frac{1}{p}\frac{d}{dt}\int_\Omega v^p + (p-1)\int_\Omega v^{p-2}|\nabla v|^2 \\
&= \frac{p-1}{p}\xi\int_\Omega v^p\Delta z + \lambda_2\int_\Omega v^p - \mu_2\int_\Omega v^{p+r_2-1} - b\int_\Omega v^p u \\
&\le \frac{p-1}{p}\xi\int_\Omega v^p|\Delta z| - \mu_2\int_\Omega v^{p+r_2-1} + \lambda_2\int_\Omega v^p \quad \forall t \in (0, T_{\max}).
\end{aligned} \tag{3.3}$$

Now, using the Young inequality, for all $t \in (0, T_{\max})$, there are constants $C_1 > 0$ and $C_2 > 0$ such that

$$\frac{(p-1)\xi}{p}\int_\Omega v^p|\Delta z| \le \frac{\mu_2}{4}\int_\Omega v^{p+r_2-1} + C_1\int_\Omega |\Delta z|^{\frac{p+r_2-1}{r_2-1}}, \tag{3.4}$$

$$\lambda_2\int_\Omega v^p \le \frac{\mu_2}{2}\int_\Omega v^{p+r_2-1} + C_2. \tag{3.5}$$

Then combining (3.3), (3.4) with (3.5), we arrive at

$$\frac{d}{dt}\int_\Omega v^p \le -\frac{\mu_2 p}{4}\int_\Omega v^{p+r_2-1} + C_1 p\int_\Omega |\Delta z|^{\frac{p+r_2-1}{r_2-1}} + C_2 p \quad \forall t \in (0, T_{\max}). \tag{3.6}$$

Furthermore, adding $\frac{p+r_2-1}{r_2-1}\int_\Omega v^p$ at the both sides of (3.6), since $r_2 > 1$ and $t \in (0, T_{\max})$, we derive that

$$\begin{aligned}
&\frac{d}{dt}\int_\Omega v^p + \frac{p+_2-1}{r_2-1}\int_\Omega v^p \\
&\le -\frac{\mu_2 p}{4}\int_\Omega v^{p+r_2-1} + C_1 p\int_\Omega |\Delta z|^{\frac{p+r_2-1}{r_2-1}} + \frac{p+r_2-1}{r_2-1}\int_\Omega v^p + C_2 p.
\end{aligned} \tag{3.7}$$

Here, we estimate the third term on the right-hand side of (3.7). Then by using the Young inequality, there exists a constant $C_3 > 0$ such that

$$\frac{p+r_2-1}{r_2-1}\int_\Omega v^p \le \frac{p\mu_2}{8}\int_\Omega v^{p+r_2-1} + C_3 \quad \forall t \in (0, T_{\max}). \tag{3.8}$$

Combining (3.7) with (3.8), for all $t \in (0, T_{\max})$, we easily obtain

$$\frac{d}{dt}\int_\Omega v^p + \frac{p+r_2-1}{r_2-1}\int_\Omega v^p \le -\frac{p\mu_2}{8}\int_\Omega v^{p+r_2-1} + C_1 p\int_\Omega |\Delta z|^{\frac{p+r_2-1}{r_2-1}} + C_4 \tag{3.9}$$

with $C_4 = pC_2 + C_3 > 0$. Consequently, for all $t \in (0, T_{\max})$, the inequality (3.9) can be rewritten as follows

$$
\begin{aligned}
&\frac{d}{dt}\Big(e^{\frac{p+r_2-1}{r_2-1}t}\|v\|^p_{L^p(\Omega)}\Big)\\
&\le \Big(-\frac{\mu_2 p}{8}\int_\Omega v^{p+r_2-1} + C_1 p\int_\Omega |\Delta z|^{\frac{p+r_2-1}{r_2-1}} + C_4\Big)e^{\frac{p+r_2-1}{r_2-1}t}.
\end{aligned}
\tag{3.10}
$$

Let s_0 be the same as (2.1). Integrating (3.10) on (s_0, t), together with Lemma 2.2, (2.1), for all $t \in (0, T_{\max})$, one has

$$
\begin{aligned}
&\|v(\cdot,t)\|^p_{L^p(\Omega)}\\
&\le e^{-\frac{p+r_2-1}{r_2-1}(t-s_0)}\|v(\cdot,s_0)\|^p_{L^p(\Omega)} - \frac{\mu_2 p}{8}e^{-\frac{p+r_2-1}{r_2-1}t}\int_{s_0}^t e^{\frac{p+r_2-1}{r_2-1}s}\int_\Omega v^{p+r_2-1}\,dx\,ds\\
&\quad + C_1 p e^{-\frac{p+r_2-1}{r_2-1}t}\int_{s_0}^t e^{\frac{p+r_2-1}{r_2-1}s}\int_\Omega |\Delta z|^{\frac{p+r_2-1}{r_2-1}}\,dx\,ds + C_4\int_{s_0}^t e^{-\frac{p+r_2-1}{r_2-1}(t-s)}ds\\
&\le -\frac{\mu_2 p}{8}e^{-\frac{p+r_2-1}{r_2-1}t}\int_{s_0}^t e^{\frac{p+r_2-1}{r_2-1}s}\int_\Omega v^{p+r_2-1}\,dx\,ds\\
&\quad + C_1 C_\gamma p e^{-\frac{p+r_2-1}{r_2-1}t}\int_{s_0}^t e^{\frac{p+r_2-1}{r_2-1}s}\|u(\cdot,s)\|^{\frac{p+r_2-1}{r_2-1}}_{L^{\frac{p+r_2-1}{r_2-1}}(\Omega)}ds + C_5\\
&\quad + C_1 C_\gamma p e^{-\frac{p+r_2-1}{r_2-1}(t-s_0)}\Big(\|z(\cdot,s_0)\|^{\frac{p+r_2-1}{r_2-1}}_{L^{\frac{p+r_2-1}{r_2-1}}(\Omega)} + \|\Delta z(\cdot,s_0)\|^{\frac{p+r_2-1}{r_2-1}}_{L^{\frac{p+r_2-1}{r_2-1}}(\Omega)}\Big)\\
&\le -\frac{\mu_2 p}{8}e^{-\frac{p+r_2-1}{r_2-1}t}\int_{s_0}^t e^{\frac{p+r_2-1}{r_2-1}s}\int_\Omega v^{p+r_2-1}\,dx\,ds\\
&\quad + C_1 C_\gamma p e^{-\frac{p+r_2-1}{r_2-1}t}\int_{s_0}^t e^{\frac{p+r_2-1}{r_2-1}s}\|u(\cdot,s)\|^{\frac{p+r_2-1}{r_2-1}}_{L^{\frac{p+r_2-1}{r_2-1}}(\Omega)}ds + C_6
\end{aligned}
\tag{3.11}
$$

with positive constants $C_5 > 0$ and $C_6 > 0$ as well as C_γ is the same as Lemma 2.2.

Multiplying the first equation of (1.1) by u^{q-1} $(q > 1)$ and integrating by parts over Ω, according to the Young inequality, for all $t \in (0, T_{\max})$, there exist positive constants C_i $(i = 7, 8, 9)$ such that

$$
\begin{aligned}
&\frac{1}{q}\frac{d}{dt}\int_\Omega u^q + (q-1)\int_\Omega u^{q-2}|\nabla u|^2\\
&= -\frac{q-1}{q}\chi\int_\Omega u^q\Delta w + \int_\Omega u^q(\lambda_1 - \mu_1 u^{r_1-1} + av)\\
&\le -\frac{\mu_1}{4}\int_\Omega u^{q+r_1-1} + C_8\int_\Omega |\Delta w|^{\frac{q+r_1-1}{r_1-1}} + C_7\int_\Omega v^{\frac{q+r_1-1}{r_1-1}} + C_9.
\end{aligned}
\tag{3.12}
$$

Similarly, adding $\frac{q+r_1-1}{r_1-1}\int_\Omega u^q$ at both sides of (3.12) and using the Young inequality again, for all $t \in (0, T_{\max})$, there is a constant $C_{10} > 0$ such that

$$
\begin{aligned}
\frac{d}{dt}\int_\Omega u^q + \frac{q+r_1-1}{r_1-1}\int_\Omega u^q \le -\frac{\mu_1 q}{8}\int_\Omega u^{q+r_1-1} + C_8 q\int_\Omega |\Delta w|^{\frac{q+r_1-1}{r_1-1}}\\
+ C_7 q\int_\Omega v^{\frac{q+r_1-1}{r_1-1}} + C_{10}.
\end{aligned}
\tag{3.13}
$$

Next, multiplying $e^{\frac{q+r_1-1}{r_1-1}t}$ on both sides of (3.13). For all $t \in (s_0, T_{\max})$, we see that

$$\begin{aligned}\frac{d}{dt}\Big(e^{\frac{q+r_1-1}{r_1-1}t}\|u\|^q_{L^q(\Omega)}\Big) &\le \Big(-\frac{\mu_1 q}{8}\int_\Omega u^{q+r_1-1} + C_7 q\int_\Omega v^{\frac{q+r_1-1}{r_1-1}} \\ &\quad + C_8 q\int_\Omega |\Delta w|^{\frac{q+r_1-1}{r_1-1}} + C_{10}\Big)e^{\frac{q+r_1-1}{r_1-1}t}.\end{aligned} \tag{3.14}$$

Integrating (3.14) over (s_0, t), for all $t \in (0, T_{\max})$, there exist positive constants C_{11} and C_{12} such that

$$\begin{aligned}\|u(\cdot,t)\|^q_{L^q(\Omega)} &\le e^{-\frac{q+r_1-1}{r_1-1}(t-s_0)}\|u(\cdot,s_0)\|^q_{L^q(\Omega)} \\ &\quad - \frac{\mu_1 q}{8}e^{-\frac{q+r_1-1}{r_1-1}t}\int_{s_0}^t e^{\frac{q+r_1-1}{r_1-1}s}\int_\Omega u^{q+r_1-1}\,dx\,ds \\ &\quad + C_7 q e^{-\frac{q+r_1-1}{r_1-1}t}\int_{s_0}^t e^{\frac{q+r_1-1}{r_1-1}s}\int_\Omega v^{\frac{q+r_1-1}{r_1-1}}\,dx\,ds \\ &\quad + C_8 q e^{-\frac{q+r_1-1}{r_1-1}t}\int_{s_0}^t e^{\frac{q+r_1-1}{r_1-1}s}\|\Delta w(\cdot,s)\|^{\frac{q+r_1-1}{r_1-1}}_{L^{\frac{q+r_1-1}{r_1-1}}(\Omega)}ds \\ &\quad + C_{10}\int_{s_0}^t e^{-\frac{q+r_1-1}{r_1-1}(t-s)}ds \\ &\le -\frac{\mu_1 q}{8}e^{-\frac{q+r_1-1}{r_1-1}t}\int_{s_0}^t e^{\frac{q+r_1-1}{r_1-1}s}\int_\Omega u^{q+r_1-1}\,dx\,ds \\ &\quad + C_{11} q e^{-\frac{q+r_1-1}{r_1-1}t}\int_{s_0}^t e^{\frac{q+r_1-1}{r_1-1}s}\|v(\cdot,s)\|^{\frac{q+r_1-1}{r_1-1}}_{L^{\frac{q+r_1-1}{r_1-1}}(\Omega)}ds + C_{12}.\end{aligned} \tag{3.15}$$

Since $r_1 > 1$, $r_2 > 1$ and $(r_1-1)(r_2-1) > 1$, we can choose the appropriate numbers q and p such that

$$\frac{q+r_1-1}{r_1-1} < p + r_2 - 1, \tag{3.16}$$

$$\frac{p+r_2-1}{r_2-1} < q + r_1 - 1. \tag{3.17}$$

Combining (3.11) with (3.15), for all $p > 1$ and $q > 1$, there exist positive constants C_{13} and C_{14} such that

$$\begin{aligned}&\|u\|^q_{L^q(\Omega)} + \|v\|^p_{L^p(\Omega)} \\ &\le -\frac{\mu_2 p}{8}e^{-\frac{p+r_2-1}{r_2-1}t}\int_{s_0}^t e^{\frac{p+r_2-1}{r_2-1}s}\int_\Omega v^{p+r_2-1}\,dx\,ds \\ &\quad + C_1 C_\gamma p e^{-\frac{p+r_2-1}{r_2-1}t}\int_{s_0}^t e^{\frac{p+r_2-1}{r_2-1}s}\|u\|^{\frac{p+r_2-1}{r_2-1}}_{L^{\frac{p+r_2-1}{r_2-1}}(\Omega)}ds \\ &\quad - \frac{\mu_1 q}{8}e^{-\frac{q+r_1-1}{r_1-1}t}\int_{s_0}^t e^{\frac{q+r_1-1}{r_1-1}s}\int_\Omega u^{q+r_1-1}\,dx\,ds \\ &\quad + C_{11} q e^{-\frac{q+r_1-1}{r_1-1}t}\int_{s_0}^t e^{\frac{q+r_1-1}{r_1-1}s}\int_\Omega v^{\frac{q+r_1-1}{r_1-1}}\,dx\,ds + C_6 + C_{12} \\ &\le -\frac{\mu_2 p}{16}e^{-\frac{p+r_2-1}{r_2-1}t}\int_{s_0}^t e^{\frac{p+r_2-1}{r_2-1}s}\int_\Omega v^{p+r_2-1}\,dx\,ds\end{aligned}$$

$$
\begin{aligned}
&- \frac{\mu_1 q}{16} e^{-\frac{q+r_1-1}{r_1-1} t} \int_{s_0}^{t} e^{\frac{q+r_1-1}{r_1-1} s} \int_\Omega u^{q+r_1-1}\, dx\, ds + C_{13} \\
&\le C_{14} \quad \forall t \in (s_0, T_{\max}).
\end{aligned}
$$

The above estimate together with (2.1) implies Lemma 3.2. □

Lemma 3.3. *Suppose that $r_1 > 1$, $r_2 > 1$, if $(r_1-1)(r_2-1) = 1$, and μ_1, μ_2 are arbitrarily big. Then for each $p > 1$, there exists $C > 0$ such that*

$$
\int_\Omega u^p + \int_\Omega v^p \le C \quad \forall t \in (0, T_{\max}). \tag{3.18}
$$

Proof. Since $(r_1-1)(r_2-1) = 1$, we can choose $r > 1$ and $\widetilde{r} > 1$ are appropriately big such that

$$
\begin{aligned}
\widetilde{r} + r_1 - 1 &= \frac{r + r_2 - 1}{r_2 - 1}, \\
r + r_2 - 1 &= \frac{\widetilde{r} + r_1 - 1}{r_1 - 1}.
\end{aligned} \tag{3.19}
$$

Multiplying both sides of the second equation in (1.1) by v^{r-1} and integrating by parts, we have

$$
\begin{aligned}
&\frac{1}{r}\frac{d}{dt}\int_\Omega v^r + (r-1)\int_\Omega v^{r-2}|\nabla v|^2 \\
&= \frac{r-1}{r}\xi \int_\Omega v^r \Delta z + \lambda_2 \int_\Omega v^r - \mu_2 \int_\Omega v^{r+r_2-1} - b\int_\Omega v^r u \\
&\le \frac{r-1}{r}\kappa \int_\Omega v^r |\Delta z| - \mu_2 \int_\Omega v^{r+r_2-1} + \lambda_2 \int_\Omega v^r \quad \forall t \in (0, T_{\max}).
\end{aligned} \tag{3.20}
$$

Define $\kappa := \max\{\xi, \chi\}$. Then, adding $\frac{r+r_2-1}{r_2-1}\int_\Omega v^r$ at both sides of the above inequality, we have

$$
\begin{aligned}
\frac{1}{r}\frac{d}{dt}\int_\Omega v^r &\le \frac{r-1}{r}\kappa \int_\Omega v^r |\Delta z| - \mu_2 \int_\Omega v^{r+r_2-1} + \lambda_2 \int_\Omega v^r \\
&\quad + \frac{r+r_2-1}{r_2-1}\int_\Omega v^r - \frac{r+r_2-1}{r_2-1}\int_\Omega v^r \\
&= \int_\Omega \Big[(\lambda_2 + \frac{r+r_2-1}{r_2-1}) v^r - \mu_2 v^{r+r_2-1}\Big] + \frac{r-1}{r}\kappa \int_\Omega v^r |\Delta z| \\
&\quad - \frac{r+r_2-1}{r_2-1}\int_\Omega v^r \quad \forall t \in (0, T_{\max}).
\end{aligned} \tag{3.21}
$$

Along with Young's inequality, this concludes that

$$
\int_\Omega (\lambda_2 + \frac{r+r_2-1}{r_2-1}) v^r - \int_\Omega \mu_2 v^{r+r_2-1} \le (\varepsilon_2 - \mu_2)\int_\Omega v^{r+r_2-1} + C_1(\varepsilon_2, r) \tag{3.22}
$$

with some constants $\varepsilon_2 > 0$ and

$$
C_1(\varepsilon_2, r) = \frac{r_2-1}{r+r_2-1}(\varepsilon_2 \frac{r+r_2-1}{r})^{-\frac{r}{r_2-1}} (\lambda_2 + \frac{r+r_2-1}{r_2-1})^{\frac{r+r_2-1}{r_2-1}} |\Omega|.
$$

Next, for each $\lambda_0 > 0$, applying Young's inequality again, we obtain

$$
\begin{aligned}
&\frac{r-1}{r}\kappa\int_\Omega v^r|\Delta z|\\
&\le \lambda_0\int_\Omega v^{r+r_2-1} + \frac{r_2-1}{r+r_2-1}\big(\lambda_0\frac{r+r_2-1}{r}\big)^{-\frac{r}{r_2-1}}\\
&\quad\times\big(\frac{r-1}{r}\kappa\big)^{\frac{r+r_2-1}{r_2-1}}\int_\Omega|\Delta z|^{\frac{r+r_2-1}{r_2-1}}\\
&= \lambda_0\int_\Omega v^{r+r_2-1} + A_1\lambda_0^{-\frac{r}{r_2-1}}\kappa^{\frac{r+r_2-1}{r_2-1}}\int_\Omega|\Delta z|^{\frac{r+r_2-1}{r_2-1}}\quad \forall t\in(0,T_{\max}),
\end{aligned}
\tag{3.23}
$$

where

$$
A_1 = \frac{r_2-1}{r+r_2-1}\big(\frac{r+r_2-1}{r}\big)^{-\frac{r}{r_2-1}}\big(\frac{r-1}{r}\big)^{\frac{r+r_2-1}{r_2-1}}. \tag{3.24}
$$

Combining (3.22), (3.23) with (3.21), for all $t\in(0,T_{\max})$, we easily obtain

$$
\begin{aligned}
\frac{1}{r}\frac{d}{dt}\int_\Omega v^r &\le (\varepsilon_2+\lambda_0-\mu_2)\int_\Omega v^{r+r_2-1} - \frac{r+r_2-1}{r_2-1}\int_\Omega v^r\\
&\quad + A_1\lambda_0^{-\frac{r}{r_2-1}}\kappa^{\frac{r+r_2-1}{r_2-1}}\int_\Omega|\Delta z|^{\frac{r+r_2-1}{r_2-1}} + C_1(\varepsilon_2,r).
\end{aligned}
\tag{3.25}
$$

Let s_0 be the same as (2.1), then for any $t\in(s_0,T_{\max})$, applying the variation-of-constants formula to the above inequality, we have

$$
\begin{aligned}
&\frac{1}{r}\int_\Omega v^r\,dx\\
&\le \frac{1}{r}e^{-\frac{r+r_2-1}{r_2-1}(t-s)}\|v(\cdot,s_0)\|^r_{L^r(\Omega)} + (\varepsilon_2+\lambda_0-\mu_2)\int_{s_0}^t e^{-\frac{r+r_2-1}{r_2-1}(t-s)}\int_\Omega v^{r+r_2-1}\\
&\quad + A_1\lambda_0^{-\frac{r}{r_2-1}}\kappa^{\frac{r+r_2-1}{r_2-1}}\int_{s_0}^t e^{-\frac{r+r_2-1}{r_2-1}(t-s)}\int_\Omega|\Delta z|^{\frac{r+r_2-1}{r_2-1}} + C_1(\varepsilon_2,r)\int_{s_0}^t e^{-\frac{r+r_2-1}{r_2-1}(t-s)}\\
&\le (\varepsilon_2+\lambda_0-\mu_2)\int_{s_0}^t e^{-\frac{r+r_2-1}{r_2-1}(t-s)}\int_\Omega v^{r+r_2-1}\\
&\quad + A_1\lambda_0^{-\frac{r}{r_2-1}}\kappa^{\frac{r+r_2-1}{r_2-1}}\int_{s_0}^t e^{-\frac{r+r_2-1}{r_2-1}(t-s)}\int_\Omega|\Delta z|^{\frac{r+r_2-1}{r_2-1}} + C_2(\varepsilon_2,r),
\end{aligned}
\tag{3.26}
$$

where

$$
C_2 := C_2(\varepsilon_2,r) = \frac{1}{r}\|v(\cdot,s_0)\|^r_{L^r(\Omega)} + C_1(\varepsilon_2,r)\frac{r_2-1}{r+r_2-1}.
$$

In view of Lemma 2.2, it follows that

$$
\begin{aligned}
&A_1\lambda_0^{-\frac{r}{r_2-1}}\kappa^{\frac{r+r_2-1}{r_2-1}}\int_{s_0}^t e^{-\frac{r+r_2-1}{r_2-1}(t-s)}\int_\Omega|\Delta z|^{\frac{r+r_2-1}{r_2-1}}\\
&= A_1\lambda_0^{-\frac{r}{r_2-1}}\kappa^{\frac{r+r_2-1}{r_2-1}}e^{-\frac{r+r_2-1}{r_2-1}t}\int_{s_0}^t e^{\frac{r+r_2-1}{r_2-1}s}\int_\Omega|\Delta z|^{\frac{r+r_2-1}{r_2-1}}\\
&\le A_1\lambda_0^{-\frac{r}{r_2-1}}\kappa^{\frac{r+r_2-1}{r_2-1}}e^{-\frac{r+r_2-1}{r_2-1}t}C_{r+1}\Big[\int_{s_0}^t\int_\Omega e^{\frac{r+r_2-1}{r_2-1}s}u^{\frac{r+r_2-1}{r_2-1}}\\
&\quad + e^{\frac{r+r_2-1}{r_2-1}s_0}\Big(\|z(\cdot,s_0)\|^{\frac{r+r_2-1}{r_2-1}}_{L^{\frac{r+r_2-1}{r_2-1}}(\Omega)} + \|\Delta z(\cdot,s_0)\|^{\frac{r+r_2-1}{r_2-1}}_{L^{\frac{r+r_2-1}{r_2-1}}(\Omega)}\Big)\Big]
\end{aligned}
\tag{3.27}
$$

with $C_{r+1} > 0$ for all $t \in (s_0, T_{\max})$. Substituting (3.27) into (3.26), for all $t \in (s_0, T_{\max})$, we obtain

$$\begin{aligned}\frac{1}{r}\int_\Omega v^r \le (\varepsilon_2+\lambda_0-\mu_2)&\int_{s_0}^t e^{-\frac{r+r_2-1}{r_2-1}(t-s)}\int_\Omega v^{r+r_2-1}\\ &+A_1\lambda_0^{-\frac{r}{r_2-1}}\kappa^{\frac{r+r_2-1}{r_2-1}}C_{r+1}\int_{s_0}^t e^{-\frac{r+r_2-1}{r_2-1}(t-s)}\int_\Omega u^{\frac{r+r_2-1}{r_2-1}}\\ &+A_1\lambda_0^{-\frac{r}{r_2-1}}\kappa^{\frac{r+r_2-1}{r_2-1}}e^{-\frac{r+r_2-1}{r_2-1}(t-s_0)}C_{r+1}\widetilde{M}+C_2(\varepsilon_2,r),\end{aligned}\tag{3.28}$$

where

$$\widetilde{M}=\|z(\cdot,s_0)\|^{\frac{r+r_2-1}{r_2-1}}_{L^{\frac{r+r_2-1}{r_2-1}}(\Omega)}+\|\Delta z(\cdot,s_0)\|^{\frac{r+r_2-1}{r_2-1}}_{L^{\frac{r+r_2-1}{r_2-1}}(\Omega)}.$$

To deal with u, multiplying both sides of the second equation in (1.1) by $u^{\widetilde{r}-1}$ and integrating by parts, for any small $\varepsilon \in (0,1)$, we derive from Young's inequality that

$$\begin{aligned}&\frac{1}{\widetilde{r}}\frac{d}{dt}\int_\Omega u^{\widetilde{r}}+(\widetilde{r}-1)\int_\Omega u^{\widetilde{r}-2}|\nabla u|^2\\ &=-\frac{\widetilde{r}-1}{\widetilde{r}}\chi\int_\Omega u^{\widetilde{r}}\Delta w+\lambda_1\int_\Omega u^{\widetilde{r}}-\mu_1\int_\Omega u^{\widetilde{r}+r_1-1}+a\int_\Omega u^{\widetilde{r}}v\\ &\le\frac{\widetilde{r}-1}{\widetilde{r}}\kappa\int_\Omega u^{\widetilde{r}}|\Delta w|+\lambda_1\int_\Omega u^{\widetilde{r}}-\mu_1\int_\Omega u^{\widetilde{r}+r_1-1}\\ &\quad+\varepsilon\int_\Omega u^{\widetilde{r}+r_1-1}+C_3\int_\Omega v^{\frac{\widetilde{r}+r_1-1}{r_1-1}}\quad\forall t\in(0,T_{\max}),\end{aligned}\tag{3.29}$$

where

$$C_3=\frac{\widetilde{r}+r_1-1}{r_1-1}(\varepsilon\frac{\widetilde{r}+r_1-1}{\widetilde{r}})^{-\frac{\widetilde{r}}{r_1-1}}a^{\frac{\widetilde{r}+r_1-1}{r_1-1}}.$$

Then we conclude that

$$\begin{aligned}\frac{1}{\widetilde{r}}\frac{d}{dt}\int_\Omega u^{\widetilde{r}}\le -\frac{\widetilde{r}+r_1-1}{r_1-1}&\int_\Omega u^{\widetilde{r}}+\frac{\widetilde{r}-1}{\widetilde{r}}\kappa\int_\Omega u^{\widetilde{r}}|\Delta w|\\ &+\int_\Omega\Big(\lambda_1u^{\widetilde{r}}+\frac{\widetilde{r}+r_1-1}{r_1-1}u^{\widetilde{r}}-\mu_1u^{\widetilde{r}+r_1-1}\Big)\\ &+\varepsilon\int_\Omega u^{\widetilde{r}+r_1-1}+C_3\int_\Omega v^{\frac{\widetilde{r}+r_1-1}{r_1-1}}\quad\forall t\in(0,T_{\max}).\end{aligned}\tag{3.30}$$

For each $\varepsilon_1 \in (0,1)$, using Young's inequality again, we have

$$\begin{aligned}&\int_\Omega\Big(\lambda_1u^{\widetilde{r}}+\frac{\widetilde{r}+r_1-1}{r_1-1}u^{\widetilde{r}}-\mu_1u^{\widetilde{r}+r_1-1}\Big)\\ &\le(\varepsilon_1-\mu_1)\int_\Omega u^{\widetilde{r}+r_1-1}+C_4(\varepsilon_1,\widetilde{r})\quad\forall t\in(0,T_{\max})\end{aligned}\tag{3.31}$$

with

$$C_4:=C_4(\varepsilon_1,\widetilde{r})=\frac{r_1-1}{\widetilde{r}+r_1-1}(\varepsilon_1\frac{\widetilde{r}+r_1-1}{\widetilde{r}})^{-\frac{\widetilde{r}}{r_1-1}}(\lambda_1+\frac{\widetilde{r}+r_1-1}{r_1-1})^{\frac{\widetilde{r}+r_1-1}{r_1-1}}|\Omega|.$$

By applying Young's inequality again, we obtain

$$\begin{aligned}
&\frac{\widetilde{r}-1}{\widetilde{r}}\kappa\int_\Omega u^{\widetilde{r}}|\Delta w| \\
&\le \lambda_0\int_\Omega u^{\widetilde{r}+r_1-1}+\frac{r_1-1}{\widetilde{r}+r_1-1}\big(\lambda_0\frac{\widetilde{r}+r_1-1}{\widetilde{r}}\big)^{-\frac{\widetilde{r}}{r_1-1}} \\
&\quad\times\big(\frac{\widetilde{r}-1}{\widetilde{r}}\kappa\big)^{\frac{\widetilde{r}+r_1-1}{r_1-1}}\int_\Omega|\Delta w|^{\frac{\widetilde{r}+r_1-1}{r_1-1}} \\
&=\lambda_0\int_\Omega u^{\widetilde{r}+r_1-1}+\tilde{A}_1\lambda_0^{-\frac{\widetilde{r}}{r_1-1}}\kappa^{\frac{\widetilde{r}+r_1-1}{r_1-1}}\int_\Omega|\Delta w|^{\frac{\widetilde{r}+r_1-1}{r_1-1}}\quad\forall t\in(0,T_{\max}),
\end{aligned}\tag{3.32}$$

where

$$\tilde{A}_1=\frac{r_1-1}{\widetilde{r}+r_1-1}\big(\frac{\widetilde{r}+r_1-1}{\widetilde{r}}\big)^{-\frac{\widetilde{r}}{r_1-1}}\big(\frac{\widetilde{r}-1}{\widetilde{r}}\big)^{\frac{\widetilde{r}+r_1-1}{r_1-1}}.\tag{3.33}$$

Combing (3.31), (3.32) with (3.30), for all $t\in(0,T_{\max})$, it follows that

$$\begin{aligned}
\frac{1}{\widetilde{r}}\frac{d}{dt}\int_\Omega u^{\widetilde{r}}&\le(\varepsilon_1+\lambda_0-\mu_1)\int_\Omega u^{\widetilde{r}+r_1-1}+\tilde{A}_1\lambda_0^{-\frac{\widetilde{r}}{r_1-1}}\kappa^{\frac{\widetilde{r}+r_1-1}{r_1-1}}\int_\Omega|\Delta w|^{\frac{\widetilde{r}+r_1-1}{r_1-1}} \\
&\quad+\varepsilon\int_\Omega u^{\widetilde{r}+r_1-1}+C_3\int_\Omega v^{\frac{\widetilde{r}+r_1-1}{r_1-1}}+C_4(\varepsilon_1,\widetilde{r}).
\end{aligned}\tag{3.34}$$

Applying the variation-of-constants formula to the above inequality again, for all $t\in(0,T_{\max})$, there exist $C_5>0$ and $\widetilde{C}_{\widetilde{r}+1}>0$ such that

$$\begin{aligned}
\frac{1}{\widetilde{r}}\int_\Omega u^{\widetilde{r}}&\le(\varepsilon_1+\lambda_0-\mu_1)\int_{s_0}^t e^{-\frac{\widetilde{r}+r_1-1}{r_1-1}(t-s)}\int_\Omega u^{\widetilde{r}+r_1-1} \\
&\quad+\tilde{A}_1\lambda_0^{-\frac{\widetilde{r}}{r_1-1}}\kappa^{\frac{\widetilde{r}+r_1-1}{r_1-1}}\widetilde{C}_{\widetilde{r}+1}\int_{s_0}^t e^{-\frac{\widetilde{r}+r_1-1}{r_1-1}(t-s)}\int_\Omega v^{\frac{\widetilde{r}+r_1-1}{r_1-1}} \\
&\quad+\tilde{A}_1\lambda_0^{-\frac{\widetilde{r}}{r_1-1}}\kappa^{\frac{\widetilde{r}+r_1-1}{r_1-1}}e^{-\frac{\widetilde{r}+r_1-1}{r_1-1}(t-s_0)}\widetilde{C}_{\widetilde{r}+1}\widetilde{N} \\
&\quad+\varepsilon\int_{s_0}^t e^{-\frac{\widetilde{r}+r_1-1}{r_1-1}(t-s)}\int_\Omega u^{\widetilde{r}+r_1-1} \\
&\quad+C_3\int_{s_0}^t e^{-\frac{\widetilde{r}+r_1-1}{r_1-1}(t-s)}\int_\Omega v^{\frac{\widetilde{r}+r_1-1}{r_1-1}}+C_5 \\
&\le(\varepsilon_1+\varepsilon+\lambda_0-\mu_1)\int_{s_0}^t e^{-\frac{\widetilde{r}+r_1-1}{r_1-1}(t-s)}\int_\Omega u^{\widetilde{r}+r_1-1} \\
&\quad+\tilde{A}_1\lambda_0^{-\frac{\widetilde{r}}{r_1-1}}\kappa^{\frac{\widetilde{r}+r_1-1}{r_1-1}}\widetilde{C}_{\widetilde{r}+1}\int_{s_0}^t e^{-\frac{\widetilde{r}+r_1-1}{r_1-1}(t-s)}\int_\Omega v^{\frac{\widetilde{r}+r_1-1}{r_1-1}} \\
&\quad+\tilde{A}_1\lambda_0^{-\frac{\widetilde{r}}{r_1-1}}\kappa^{\frac{\widetilde{r}+r_1-1}{r_1-1}}e^{-\frac{\widetilde{r}+r_1-1}{r_1-1}(t-s_0)}\widetilde{C}_{\widetilde{r}+1}\widetilde{N}+C_5
\end{aligned}\tag{3.35}$$

with

$$\widetilde{N}=\|w(\cdot,s_0)\|^{\frac{\widetilde{r}+r_1-1}{r_1-1}}_{L^{\frac{\widetilde{r}+r_1-1}{r_1-1}}(\Omega)}+\|\Delta w(\cdot,s_0)\|^{\frac{\widetilde{r}+r_1-1}{r_1-1}}_{L^{\frac{\widetilde{r}+r_1-1}{r_1-1}}(\Omega)}.$$

Adding (3.28) to (3.35) yields

$$
\begin{aligned}
\frac{1}{\widetilde{r}}\int_\Omega u^{\widetilde{r}} + \frac{1}{r}\int_\Omega v^r \le{}& (\varepsilon_1+\varepsilon+\lambda_0-\mu_1)\int_{s_0}^t e^{-\frac{\widetilde{r}+r_1-1}{r_1-1}(t-s)}\int_\Omega u^{\widetilde{r}+r_1-1}\\
&+ \tilde{A}_1\lambda_0^{-\frac{\widetilde{r}}{r_1-1}}\kappa^{\frac{\widetilde{r}+r_1-1}{r_1-1}}\widetilde{C}_{\widetilde{r}+1}\int_{s_0}^t e^{-\frac{\widetilde{r}+r_1-1}{r_1-1}(t-s)}\int_\Omega v^{\frac{\widetilde{r}+r_1-1}{r_1-1}}\\
&+ (\varepsilon_2+\lambda_0-\mu_2)\int_{s_0}^t e^{-\frac{r+r_2-1}{r_2-1}(t-s)}\int_\Omega v^{r+r_2-1}\\
&+ A_1\lambda_0^{-\frac{r}{r_2-1}}\kappa^{\frac{r+r_2-1}{r_2-1}}C_{r+1}\int_{s_0}^t e^{-\frac{r+r_2-1}{r_2-1}(t-s)}\int_\Omega u^{\frac{r+r_2-1}{r_2-1}}\\
&+ \tilde{A}_1\lambda_0^{-\frac{\widetilde{r}}{r_1-1}}\kappa^{\frac{\widetilde{r}+r_1-1}{r_1-1}}e^{-\frac{\widetilde{r}+r_1-1}{r_1-1}(t-s_0)}\widetilde{C}_{\widetilde{r}+1}\widetilde{N}\\
&+ A_1\lambda_0^{-\frac{r}{r_2-1}}\kappa^{\frac{r+r_2-1}{r_2-1}}e^{-\frac{r+r_2-1}{r_2-1}(t-s_0)}C_{r+1}\widetilde{M}+C_6
\end{aligned}
\tag{3.36}
$$

with $C_6>0$ and for all $t\in(0,T_{\max})$. Recalling (3.19), we can rewrite (3.36) as

$$
\begin{aligned}
&\frac{1}{\widetilde{r}}\int_\Omega u^{\widetilde{r}} + \frac{1}{r}\int_\Omega v^r\\
&\le (\varepsilon_1+\varepsilon+\lambda_0+A_1\lambda_0^{-\frac{r}{r_2-1}}\kappa^{\frac{r+r_2-1}{r_2-1}}C_{r+1}-\mu_1)\int_{s_0}^t e^{-\frac{\widetilde{r}+r_1-1}{r_1-1}(t-s)}\int_\Omega u^{\widetilde{r}+r_1-1}\\
&\quad+ (\varepsilon_2+\lambda_0+\tilde{A}_1\lambda_0^{-\frac{\widetilde{r}}{r_1-1}}\kappa^{\frac{\widetilde{r}+r_1-1}{r_1-1}}\widetilde{C}_{\widetilde{r}+1}-\mu_2)\int_{s_0}^t e^{-\frac{r+r_2-1}{r_2-1}(t-s)}\int_\Omega v^{r+r_2-1}\\
&\quad+ \tilde{A}_1\lambda_0^{-\frac{\widetilde{r}}{r_1-1}}\kappa^{\frac{\widetilde{r}+r_1-1}{r_1-1}}e^{-\frac{\widetilde{r}+r_1-1}{r_1-1}(t-s_0)}\widetilde{C}_{\widetilde{r}+1}\widetilde{N}\\
&\quad+ A_1\lambda_0^{-\frac{r}{r_2-1}}\kappa^{\frac{r+r_2-1}{r_2-1}}e^{-\frac{r+r_2-1}{r_2-1}(t-s_0)}C_{r+1}\widetilde{M}+C_6\quad\forall t\in(s_0,T_{\max}).
\end{aligned}
$$

Moreover, we define

$$
g_1(\lambda_0,r)=\lambda_0+\max\{A_1\kappa^{\frac{r+r_2-1}{r_2-1}}C_{r+1},\tilde{A}_1\kappa^{\frac{\widetilde{r}+r_1-1}{r_1-1}}\widetilde{C}_{\widetilde{r}+1}\}[\lambda_0^{-\frac{r}{r_2-1}}+\lambda_0^{-\frac{\widetilde{r}}{r_1-1}}].
$$

After performing some basic calculations, we can obtain there is $\eta_0>0$ such that

$$
g_1(\eta_0,r):=\min_{\lambda_0>0}g_1(\lambda_0,r).
$$

For the above $\lambda_0:=\eta_0$, we choose μ_1 and μ_2 is appropriately large such that

$$
\begin{aligned}
\lambda_0+A_1\lambda_0^{-\frac{\frac{N}{2}}{r_2-1}}\kappa^{\frac{\frac{N}{2}+r_2-1}{r_2-1}}C_{\frac{N}{2}+1}<\mu_1,\\
\lambda_0+\tilde{A}_1\lambda_0^{-\frac{\frac{N}{2}}{r_1-1}}\kappa^{\frac{\frac{N}{2}+r_1-1}{r_1-1}}\widetilde{C}_{\frac{N}{2}+1}<\mu_2.
\end{aligned}
$$

Therefore, one can pick $r>\frac{N}{2}$ and $\widetilde{r}>\frac{N}{2}$ and $\varepsilon_1,\varepsilon,\varepsilon_2$ small enough such that

$$
\begin{aligned}
\varepsilon_1+\varepsilon+\lambda_0+A_1\lambda_0^{-\frac{r}{r_2-1}}\kappa^{\frac{r+r_2-1}{r_2-1}}C_{r+1}<\mu_1,\\
\varepsilon_2+\lambda_0+\tilde{A}_1\lambda_0^{-\frac{\widetilde{r}}{r_1-1}}\kappa^{\frac{\widetilde{r}+r_1-1}{r_1-1}}\widetilde{C}_{\widetilde{r}+1}<\mu_2.
\end{aligned}
$$

Then in light of (3.36), there exist positive constants C_7 and C_8 such that

$$
\int_\Omega u^{\widetilde{r}}\le C_7\quad\forall t\in(s_0,T_{\max}),
\tag{3.37}
$$

$$\int_\Omega v^r \le C_8 \quad \forall t \in (s_0, T_{\max}). \tag{3.38}$$

Let $q_0 = \min\{r, \widetilde{r}\}$, since $r > \frac{N}{2}$, $\widetilde{r} > \frac{N}{2}$, then we can get $q_0 > \frac{N}{2}$. Now, we fix $q < \frac{Nq_0}{(N-q_0)_+}$ and choose some $\alpha > \frac{1}{2}$ such that

$$q < \frac{1}{\frac{1}{q_0} - \frac{1}{N} + \frac{2}{N}(\alpha - \frac{1}{2})} \le \frac{Nq_0}{(N-q_0)_+}. \tag{3.39}$$

Applying the variation-of-constants formula to w, we obtain

$$w(\cdot,t) = e^{-(t-s_0)(A+1)} w(\cdot, s_0) + \int_{s_0}^t e^{-(t-s)(A+1)} v(\cdot, s) ds \tag{3.40}$$

for all $t \in (s_0, T_{\max})$, where $A := A_p$ denotes the sectorial operator defined by

$$A_p w := -\Delta w \ for\ all\ w \in D(A_p) := \{\varphi \in W^{2,p}(\Omega) | \frac{\partial \varphi}{\partial \nu}|_{\partial\Omega = 0}\}.$$

According to (2.1) and (3.40), there is a positive constant C_9 such that

$$\begin{aligned}
&\|(A+1)^\alpha w(\cdot,t)\|_{L^q(\Omega)} \\
&\le C_9 \int_{s_0}^t (t-s)^{-\alpha - \frac{N}{2}(\frac{1}{q_0} - \frac{1}{q})} e^{-\lambda_1 (t-s)} \|v(\cdot,s)\|_{L^{q_0}(\Omega)} ds \\
&\quad + C_9 s_0^{-\alpha - \frac{N}{2}(1-\frac{1}{q})} \|w(\cdot,s_0)\|_{L^1(\Omega)} ds \\
&\le C_8 C_9 \int_{s_0}^\infty \delta^{-\alpha - \frac{N}{2}(\frac{1}{q_0} - \frac{1}{q})} e^{-\lambda_1 \delta} d\delta + C_9 s_0^{-\alpha - \frac{N}{2}(1-\frac{1}{q})} K,
\end{aligned} \tag{3.41}$$

where s_0 is same as the parameter in (2.1). From (3.39) and (3.41), for a positive constant C_{10}, we have

$$\int_\Omega |\nabla w|^q \le C_{10} \quad \forall t \in (0, T_{\max}) \text{ and } q \in [1, \frac{Nq_0}{(N-q_0)_+}). \tag{3.42}$$

Similarly, for $C_{11} > 0$, we obtain

$$\int_\Omega |\nabla z|^q \le C_{11} \quad \forall t \in (0, T_{\max}) \text{ and } q \in [1, \frac{Nq_0}{(N-q_0)_+}). \tag{3.43}$$

Multiplying the second equation of (1.1) by v^{p-1} and using Young's inequality, for all $t \in (0, T_{\max})$, we have

$$\begin{aligned}
&\frac{1}{p}\frac{d}{dt}\int_\Omega v^p + (p-1)\int_\Omega v^{p-2}|\nabla v|^2 \\
&= -(p-1)\xi \int_\Omega v^{p-1} \nabla v \cdot \nabla z + \lambda_2 \int_\Omega v^p - \mu_2 \int_\Omega v^{p+r_2-1} - b\int_\Omega v^p u \\
&\le \frac{p-1}{2}\int_\Omega v^{p-2}|\nabla v|^2 + \frac{\xi^2(p-1)}{2}\int_\Omega v^p |\nabla z|^2 - \mu_2 \int_\Omega v^{p+r_2-1} + \lambda_2 \int_\Omega v^p \\
&\le \frac{p-1}{2}\int_\Omega v^{p-2}|\nabla v|^2 + \frac{\xi^2(p-1)}{2}\int_\Omega v^p |\nabla z|^2 - \frac{\mu_2}{2}\int_\Omega v^{p+r_2-1} + C_{12}
\end{aligned} \tag{3.44}$$

with a constant $C_{12} > 0$. Since $q_0 > \frac{N}{2}$ implies $q_0 < \frac{Nq_0}{2(N-q_0)_+}$, there is a constant $C_{13} > 0$ such that

$$\begin{aligned}\frac{\xi^2(p-1)}{2}\int_\Omega v^p|\nabla z|^2 &\le \frac{\xi^2(p-1)}{2}\Big(\int_\Omega v^{\frac{q_0}{q_0-1}p}\Big)^{\frac{q_0-1}{q_0}}\Big(\int_\Omega |\nabla z|^{2q_0}\Big)^{1/q_0}\\ &\le C_{13}\|v^{\frac{p}{2}}\|^2_{L^{\frac{2q_0}{q_0-1}}(\Omega)} \quad \forall t\in(0,T_{\max}).\end{aligned} \tag{3.45}$$

Let $p > q_0 - 1$ and it follows that

$$\frac{q_0}{p} < \frac{q_0}{q_0-1} < \frac{N}{(N-2)_+}.$$

Together with the Gagliardo-Nirenberg inequality and (3.38) implies that

$$\begin{aligned}C_{13}\|v^{\frac{p}{2}}\|^2_{L^{\frac{2q_0}{q_0-1}}(\Omega)} &\le C_{14}(\|\nabla v^{\frac{p}{2}}\|^{2\theta}_{L^2(\Omega)}\|v^{\frac{p}{2}}\|^{2(1-\theta)}_{L^{\frac{2q_0}{p}}(\Omega)} + \|v^{\frac{p}{2}}\|^2_{L^{\frac{2q_0}{p}}(\Omega)})\\ &\le C_{15}(\|\nabla v^{\frac{p}{2}}\|^2_{L^2(\Omega)}\| + 1) \quad \forall t\in(0,T_{\max})\end{aligned} \tag{3.46}$$

with some $C_{14} > 0$, $C_{15} > 0$ and

$$\theta = \frac{\frac{Np}{2q_0} - \frac{N(q_0-1)}{2q_0}}{1 + \frac{Np}{2q_0} - \frac{N}{2}} \in (0,1).$$

In summary, there is a $C_{16} > 0$ such that

$$\frac{\xi^2(p-1)}{2}\int_\Omega v^p|\nabla z|^2 \le \frac{p-1}{4}\int_\Omega v^{p-2}|\nabla v|^2 + C_{16} \quad \forall t\in(0,T_{\max}). \tag{3.47}$$

Combining (3.47) with (3.44), for some $C_{17} > 0$ and for all $t \in (0, T_{\max})$, we have

$$\frac{1}{p}\frac{d}{dt}\int_\Omega v^p + \frac{(p-1)}{4}\int_\Omega v^{p-2}|\nabla v|^2 + \frac{\mu_2}{2}\int_\Omega v^{p+r_2-1} \le C_{17}. \tag{3.48}$$

According to a standard ODE comparison argument, it implies

$$\|v(\cdot,t)\|_{L^p(\Omega)} \le C_{18} \quad \forall p > 1 \text{ and } t \in (0,T_{\max}) \tag{3.49}$$

with $C_{18} > 0$. As for u, the only difference is the term av in the first equation of (1.1). Recalling the Young inequality and (3.49), there exist positive constants C_{19} and C_{20} such that

$$\begin{aligned}a\int_\Omega u^p v &\le \frac{\mu_1}{4}\int_\Omega u^{p+r_1-1} + C_{19}\int_\Omega v^{\frac{p+r_1-1}{r_1-1}}\\ &\le \frac{\mu_1}{4}\int_\Omega u^{p+r_1-1} + C_{20} \quad \forall t\in(0,T_{\max}).\end{aligned} \tag{3.50}$$

We repeat (3.44)-(3.48) and Lemma 2.4, for a constant $C_{21} > 0$, we conclude that

$$\|u(\cdot,t)\|_{L^p(\Omega)} \le C_{21} \quad \forall p \ge 1 \text{ and } t \in (0,T_{\max}). \tag{3.51}$$

This completes the proof. □

4. Proof of main result

Now a standard procedure enables us to prove the finial step from $L^1(\Omega)$ to $L^\infty(\Omega)$. Next, we will give the proof of the existence and boundedness of global solutions to system (1.1) by using Neumann heat semigroup theory.

Lemma 4.1. *Let (u,v,w,z) be a classical nonnegative solution of system* (1.1) *in $\Omega\times(0,T_{\max})$. Then we can conclude that there exists $C>0$ such that*

$$\begin{aligned}\sup_{t\in(0,T_{\max})}\Big(&\|u(\cdot,t)\|_{L^\infty(\Omega)}+\|v(\cdot,t)\|_{L^\infty(\Omega)}+\|w(\cdot,t)\|_{W^{1,\infty}(\Omega)}\\&+\|z(\cdot,t)\|_{W^{1,\infty}(\Omega)}\Big)\le C.\end{aligned}$$

Proof. According to the third equation in (1.1) and an associated variation-of-constants formula, we can represent the formula of w

$$\nabla w(\cdot,t)=\nabla e^{t(\Delta-1)}w(\cdot,s_0)+\nabla\int_0^t e^{(t-s)(\Delta-1)}v(\cdot,s). \tag{4.1}$$

Then using Lemma 2.4 and $-\frac12-\frac N2(\frac1{2N}-\frac1\infty)>-1$, we have

$$\begin{aligned}&\|\nabla w(\cdot,t)\|_{L^\infty(\Omega)}\\&\le\|\nabla e^{t(\Delta-1)}w(\cdot,s_0)\|_{L^\infty(\Omega)}+\int_0^t\|\nabla e^{(t-s)(\Delta-1)}v(\cdot,s)\|_{L^\infty(\Omega)}\\&\le C_1+\int_0^t(1+(t-s)^{-\frac12-\frac N2(\frac1{2N}-\frac1\infty)})\|v(\cdot,s)\|_{L^{2N}(\Omega)}\\&\le C_1+\int_0^{+\infty}(1+\sigma^{-\frac12-\frac N2(\frac1{2N}-\frac1\infty)})\|v(\cdot,s)\|_{L^{2N}(\Omega)})\\&\le C_2\quad\forall t\in(0,T_{\max})\end{aligned} \tag{4.2}$$

with some constants $C_1>0$ and $C_2>0$. This implies that

$$\sup_{t\in(0,T_{\max})}\|w(\cdot,t)\|_{W^{1,\infty}(\Omega)}\le C_3 \tag{4.3}$$

with $C_3>0$.

Similarly, there exists a positive constant C_4 such that

$$\sup_{t\in(0,T_{\max})}\|z(\cdot,t)\|_{W^{1,\infty}(\Omega)}\le C_4. \tag{4.4}$$

To derive the L^∞ estimates of u and v, for convenience, we use the corresponding general result from Tao and Winkler [17] rather than the standard Moser-type iteration technique. According to the [17, Lemma A.1], the first equation in the system (1.1) can be rewritten as

$$u_t=\nabla\cdot(D(x,t,u)\nabla u)-\chi\nabla\cdot f(x,t)+g(x,t),\quad x\in\Omega,\ t>0, \tag{4.5}$$

where $D(x,t,u)=1$, $f(x,t)=u\nabla w$ and $g(x,t)=u(\lambda_1-\mu_1u^{r_1-1}+av)$. Combining the L^p estimate of u, (4.3), (4.4) and [17, Lemma A.1], we can obtain the L^∞ estimate of u directly, i.e., there exists a constant $C_5>0$ such that

$$\|u(\cdot,t)\|_{L^\infty(\Omega)}\le C_5. \tag{4.6}$$

We can also obtain the L^∞ estimate of v in the similar way

$$\|v(\cdot,t)\|_{L^\infty(\Omega)}\le C_6 \tag{4.7}$$

with a constant $C_6 > 0$. Thus, the proof is complete. □

5. Proof of Theorem 1.1

On the basis of the extensibility criterion in Lemma 2.1, we can assert $T_{\max} = \infty$ according to the estimates of Lemmas 3.2 and Lemma 4.1. Thus the solution (u, v, w, z) of the model (1.1)-(1.2) is global-in-time and bounded.

Acknowledgments. The authors were partially supported by National Natural Science Foundation of China (No. 12101534) and by the Shandong Provincial Natural Science Foundation (No. ZR2022JQ06 and No. ZR2021QA052).

References

[1] P. Amorim, B. Telch, L. Villada; A reaction-diffusion predator-prey model with pursuit, evasion, and nonlocal sensing, *Math. Biosci. Eng.*, **16** (2019), 5114-5145.
[2] P. Amorim, B. Telch; A chemotaxis predator-prey model with indirect pursuit-evasion dynamics and parabolic signal, *J. Math. Anal. Appl.*, **500** (2021), 125128.
[3] X. Cao; Boundedness in a quasilinear parabolic-parabolic Keller-Segel system with logistic source, *J. Math. Anal. Appl.*, **412** (2014), 181–188.
[4] X. Cao; Large time behavior in the logistic Keller-Segel model via maximal Sobolev regularity, *Discrete Contin. Dyn. Syst. Ser. B*, **9** (2017), 3369–3378.
[5] T. Goudon, L. Urrutia; Analysis of kinetic and macroscopic models of pursuit-evasion dynamics, *Commun. Math. Sci.*, **14** (2016), 2253–2286.
[6] M. Herrero, J. Velázquez; A blow-up mechanism for a chemotaxis model, *Ann. Scuola Norm. Sup. Pisa CI Sci.*, **24** (1997), 663–683.
[7] M. Hieber, J. Prüss; Heat kernels and maximal L^p-L^q estimates for parabolic evolution equations, *Comm. Part. Differ. Equ.*, **22** (1997), 1647–1669.
[8] D. Horstmann, G. Wang; Blow-up in a chemotaxis model without symmetry assumptions, *European J. Appl. Math.*, **12** (2001), 159–177.
[9] E. Keller, L. Segel; Initiation of slime mold aggregation viewed as an instability, *J. Theor. Biol.*, **26** (1970), 399–415.
[10] G. Li, Y. Tao, M. Winkler; Large time behavior in a predator-prey system with indirect pursuit-evasion interaction, *Discrete Contin. Dyn. Syst. Ser. B*, **25** (2020), 4383–4396.
[11] X. Liu, J. Zheng; Convergence rates of solutions in a predator-prey system with indirect pursuit-evasion interaction in domains of arbitrary dimension, *Discrete Contin. Dyn. Syst. Ser. B*, **28** (3) (2023), 2269–2293.
[12] T. Nagai, T. Senba, T. Suzuki; Chemotactic collapse in a parabolic system of mathematical biology, *Hiroshima Math. J.*, **30** (2000), 463–497.
[13] D. Qi, Y. Ke; Large time behavior in a predator-prey system with indirect pursuit-evasion interaction, *Discrete Contin. Dyn. Syst. Ser. B*, **27** (2022), 4531–4549.
[14] H. Qiu, S. Guo; Global existence and stability in a two-species chemotaxis system, *Discrete Contin. Dyn. Syst. Ser. B*, **24** (2019), 1569–1587.
[15] A. Rehman; Boundedness of a two-species chemotaxis system with Lotka-Volterra type competition and two signals, *Int. J. Math.*, **15** (2021), 393–405.
[16] H. Tang, J. Zheng, K. Li; Global and bounded solution to a quasilinear parabolic-elliptic pursuit-evasion system in N-dimensional domain, *J. Math. Anal. Appl.*, **527** (2023), 127406.
[17] Y. Tao, M. Winkler; Boundedness in a quasilinear parabolic-parabolic Keller-Segel system with subcritical sensitivity, *J. Differ. Equ.*, **252** (2012), 692–715.
[18] Y. Tao, Z. Wang; Competing effects of attraction vs. repulsion in chemotaxis, *Math. Models Methods Appl. Sci.*, **23** (2013), 1–36.
[19] L. Wang, C. Mu; A new result for boundedness and stabilization in a two-species chemotaxis system with two chemicals, *Discrete Contin. Dyn. Syst. Ser. B*, **25** (2020), 4585–4601.
[20] M. Winkler; Aggregation vs. global diffusive behavior in the higher-dimensional Keller-Segel model, *J. Differ. Equ.*, **248** (2010), 2889–2905.
[21] M. Winkler; Boundedness in the higher-dimensional parabolic-parabolic chemotaxis system with logistic source, *Comm. Part. Differ. Equ.*, **35** (2010) , 1516–1537.

[22] M. Winkler; Finite-time blow-up in the higher-dimensional parabolic-parabolic Keller-Segel system, *J. Math. Pures Appl.*, **100** (2013), 748–767.
[23] M. Winkler; Global asymptotic stability of constant equilibria in a fully parabolic chemotaxis system with strong logistic dampening, *J. Differ. Equ.*, **257** (2014), 1056–1077.
[24] J. Xie, J. Zheng; A new result on existence of global bounded classical solution to a attraction-repulsion chemotaxis system with logistic source, *J. Differ. Equ.*, **298** (2021), 159–181.
[25] J. Zheng; Boundedness of solutions to a quasilinear parabolic-elliptic Keller-Segel system with logistic source, *J. Differ. Equ.*, **259** (2015), 120–140.
[26] J. Zheng, Y. Li, G. Bao, X. Zou; A new result for global existence and boundedness of solutions to a parabolic-parabolic Keller-Segel system with logistic source, *J. Math. Anal. Appl.*, **462** (2018), 1–25.
[27] J. Zheng, P. Zhang; Blow-up prevention by logistic source an N-dimensional parabolic-elliptic predator-prey system with indirect pursuit-evasion interaction, *J. Math. Anal. Appl.*, **519** (2023), 126741.
[28] J. Zheng, X. Liu, P. Zhang; Existence and boundedness of solutions for a parabolic-elliptic predator-paey chemotaxis system, *Discrete Contin. Dyn. Syst. Ser. B*, **28** (2023), 5437–5446.

Fengxiang Zhao
School of Mathematical and Informational Sciences, Yantai University, Yantai, 264005, Shandong, China
Email address: zfx2037@163.com

Haotian Tang
Department of Mathematics, Faculty of Science and Technology, University of Macau, Taipa, Macau, China
Email address: haotiantang2022@163.com

Jiashan Zheng
School of Mathematical and Informational Sciences, Yantai University, Yantai, 264005, Shandong, China
Email address: zhengjiashan2008@163.com

Kaiqiang Li (corresponding author)
School of Mathematical and Informational Sciences, Yantai University, Yantai, 264005, Shandong, China
Email address: kaiqiangli19@163.com